THE
PRINCIPLES
OF
MECHANICS.

EXPLAINING and DEMONSTRATING
The general LAWS of MOTION,

The LAWS of GRAVITY,
MOTION of DESCENDING BODIES,
PROJECTILES,
MECHANIC POWERS,
PENDULUMS,
CENTERS of GRAVITY, &c.
STRENGTH and STRESS of TIMBER,
HYDROSTATICS, and
CONSTRUCTION of MACHINES.

A WORK very neceſſary to be known, by all Gentlemen, and Others, that deſire to have an Inſight into the Works of NATURE and ART. And extremely uſeful to all Sorts of ARTIFICERS; particularly to ARCHITECTS, ENGINEERS, SHIPWRIGHTS, MILLWRIGHTS, WATCHMAKERS, &c. or any that Work in a Mechanical Way.

The THIRD EDITION, CORRECTED.

Illuſtrated with FORTY-THREE COPPER-PLATES.

Multáque per Trochleas, et Tympana pondere magno
Commovet, atque levi ſuſtollit Machina niſu.

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MDCCLXXIII.

THE

PREFACE.

THE *art of* MECHANICS *being the firſt that men had occaſion to make uſe of, it is reaſonable to ſuppoſe that it took its beginning with man; and was ſtudied in the earlieſt ages of the world. For no ſooner did mankind begin to people the earth, than they wanted houſes to dwell in, cloaths to wear, and utenſils to till the ground, to get them bread, with other neceſſaries of life; and being thus deſtitute of proper habitations, and other conveniences of living, their wants muſt immediately put them upon the ſtudy of mechanics. At their firſt ſetting out, they would be content with very little theory; endeavouring to get that more by experience than reaſoning, and being unacquainted with numbers, or any ſort of calculation; and having neither rule nor compaſs to work by, nor inſtruments to work with, but ſuch as they muſt invent firſt of all, nor any methods of working: with all theſe diſadvantages, we may judge what ſort of work they were likely to make. All their contrivances muſt be mere gueſſing, and they could but ill execute what they had ſo badly contrived; and muſt be continually mending their work by repeated trials, till they got it to ſuch a form as to make a ſhift to ſerve for the uſe deſigned. And this is the firſt and loweſt ſtate of mechanics, which was enough to give a beginning to it; and in this ſtate it doubtleſs remained for a long time, without much improvement. But at length as men found more leiſure and opportunity, and gained more experience, manual arts began to take their riſe, and by degrees to make ſome progreſs in the world.*

But we meet with no conſiderable inventions in the mechanical way, for a long ſeries of ages; or if there had been any, the accounts of them are now loſt, through the length of time; for we have nothing upon record for two or three thouſand years forward. But afterwards we find an account of ſeveral machines that were in uſe. For we read in Geneſis *that ſhips were as old, even on the Mediterranean, as the days of* Jacob. *We likewiſe read that the* Philiſtines *brought* 30 *thouſand chariots into the field againſt* Saul; *ſo that chariots were in uſe* 1070 *years before* Chriſt. *And about the ſame time architecture was brought into* Europe. *And*

1030 *years before* Chriſt, Ammon *built long and tall ſhips with ſails, on the Red Sea and the Mediterranean. And about* 90 *years after, the ſhip* Argo *was built; which was the firſt* Greek *veſſel that ventured to paſs through the ſea, by help of ſails, without ſight of land, being guided only by the ſtars.* Dædalus *alſo, who lived* 980 *years before* Chriſt, *made ſails for ſhips, and invented ſeveral ſorts of tools, for carpenters and joiners to work with. He alſo made ſeveral moving ſtatues, which could walk or run of themſelves. And about* 800 *years before* Chriſt, *we find in* 2 Chron. xv. *that* Uzziah *made in* Jeruſalem, *engines invented by cunning men, to be on the towers and upon the bulwarks, to ſhoot arrows and great ſtones withal. Corn mills were early invented; for we read in* Deuteronomy, *that it was not lawful for any man to take the nether or the upper mill ſtone to pledge; yet water was not applied to mills before the year of* Chriſt 600, *nor wind mills uſed before the year* 1200. *Likewiſe* 580 *years before* Chriſt, *we read in* Jeremiah xviii. *of the potter's wheel.* Architas *was the firſt that applied mathematics to mechanics, but left no mechanical writings behind him: he made a wooden pigeon that could fly about.* Archimedes, *who lived about* 200 *years before* Chriſt, *was a moſt ſubtle geometer and mechanic. He made engines that drew up the ſhips of* Marcellus *at the ſiege of* Syracuſe; *and others that would caſt a ſtone of a prodigious weight to a great diſtance, or elſe ſeveral leſſer ſtones, as alſo darts and arrows; but there have been many fabulous reports concerning theſe engines. He alſo made a ſphere which ſhowed the motions of the ſun, moon, and planets. And* Poſidonius *afterwards made another which ſhewed the ſame thing. In theſe days the liberal arts flouriſhed, and learning met with proper encouragement; but afterwards they became neglected for a long time.* Ariſtotle, *who lived about* 290 *years before* Chriſt, *was one of the firſt that writ any methodical diſcourſe of mechanics. But at this time the art was contained in a very little compaſs, there being ſcarce any thing more known about it, than the* 6 *mechanical powers. In this ſtate it continued till the* 16*th century, and then clock work was invented, and about* 1650 *were the firſt clocks made. At this time ſeveral of the moſt eminent mathematicians began to conſider mechanics. And by their ſtudy and induſtry have prodigiouſly enlarged its bounds, and made it a moſt comprehenſive ſcience. It extends through heaven and earth, the whole univerſe, and every part of it is its ſubject. Not one particle of matter but what comes under its laws. For what elſe is there in the viſible world, but matter and motion; and the properties and affections of both theſe, are the ſubject of mechanics.*

To the art of mechanics is owing all ſorts of inſtruments to work with, all engines of war, ſhips, bridges, mills, curious roofs and arches, ſtately theatres, columns, pendent galleries, and all other grand works in building. Alſo clocks, watches, jacks, chariots, carts and carriages, and even the wheel barrow. Architecture, navigation, huſbandry, and military affairs, owe their invention and uſe to this art. And whatever hath artificial motion by air, water, wind, or cords; as all manner of muſical inſtruments, water-works, &c. This is a ſcience of ſuch importance, that without it we could hardly eat our bread, or lie dry in our beds.

By mechanics we come to underſtand the motions of the parts of an animal body; the uſe of the nerves, muſcles, bones, joints, and veſſels. All which have been made ſo plain, as proves an animal body to be nothing but a mechanical engine. But this part of mechanics, called anatomy, is a ſubject of itſelf. Upon mechanics are alſo founded the motions of all the celeſtial bodies, their periods, times, and revolutions. Without mechanics, a general cannot go to war, nor beſiege a town, or fortify a place. And the meaneſt artificer muſt work mechanically, or not work at all. So that all perſons whatever are indebted to this art, from the king down to the cobler.

Upon mechanics is alſo founded the Newtonian, *or only true philoſophy in the world. For all the difficulty of philoſophy conſiſts in this; from ſome of the principal phænomena of motions to inveſtigate the forces of nature. And then from theſe forces to demonſtrate the other phænomena; all which is to be done upon mechanical principles. Thus from the diſtances and revolutions of the heavenly bodies, the forces of gravity are derived; and from theſe forces thus known, are deduced the motions of the planets, comets, the moon, and the ſea; as well as the motions of bodies upon the ſurface of the earth. Theſe relate to the viſible bodies of the univerſe. But there are alſo certain forces belonging to the ſmall particles of matter, which we are ſtill ignorant of; by which they are either impelled towards one another, and cohere in regular figures; or are repelled, and ſo recede from each other. For the particles of different ſorts of bodies have different laws; ſince the ſmall particles of ſome bodies attract one another, whilſt thoſe of other ſorts repel each other; and that by forces almoſt infinitely various. Upon theſe forces the coheſion, ſolidity, and fluidity of bodies depend. The nature of elaſticity, electricity, and magnetiſm. Upon theſe alſo depend the principles of fermentation, putrefaction, generation, vegetation, and diſſolution of bodies; digeſtion and ſecretion in animal bodies; the motion of the blood and fluids in animals,*

mals, and the moving of the members by the command of the will; the exciting ſenſations in the mind, the emiſſion, reflection refraction, and inflection of light; freezing by cold burning by fire; all operations in chemiſtry, &c. If these forces could be found out, it would open to us a new field in the ſcience of mechanics. But for want of proper experiments, theſe forces among the inviſible and imperceptible particles of matter, are utterly unknown, and exceeding difficult to be diſcovered, and therefore make no part of the enſuing Treatiſe. Nor ſhall I meddle with aſtronomy, as being a ſubject of itſelf: nor with experimental philoſophy, any further than concerns mechanics.

And although architecture has a great dependance upon mechanics; yet there are a great many precarious rules in this art, invented purely for ornament, and the ſake of beauty; which have nothing to do with mechanics. And therefore mechanical beauty *(that is, ſtrength in due proportion) is all that I have any buſineſs to meddle with here.*

It has been ignorantly objected by ſome, that the Newtonian *philoſophy, like all others before it, will grow old and out of date, and be ſucceeded by ſome new ſyſtem, which will then be as much cried up as this is now. But this objection is very falſly made. For never a philoſopher before* Newton *ever took the method that he did. For whilſt their ſyſtems are nothing but hypotheſes, conceits, fictions, conjectures, and romances, invented at pleaſure, and without any foundation in the nature of things. He on the contrary, and by himſelf alone, ſet out upon a quite different footing. For he admits nothing but what he gains from experiments, and accurate obſervations. And from this foundation, whatever is further advanced, is deduced by ſtrick mathematical reaſoning. And where this thread does not carry him, he ſtops, and proceeds no further; not pretending to be wiſe above what is written in nature. Being rather content with a little true knowledge, than, by aſſuming to know every thing, run the hazard of error. Contrary to all this, theſe ſcheming philoſophers, being men of ſtrong imaginations and weak judgments, will run on ad infinitum, and build one fiction upon another, till their* Babel *thus erected, proves to be nothing but a heap of endleſs confuſion and contradiction. And then it is no wonder, if the whole airy fabric tumbles down, and ſinks into ruin. And yet it ſeems, ſuch romantic ſyſtems of philoſophy, will pleaſe ſome people as well as the ſtricteſt truth, or moſt regular ſyſtem. As if philoſophy, like religion, was to depend on the faſhion of the country, or on the fancies and caprice of weak people. But ſurely this is nothing but rambling in the dark, and ſaying that the nature of things depends upon no ſteady principles at all. But* *in truth, the buſineſs of true philoſophy is to derive the nature of things from cauſes truly exiſtent; and to enquire after thoſe laws on which the Creator chooſed to found the world; not thoſe by which he might have done the ſame, had he ſo pleaſed. It is reaſonable to ſuppoſe that from ſeveral cauſes, ſomething differing from each other, the ſame effect may ariſe. But the true cauſe will always be that from which it truly and actually does ariſe; the others have no place in true philoſophy. And this can be known no way, but by obſervations and experiments. Hence it evidently follows, that the* Newtonian *philoſophy, being thus built upon this ſolid foundation, muſt ſtand firm and unſhaken; and being once proved to be true, it muſt eternally remain true, until the utter ſubverſion of all the laws of nature. It is therefore a mere joke to talk of a new philoſophy. The foundation is now firmly laid: the* Newtonian *philoſophy may indeed be improved, and further advanced; but it can never be overthrown: notwithſtanding the efforts of all the* Bernouilli'*s, the* Leibnitz'*s, the* Green'*s, the* Berkley'*s, the* Hutchinſon'*s, &c. And even the* French *themſelves have at laſt adopted it, and given up the* Carteſian *ſcheme.*

Practical mechanics might be very much improved, if the ſecrets of all trades were to lie open; and the ſeveral machines uſed in each trade, duly explained. And experimental philoſophy would be thereby much improved, as well as the trades themſelves. And one trade might borrow many great helps in working, from another trade. It is not my deſign to treat at all on the loweſt part of mechanics, which concerns manual arts or working by hand. For there is no theory required here, but only a habit of working, to be acquired by frequent practice.

I have, in the following book, confined myſelf entirely to the mathematical part, and what depends on it, and is deduced from it. And therefore I have firſt of all laid down and demonſtrated the general laws of motion, as a foundation for all the reſt. Then follows the laws of gravity, the deſcent of bodies, and motion of projectiles in free ſpace; the mechanical powers; the deſcent of bodies upon inclined planes, the vibration of pendulums; center of gravity, and others; the preſſure, ſtrength, and ſtreſs, of beams of timber; then you have the principles of hydroſtatics, hydraulics, and pneumatics; the reſiſtance of fluids; the powers of engines, and the deſcription of machines. In each of theſe, I have delivered all the fundamental principles both in theory and practice. And to make it more univerſally uſeful, I have demonſtrated every thing geometrically; or at moſt, by the help only of the loweſt and eaſieſt rules of algebra, for the ſake of brevity; avoiding all operations by fluxions: ſo that the reader need not be ſcared with the thoughts of any difficulty of that kind.

Concerning the machines, I have given an account of their ſtructure, as far as is neceſſary to explain their motions and effects; omitting the deſcription of their minuter parts, not ſo neceſſary for this end, and which are eaſily underſtood of themſelves. I hope the reader will pardon my inſerting among the reſt, ſome machines that may ſeem trifling, put in here and there to fill up vacant places in the ſchemes. Yet even in theſe, there is ſomething curious in their ſtructure or motion, that may be worth obſerving. I might have given the cuts of many more machines; but perhaps what I have already done, may be thought too much, in ſuch a nation as this, where natural knowledge wants due encouragement, and where no Mæcenas *appears to patronize and protect it; and where arts and ſciences hang, as it were, in ſuſpence, whether they ſhall ſtand or fall; and where public ſpirit and* Engliſh *generoſity are juſt expiring. This decline of arts and ſciences, is wholly, or in a great meaſure owing to the ambition, and moſt extreme avarice of the preſent age. Where men, not being able to lift their eyes above this earth, think nothing worth their care, but raking together the droſs it affords; ſtriving. like the toad, who ſhall die with the moſt earth in his paws. The duller part of mankind are entirely engaged in the purſuit of filthy lucre; and the brighter ſort are wholly devoted to low, trifling, and often barbarous diverſions. In ſuch momentous concerns as theſe, it is no wonder if arts and ſciences flag; and natural knowledge meet with nothing but contempt; and* Minerva *give place to* Plutus. *And indeed, if the general temper and diſpoſition of men had been the ſame in all ages, as it is in this; I am in doubt whether we had ever had any ſuch thing as a mill to grind us corn for bread, or a pump to draw us water. It is a trifling excuſe for men of exalted ſtation, to urge, that they are unacquainted with ſuch arts or ſciences. For learning has always been eſteemed to be under the peculiar care and ſuperintendency of the* great; *who ought to protect and encourage both that, and the profeſſors of it; or elſe arts and ſciences can never flouriſh. And as the encouragement of theſe evidently tends to the benefit of mankind, and the promoting of the public good; nothing can excuſe ſo groſs a neglect, or ſuch a manifeſt diſregard, as they ſhew, for the happineſs of their fellow creatures. The induſtrious ſtudents only have the fatigue; whilſt all the world reap the advantage of their labours:*

Scire volunt omnes, mercædem ſolvere nemo. JUVEN.

It is reported of Alexander *that he allowed* 800 *talents a year to* Ariſtotle, *to defray the expences of procuring all ſorts of living creatures; ſo that by his own particular experience, he might be* *enabled to write of the nature and properties of them. And the reaſon why the world hath now ſo few* Ariſtotles, *is becauſe there are no* Alexanders.

But as to the fate of this book, it is indifferent to me what reception it ſhall meet with in the world. Not that I am in the leaſt diffident of the principles here delivered; for I know they will ſtand the ſtricteſt examination. Nor would I be thought careleſs concerning the advantage, which my few readers may receive from it: for on the contrary, I have done all I could to inſtruct them, and lead them regularly through this noble and uſeful ſcience. But in a mercenary age, where there is ſo little encouragement for works of this nature, I am under no concern what judgment may be paſſed upon it by the ignorant multitude. Yet I ſincerely wiſh, that my more ingenious readers may find what they expect here; and am in hopes, that they will meet with no difficulties, but what they will eaſily ſurmount. To effect which, I have made every thing in this book as full and clear as my own abilities, and the nature of the thing would permit me.

W. EMERSON.

P. S. *The kind reception the former editions of this book has met with from the publick, has induced me to reviſe and correct it; and to make ſuch further additions thereto, as I thought neceſſary for completing ſo uſeful a ſcience. Accordingly I have made ſuch alterations and improvements in this third edition, as to me ſeemed abſolutely neceſſary for the benefit of my readers.*

I cannot find that I have omitted any thing material in this Treatiſe pertinent to my ſubject. And as my profeſſed deſign was to write a book of Principles, *no body of common ſenſe would expect that I ſhould go contrary to that deſign, and ſtuff my book with calculations of difficult problems, fit only for books of Algebra and Fluxions; and not at all proper for an* elementary Treatiſe. *If I had done this, I had not only deſerted my ſubject, and all good method; but alſo impoſed upon my readers, by ſwelling the book, and making them pay dear for* ſuperfluities; *and that to pleaſe a few trifling critics, who know no better. Theſe ſort of animals live by detraction, and are always* ſnarling *at what they don't underſtand, and cannot mend.*

If any ſuch ſet of critics, *aſſuming the privilege of being* dictators *to the public, and* cenſors *of other men's writings, ſhould ſtart up, and cry down my book, becauſe not written after their crude notions, I would not have my readers be at all ſurprized at* *this : for it would be more ſurprizing if any thing of value ſhould eſcape, without being degraded and condemned by them. And for my own part, the only concern it will give me, will be to deſpiſe their dull criticiſms, and laugh at their ignorance.*

> Men' moveat cimex Pantilius? aut crucier, quòd
> Vellicit abſentem Demetrius? aut quòd ineptus
> Fannus Hermogenis lœdat Conviva Tigelli?
> Plotius, & Varius, Mæcenas, Virgiliuſque,
> Valgius, —— probet hæc ————
>
> HOR.

W. E.

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EXPLANATION of the CHARACTERS.

$\propto$ as, or in a given proportion to.
Perp. perpendicular.
Rad. radius.
S. the ſine.
Coſ. the coſine.
Tan. the tangent.
Cotan. the cotangent.
Sec. the ſecant.

MECHANICS.

DEFINITIONS.

1. MECHANICS is a ſcience, which teaches the proportion of the forces, motions, velocities, and in general the actions of bodies upon one another.

2. *Body* is the maſs or quantity of matter. If a body yields to a ſtroke and recovers its former figure again, it is called an *elaſtic body:* If not, it is *inelaſtic*.

3. *Denſity* of a body is the proportion, of the quantity of matter contained in it, to the quantity of matter in another body of the ſame bigneſs. Thus the denſity is ſaid to be double or triple, when the quantity of matter contained in the ſame ſpace is double or triple.

4. *Force* is a power exerted on a body to move it. If it act but for a moment it is called the force of *percuſſion* or *impulſe*. If it act conſtantly, it is called an *accelerative force:* If conſtantly and equally, it is called a *uniform accelerative force*.

5. *Velocity* is an affection of motion, by which a body paſſes over a certain ſpace in a given time. The velocity is ſaid to be greater or leſs, according as the body paſſes over a greater or leſs ſpace in the ſame time.

6. *Motion* is a continual and ſucceſſive change of place. If a body moves through equal ſpaces in equal times, it is called *equable motion*. If its velocity continually increaſes, it is called *accelerated motion*; if it decreaſes, it is *retarded motion*. If it increaſes or decreaſes uniformly, it is *equably accelerated* or *retarded*. Likewiſe if its motion be conſidered in regard to ſome other body at reſt, it is called *abſolute motion*. But if its motion be conſidered with reſpect to other bodies alſo in motion, then it is *relative motion*.

7. *Direction of motion* is the way the body tends, or the right line it moves in.

8. *Quantity of motion*, is the motion a body has, both in regard to its velocity, and quantity of matter. This is called the *momentum* of a body, by ſome mechanical writers.

9. *Vis inertiæ*, is that innate force of matter by which it reſiſts any change, and endeavours to preſerve its preſent ſtate of motion or reſt.

10. *Gravity* is that force wherewith a body endeavours to deſcend towards the center of the earth. This is called *abſolute gravity*, when the body tends downwards in free ſpace: and *relative gravity* is the force it endeavours to deſcend with in a fluid.

11. *Specific gravity* of bodies, is the greater or leſs weight of bodies of the ſame magnitude; or the proportion between theſe weights. The ſpecific gravity is ſaid to be double or triple, when the weight of the ſame bulk of matter is double or triple.

13. *Center of gravity* of a body is a certain point in it, upon which the body being freely ſuſpended, it would reſt in any poſition.

13. *Center of motion* of a body, is a fixed point about which the body is moved. And the *axis of motion*, is the fixed axis it moves about.

14. *Weight and power* when oppoſed to one another, ſignify the body to be removed, and the body that moves it. That body which communicates the motion is called the *power*; and that which receives it, the *weight*.

15. *Equilibrium* is when two or more forces acting againſt one another, none of them overcome the others, but deſtroy one another's effects, and remain at reſt.

16. *A fluid* is a body whoſe parts yield to any impreſſed force; and by yielding are eaſily moved among themſelves.

17. *Hydroſtatics* is a ſcience that treats of the properties of fluids.

18 *Hydraulics* is the art of raiſing or conveying water by the help of engines.

19. *Pneumatics* is a ſcience that treats of the properties of the air.

20. *Machine* is any mechanical inſtrument contrived to move bodies, or to perform ſome particular motions. The mechanical powers are ſimple machines.

21. *Engine* is a mechanic inſtrument compoſed of levers, wheels, pullies, ſcrews, &c. in order to move, lift, or ſuſtain ſome great weight, or perform ſome great effect. This is the largeſt and moſt compounded ſort of machines.

22. *Mechanic powers*, are the ballance, the lever, the wheel, the pulley, the ſcrew, and the wedge. To which ſome add the inclined plane.

23. *Streſs* is the effect of a force acting againſt a beam, or any thing to break it, or the violence it ſuffers by that force. The contrary to this is *ſtrength*, which is the reſiſtance any beam is able to make againſt a force endeavouring to break it.

24. *Friction* is the reſiſtance ariſing from the parts of machines, or of any bodies rubbing againſt one another.

POSTULATA.

1. THAT a ſmall part of the ſurface of the earth, or the horizon, may be looked upon as a plane. Though this is not ſtrictly true, yet it differs inſenſibly in ſo ſmall a ſpace as we have any occaſion to conſider it.

2. Heavy bodies deſcend in lines parallel to one another, and perpendicular to the horizon: And they always tend perpendicular to the horizon by their weight. For this is true as to ſenſe, becauſe the lines of their direction meet only at the center of the earth, taken as a perfect ſphere.

3. The weight of any body is the ſame in all places at or near the ſurface of the earth. For the difference is inſenſible at any heights to which we can aſcend. Though in ſtrictneſs the force of gravity decreaſes in aſcending, from the earth's ſurface, in the reciprocal ratio of theſquares of the heights from the earth's center.

4. We are to ſuppoſe all planes perfectly even and regular, all bodies perfectly ſmooth and homogeneous; and moving without friction or reſiſtance; lines perfectly ſtreight, and inflexible, without weight or thickneſs; cords extremely pliable; &c. For tho' bodies are defective in all theſe; and the parts or matter, whereof engines are made, ſubject to many imperfections; yet me muſt ſet aſide all theſe irregularities, till the theory is eſtabliſhed; and afterwards make ſuch allowance as is proper.

AXIOMS.

1. EVERY body preſerves in its preſent ſtate, whether of reſt, or moving uniformly in a right line; till it is compelled to change that ſtate, by ſome external force.

2. The alteration of motion, or the motion generated, or deſtroyed in any body, is proportional to the force applied: And is made in the direction of that right line in which the force acts.

3. The action and re-action between two bodies are equal, and in contrary directions.

4. The motion of the whole body is made up of the ſum of the motions of all the parts.

5. The weights of all bodies in the ſame place, are proportional to the quantities of matter they contain; without any regard to their bulk, figure, or kind. For twice the matter will be twice as heavy, and thrice the matter thrice as heavy; and ſo on.

6. The vis inertiæ of all bodies, is proportional to the quantity of matter.

7. Every body will deſcend to the loweſt place it can get to.

8. Whatever ſuſtains a heavy body, bears all the weight of it.

9. Two equal forces acting againſt one another in contrary directions deſtroy one another's effects.

10. If a body is acted on with two forces in contrary directions; it is the ſame thing as if it were only acted on with the difference of theſe forces, in direction of the greater.

11. If a body is kept in equilibrio; the contrary forces, in any one line of direction, are equal, and deſtroy one another.

12. Whatever quantity of motion any force generates in a given time; the ſame quantity of motion will an equal force deſtroy in the ſame time; acting in a contrary direction.

13. Any active force will ſooner or more eaſily overcome a leſſer reſiſtance than a greater.

14. If a weight be drawn or puſhed by any power; it puſhes or draws all points of the line of direction equally. And it is the ſame thing, whatever point of that line the force is applied to.

15. If two bodies be moving the ſame way in any right line; their relative motion will be the ſame, as if one body ſtood ſtill, and the other approached, or receded from it with the difference of their motions: or with the ſum of their motions, if they move contrary ways.

16. If a body is drawn or urged by a rope, the direction of that force is the ſame as the direction of that part of the rope next adjoining to the body.

17. If any force is applied to move or ſuſtain a body, by means of a rope; all the intermediate parts of the rope are equally diſtended, and that in contrary directions.

18. If a running rope go freely over ſeveral pullies; all the parts of it are equally ſtretched.

19. If any forces be applied againſt one end of a free lever or beam; the other end will thruſt or act with a force, in direction of its length.

20. The parts of a fluid will yield, and recede towards that part where it is leaſt preſſed.

21. The upper part of a fluid is ſuſtained by the lower part.

SECT.

SECT. I.

The general laws of MOTION.

PROP. I.

The quantities of matter in all bodies are in the complicate ratio of their magnitudes and denſities.

FOR by Def. 3. if the magnitudes be equal, the matter will be as the denſities. And if the denſities be equal, the matter will be as the magnitudes. Therefore the matter is univerſally in the compound ratio of both.

Cor. 1. *The quantities of matter in ſimilar bodies, are as the denſities and cubes of their like dimenſions, in a ſphere.* The magnitude is as the cube of the diameter.

Cor. 2. *The quantities of matter are as the magnitudes and ſpecific gravities.* For the ſpecific gravities are as the denſities, by Ax. 5.

PROP. II.

The quantities of motion, in all moving bodies whatever, are in the complicate ratio of the quantities of matter and the velocities.

For if the velocities are equal, it it manifeſt (by Ax. 4.) that the quantities of motion will be as the quantities of matter. And if the quantities of matter are equal, the motions will be as the velocities. Therefore univerſally, the quantities of motion are in the compound ratio of the velocities and quantities of matter.

Cor. In any ſort of motion, the quantity of motion is as the ſum of all the products of every particle of matter multiplied by its reſpective velocity.

For the quantity of motion of any particle is as that particle multiplied by its velocity: And as each particle of the ſame body moves with equal velocity, the motion of the whole will be the ſum of the motions of all the parts.

PROP. III.

In all uniform motions, the ſpace deſcribed is in the complicate ratio of the time and velocity.

For it is evident, if the velocity be given, the ſpace deſcribed by any body, will be as the time of its moving. And if the time be given, the ſpace deſcribed will be greater or leſs, according as the velocity is greater or leſs; that is, the ſpace will be as the velocity. Therefore if neither be given, the ſpace will be in the compound ratio of both the time and velocity.

Cor. The time is as the ſpace directly and velocity reciprocally.

PROP. IV.

The motion generated by any momentary force, is as the force that generates it.

For if a certain quantity of force generates any motion, a double quantity of force, will generate double the motion; and a triple force, triple the motion, and ſo on.

Cor. The ſpace deſcribed is as the force and time directly, and quantity of matter reciprocally.

For by this Prop. the force is as the motion, that is (by Prop. II.) as the matter and velocity; therefore the velocity is as the force directly and matter reciprocally.. Alſo (by Prop. III.) the ſpace is as the time and velocity, and therefore as the time and force directly, and matter reciprocally.

SCHOLIUM.

Let b = *body or quantity of matter to be moved.*
f = *force of impulſe acting on the body b.*
m = *momentum or quantity of motion generated in b.*
v = *velocity generated in b.*
s = *ſpace deſcribed by the body b.*
t = *time of deſcribing the ſpace s with the velocity v.*

Then by the three laſt Props. we ſhall have $m \propto bv$, $s \propto tv$, and $f \propto m$. By the help of theſe three general proportions, the relation of the ſpaces, times, velocities, *&c.* may be found, upon all ſuppoſitions. And thence all the laws and proportions belonging to uniform motion, may be readily and univerſally reſolved: Expunging ſuch as are not concerned in the queſtion; and rejecting all thoſe that are given or conſtant; and alſo thoſe that are in both terms of the proportion. Thus we ſhall get in general,

$$\begin{array}{l} f \propto m \propto bv \propto \frac{bs}{t}. \\ m \propto f \propto bv \propto \frac{bs}{t}. \\ s \propto tv \propto \frac{tm}{b} \propto \frac{tf}{b}. \\ v \propto \frac{m}{b} \propto \frac{s}{t} \propto \frac{f}{b}. \\ t \propto \frac{s}{v} \propto \frac{sb}{m} \propto \frac{sb}{f}. \\ b \propto \frac{m}{v} \propto \frac{f}{v} \propto \frac{mt}{s} \propto \frac{ft}{s}. \end{array}$$

PROP. V.

In any motion, generated by a uniformly accelerating force: the motion generated in any time, is in the complicate ratio of the force and time.

For in any given time, the motion generated will be proportional to the force that generates it, this being its natural and genuine effect. And ſince in all the ſeveral parts of time, the force is the ſame, and has the ſame efficacy; therefore the motion generated will alſo be as the time: whence univerſally, the motion generated is in the compound ratio of both the force and the time of acting.

Cor. 1. *This Prop. is equally true in reſpect to the motion loſt or deſtroyed in a moving body, by a force acting in a contrary direction.* By Ax. 12.

Cor. 2. *If the ſpace thro' which a body is moved by any force, be divided into an indefinite number of ſmall equal parts; and if, in each part, the accelerative force acts differently upon the body, according to any certain law. And if there be taken the product of the accelerating force in each part multiplied by the time of paſſing through it. Then I ſay,*

As any uniform accelerative force × by the time of acting (FT):
To the motion generated in that time (M) ::
So the ſum of all the products of each particular force and time:
To the motion generated in this whole time, by the variable force.

For (by this Prop.) the time of deſcribing any part × by the force: motion generated in that time :: $F \times T : M$. therefore by compoſition, the ſum of all the products, to the whole motion generated; is alſo as FT, to M.

Cor. 3. *The velocity generated (or deſtroyed) in any time, is as the force and time directly, and the quantity of matter reciprocally.*

For (by Prop. II.) the quantity of motion is as the matter and velocity; therefore the velocity is as the motion directly and the matter reciprocally, that is, (by this Prop.) as the force and time directly, and the matter reciprocally.

Cor. 4. *The increaſe or decreaſe of any velocity, generated or deſtroyed in any time; is as the force and time directly, and the matter reciprocally.*

This follows from Cor. 3. becauſe all effects are proportional to their cauſes.

PROP. VI.

In any motion generated by a uniform accelerative force; the ſpace deſcribed from the beginning of the motion, is in the complicate ratio of the laſt velocity, and the time wherein it is generated.

For ſuppoſe the time divided into an infinite number of equal parts, each of which call 1, and let the times from the beginning be 0, 1, 2, 2, 4 .... to t. And let c be the velocity generated at the end of the time 1; then by Cor. 3. of the laſt Prop. the velocity generated in the ſame body, by the ſame force, in the times 0, 1, 2, 3, 4 .... t, will be reſpectively 0, c, $2c$, $3c$, $4c$ ... to tc. And by Prop III. the ſpaces deſcribed in each given part of time, will be as the time (1), and the velocity;

and therefore theſe ſpaces will be 0, c, $2c$, $3c$ . . . tc; and the ſum of all theſe is the whole ſpace deſcribed by the body. But the ſum of the arithmetic progreſſion, 0, c, $2c$, $3c$, $4c$ . . . tc where the number of terms is very great $\frac{0+tc}{2}\times t$, or $\frac{tc}{2}\times t$. but tc is the laſt velocity, and t the whole time, therefore the whole ſpace is as ($tc\times t$) the velocity and time conjunctly.

Cor. 1. *If a body moves uniformly forward with the velocity acquired by a uniform accelerative force; it will deſcribe twice the ſpace in the ſame or an equal time, that it deſcribed by the accelerative force.*

For here the ſpaces deſcribed in the ſeveral ſmall parts of time, will be tc, tc, tc . . &c. to t terms. whoſe ſum is $tc\times t$. And the ſum of the former ſpaces, deſcribed by the accelerating force, was $\frac{tc}{2}\times t$. but $tc\times t$ is to $\frac{tc}{2}\times t$, as 2 to 1.

Cor. 2. *Since any active force is an endeavour of putting a body into motion; therefore the adequate and immediate effect of an accelerating force is either motion, or preſſure, or both.*

SCHOL.

Let b = *body or quantity of matter.*
F = *accelerative force acting uniformly and equally on the body b.*
v = *velocity generated in b by the force F.*
m = *motion generated in b.*
s = *ſpace deſcribed by b.*
t = *time of deſcribing the ſpace s.*

Then the two laſt propoſitions together with Prop. II. will reſolve all queſtions relating to the times, forces, velocities, *&c.* in uniformly accelerated motions. Thus $m \propto bv$, $m \propto Ft$, and $s \propto tv$. whence we have in general,

$$b \propto \frac{m}{v}, \text{&c.}$$

$$m \propto bv \propto Ft, \text{\&c.}$$

$$v \propto \frac{s}{t} \propto \frac{Ft}{b} \propto \sqrt{\frac{Fs}{b}} \propto \frac{m}{b},$$

$$s \propto tv \propto \frac{bvv}{F} \propto \frac{Ftt}{b},$$

t ∝

$$t \propto \frac{s}{v} \propto \frac{bv}{F} \propto \sqrt{\frac{bs}{F}} \propto \frac{m}{F},$$

$$F \propto \frac{bv}{t} \propto \frac{bs}{tt} \propto \frac{bvv}{s} \propto \frac{m}{t}, \text{ \&c.}$$

Whence, if m, b, &c. be given; ſuch quantities muſt be left out.

PROP. VII.

If any force acting on a body at A in direction AB, cauſe the body to paſs through the ſpace AB in any time; and another ſimilar force acting in direction AC, would move it through the ſpace AC in the ſame time: I ſay, by both forces acting together, in their proper directions, the body will, in the ſame time, be moved through the ſpace AD, the diagonal of the parallelogram ABDC. 1.

CASE I.

Let the forces at *A* cauſe the body to move uniformly along the lines *AB*, *AC*. Then ſince the force acting in direction *AC* parallel to *BD*, by Ax. 2, will not alter the velocity towards the line *BD*; the body therefore will arrive at *BD* in the ſame time, whether the force in direction *AC* be impreſſed or not: Therefore, at the end of the time, it will be found ſomewhere in *BD*. By the ſame argument, it will be found ſomewhere in the line *CD*; therefore it will be found in *D* the point of interſection; and by Ax. 1, it will move in a right line from *A* to *D*.

Otherwiſe.

Suppoſe the line *AC* to move parallel to itſelf into the place *BD*, whilſt *A* moves from *A* to *C*. Then ſince this line and the body are both equally moved towards *BD*, it is plain the body muſt be always in the moveable line *AC*. Therefore when *AC* comes to the poſition *bg*, let the body be arrived at *d*; then, ſince both the line *AC* moves uniformly along *AB*, and the body *A* along *AC*; therefore it will be as *Ab* : *bd* :: *AB* : *BD*, therefore *AdD* is a right line

CASE II.

Let the body be carried through *AB*, *AC* by an accelerative force. Then by Prop. VI. the ſpace deſcribed will be as the time and velocity, and therefore the velocity will be as the ſpace directly

FIG. 1. directly and time reciprocally. Alſo by Cor. 3. Prop. V. when the force and the body is given, the velocity is as the time. Whence the time will be as the ſpace directly and time reciprocally, and the ſpace as the ſquare of the time. That is, the ſame body acted on by the ſame force will deſcribe ſpaces which are as the ſquares of the times. Now let the time of deſcribing AB or AC be 1; and let the line AC move along with the body, always parallel to itſelf, and in the time t, let it arrive at bg, and the body at d, moving towards g. Then, from what has been ſaid, it is as $1 : tt :: AB : Ab$, and alſo as $1 : tt :: bg$ or $BD : bd$. whence $AB : Ab :: BD : bd$. therefore AdD is a right line.

And if you ſuppoſe the ſpace to be as the nth power of the time, it will ſtill be, $1 : t^n :: AB : Ab :: BD : bd$; and AdD is ſtill a right line for any ſimilar forces.

CASE III.

6. But if the body A be carried through AD by a uniform force, in the ſame time that it would be carried through AM by an accelerative force; then by both forces acting together, it will, at the end of that time, be found in the point H, of the parallelogram $MADH$; but then the line it deſcribes AGH, will not be a right line.

1. *Cor.* 1. *The forces, in the directions AB, AC, AD, are reſpectively proportional to the lines, AB, AC, AD; and in theſe directions.*

For by Cor. Prop. IV. the time and the quantity of matter being given; the force is directly as the ſpace deſcribed. And in accelerated motion, the ſame is true, by Prop. V. Cor. 3. and Prop. VI.

Cor. 2. *The two oblique forces AB, AC is equivalent to the ſingle direct force AD, which may be compounded of theſe two, by drawing the diagonal of the parallelogram AD.*

Cor. 3. *Any ſingle direct force AD, may be reſolved into the two oblique forces whoſe quantities and directions are AB, AC, having the ſame effect; by deſcribing any parallelogram whoſe diagonal is AD.*

Cor. 4. *A body being agitated by two forces at once, will paſs through the ſame point, as it would do if the two forces were to act ſeparately and ſucceſſively. And if any new motion be impreſſed on*

a body already in motion, is does not alter its motion in lines parallel FIG.
to its former direction.

Cor. 5. *If two forces as AB, AC, act in the directions AB, AC,* 2.
reſpectively; draw AR to the middle of the right line BC, and 2 *AR is the force compounded out of theſe, and AR its direction.*

PROP. VIII.

Let there be three forces A, B, C of the ſame kind, acting againſt one 3.
another, at the point D, and whoſe directions are all in one plane; and if they keep one another in equilibrio, theſe forces will be to each other reſpectively, as the three ſides of a triangle drawn parallel to their lines of direction, DI, CI, CD.

Let *DC* repreſent the force *C*, and produce *AD*, *BD*, and complete the parallelogram *DICH*: And by the laſt Prop. the force *DC* is equivalent to the two forces *DH*, *DI*; put therefore, the forces *DH*, *DI* inſtead of *DC*, and all the forces will ſtill be in equilibrio. Therefore by Ax. 11. *DI* is equal to its oppoſite force *A*; and *DH* or *CI* equal to its oppoſite force *B*. Therefore the three forces *A*, *B*, *C* are reſpectively as *DI*, *CI*, *CD*.

Cor. 1. *Hence the forces A, B, C are reſpectively as the three ſides of a triangle, drawn perpendicular to their lines of direction, or in any given angle to them, on the ſame ſide.* For ſuch a triangle will be ſimilar to the former triangle.

Cor. 2. *The three forces ABC will be to each other as the ſines of the angles through which their reſpective lines of direction do paſs, when produced:*
For *DI* : *CI* : : *S.DCI* or *CDB* : *S.DCI* or *CDA*.
And *CI* : *CD* : : *S.CDI* or *CDA* : *S.CHD* or *HDI* or *ADB*.

Cor. 3. *If there be never ſo many forces acting againſt any point in one plane, and keep one another in equilibrio; they may be all reduced to the action of three, or even of two equal and oppoſite ones.*

For if *HD*, *ID* be two forces, they are equivalent to the ſingle force *DC*. and in like manner *A* and *B* may be reduced to a ſingle force.

Cor. 4. *And if ever ſo many forces in different planes, acting againſt one point, keep one another in equilibrio; they may be all reduced*

to

FIG. *to the actions of ſeveral forces in one plane, and conſequently to two equal and oppoſite ones.*

3. For if the four forces *A*, *B*, *H*, *I*, act againſt the point *D*; and *H*, *I*, be out of the plane *ABD*. Let *DC* be the common ſection of the planes *ADB*, *HDI*; then the forces *H*, *I*, are reduced to the force *C*, in the plane *ADB*.

SCHOL.

This Prop. holds true of all forces whatever; whether of impulſe or percuſſion, thruſting, pulling, preſſing; or whether inſtantaneous or continual; provided they be all of the ſame kind.

Hence if three forces act in one plane, their proportions are had; and if one force be given, the reſt may be found. And if four forces act, and two be given, the other two may be found; but if only one be given, the reſt cannot be found; for in the three forces, *A*, *B*, *C*, the force *C* may be divided into other two, an infinite number of ways, by drawing any parallelogram *DICH* about the diagonal *DC*. And in general if there be any number of forces acting at *D*, and all be given but two, theſe two may be found: otherwiſe not, though the poſitions of them all be given.

PROP. IX.

If one body acts againſt another body, by any kind of force whatever, it exerts that force in the direction of a line perpendicular to the ſurface whereon it acts.

4. Let the body *B* be acted on by the force *AB* in the direction *AB*. Let the body *C* and the obſtacle *O*, hinder the body *B* from moving; divide the force *AB* into the two forces *AD*, *AE*, or *EB*, *DB*, by Cor. 5. Prop. VII. the one perpendicular, the other parallel to the ſurface *DB*; then the ſurface *DB* receives the perpendicular force *EB*, and the obſtacle *O* the parallel force *DB*; take away the obſtacle *O*, and the force *DB* will move the body *B* in a direction parallel to the ſurface, with no other effect than what ariſes from the friction of *B* againſt that ſurface, occaſioned by the preſſure of *B* againſt it by the force *EB*; which, if the ſurface be perfectly ſmooth and void of tenacity, will be nothing. The force *DB* therefore having no effect, the remaining force *EB* will be the only one, whereby the body *B* acts

againſt the ſurface DB, and that in direction EB perpendicular to it. FIG.

Cor. 1. *If a given body B ſtrike another body C obliquely, at any angle ABD, the magnitude of the ſtroke will be directly as the velocity and the ſine of the angle of incidence ABD*; *and the body C receives that ſtroke in the direction EB perpendicular to the ſurface DB.* 4.

For if the angle ABD be given, the ſtroke will be greater in proportion to the velocity; and if the velocity AB be given, the ſtroke will be as AD, or $S. < ABD$: Or the magnitude of the ſtroke is as the velocity wherewith the body approaches the plane.

Cor. 2. *If a perfectly elaſtic body A impinges on a hard or elaſtic body CB at B, it will be reflected from it, ſo that the angle of reflection will be equal to the angle of incidence.* 5.

For the motion at B parallel to the ſurface is not at all changed by the ſtroke. And becauſe the bodies are elaſtic they recover their figure, in the ſame time they loſe it by the ſtroke; therefore the velocity in direction BE is the ſame after as before the ſtroke. Let AE, BE repreſent the velocities before the ſtroke, and ED ($= AE$) and BE the reſpective velocities after the ſtroke; then in the two ſimilar and equal triangles AEB, BED, $< ABE$ is equal to EBD.

But ſince no bodies in nature are perfectly elaſtic, they are ſomething longer of regaining their figure; and therefore the angle DBF will be ſomething more acute than the $< ABG$.

Cor. 3. *If one given body impinges upon another given body*; *the magnitude of the ſtroke will be as the relative velocity between the bodies.*

For the magnitude of the ſtroke is as the line BE, or the velocity wherewith the bodies approach each other; that is, as the relative velocity. 4.

Cor. 4. *And in any bodies whatever, if a body in motion ſtrike againſt another, the magnitude of the ſtroke will be as the motion loſt by the ſtriking body.*

For the motion impreſſed on the body that receives the ſtroke is equal to the magnitude of the ſtroke. And the ſame motion by Ax. 3. is equal to the motion loſt in the ſtriking body.

Cor. 5. A non-elaſtic body ſtriking another non-elaſtic body, only loſes half as much motion as if the bodies were perfectly elaſtic.

For the non-elaſtic bodies only ſtop; but elaſtic bodies recede with the ſame velocity they meet with.

Cor. 6. Hence alſo it follows, that if one body acts upon another, by ſtriking, preſſing, &c. the other re-acts upon this in the direction of a line perpendicular to the ſurface whereon they act. By Ax. 3.

SCHOL.

Though the momentum or quantity of motion in a moving body, is a quite diſtinct thing from the force that generates it; yet when it ſtrikes another body and puts it into motion, it may with reſpect to that other body, he conſidered as a certain quantity of force proportional to the motion it generates in the other body.

Alſo, although the motion generated by the impulſe of another body is conſidered as generated in an inſtant, upon account of the very ſmall time it is performed in; yet in mathematical ſtrictneſs it is abſolutely impoſſible that any motion can be generated in an inſtant, by impulſe or any ſort of finite force whatever. For when we conſider that the parts of the body which yield to the ſtroke, are forced into a new poſition; there will be required ſome time for the yielding parts to be moved through a certain ſpace into this new poſition. Now during this time, the two bodies are acting upon each other with a certain accelerative force, which in that time generates that motion, which is the effect of their natural impulſe. So that it is plain that this is an effect produced in time; and the leſs the time, the greater the force; and if the time be infinitely ſmall, the force ought to be infinitely great, which is impoſſible. But by reaſon that this effect is produced in ſo ſmall a time as to be utterly imperceptible, ſo that it cannot be brought to any calculation; upon this account the time is entirely ſet aſide, and the whole effect imputed to the force only, which is therefore ſuppoſed to act but for a moment.

The quantity of motion in bodies has been proved to be as the velocity and quantity of matter. But the momentum or quantity of motion may be the ſame in different bodies, and yet may have very different effects upon other bodies, on which they impinge. For if a ſmall body with a great velocity impinge upon another body; and if, by reaſon of its great velocity, it act more ſtrongly upon the ſmall part of the body upon which it impinges, than the force of coheſion of the parts of that body; then the part acted on, will, by this vigorous action, be ſeparated from the reſt; whilſt, by reaſon of the very ſmall time of acting, little or no motion is communicated to the reſt of the body. But if a great body with a ſmall velocity ſtrike another body, and if by reaſon of its ſlow motion it does not act ſo vigorouſly as to exceed the force of coheſion, the part ſtruck will communicate the motion to the reſt of the body, and the whole body will be moved together. Thus if a bullet be ſhot out of a gun, the momentum of the bullet and piece are equal; but the bullet will ſhoot through a board, and the gun will only jump a little againſt him that diſcharges it. Therefore, ſmall bodies with great velocity, are more proper to tear in pieces: and great bodies with ſmall velocity, to ſhake or move the whole.

PROP. X.

The ſum of the motions of any two bodies in any one line of direction, towards the ſame part, cannot be changed by any action of the bodies upon each other, whatever forces theſe actions are cauſed by, or the bodies exert among themſelves.

Here I eſteem progreſſive motions, or motions towards the ſame part, affirmative; and regreſſive ones, negative.

CASE I.

Let two bodies move the ſame way, and ſtrike one another directly. Now, ſince action and re-action (by Ax. 3.) are equal and contrary, and this action and re-action is the very force by which the new motions are generated in the bodies; therefore (by Ax. 2.) there will be produced equal changes towards contrary parts: And therefore whatever quantity of motion is gained by the preceding body will be loſt by the following one; and conſequently their ſum is the ſame as before.

And if the bodies do not ſtrike each other, but are ſuppoſed to act any other way, as by preſſure, attraction, repulſion, &c. yet ſtill, ſince action and re-action are equal and contrary, there will be induced an equal change in the motion of the bodies, and in contrary directions; ſo that the ſum of the motions will ſtill remain the ſame.

CASE II.

Suppoſe the bodies to ſtrike each other obliquely; then ſince (by the laſt Prop.) they act upon each other in a direction perpendicular to the ſurface in which they ſtrike; the action and re-action in that direction being equal and contrary; the ſum of the motions, the ſame way, in that line of direction, muſt remain the ſame as before. And ſince the bodies do not act upon each other in a direction parallel to the ſtriking ſurface; therefore there is induced no change of motion in that direction. And therefore univerſally the ſum of the motions will remain the ſame, conſidered in any one line of direction whatever. And if the bodies act upon one another by any other forces whatever, ſtill (by Ax. 2. and 3.) the changes of motion will be equal and contrary, and their ſum the ſame as before.

Cor. 1. *The ſum of the motions of any ſyſtem or number of bodies, in any one line of direction, taken the ſame way, remains always the ſame, whatever forces theſe bodies exert upon each other; eſteeming contrary motions to be negative. And therefore,*

Cor. 2. *The ſum of the motions of all the bodies in the world, eſtimated in one and the ſame line of direction, and always the ſame way, is eternally and invariably the ſame; eſteeming theſe motions affirmative which are progreſſive, or directed the ſame way; and the regreſſive motions negative. And therefore in this ſenſe motion can neither be increaſed nor diminiſhed. But,*

Cor. 3. *If you reckon the motions in all directions to be affirmative, then the quantity of motion way be increaſed or decreaſed an infinite number of mays. As ſuppoſe two equal non-elaſtic bodies, to meet one another with equal velocities, they will both ſtop and loſe all their motions.*

For let M be the motion of each, then before meeting the ſum of their motions is $M+M$; and after their meeting, it is 0. But in the ſenſe of this Prop. $M—M$ is the ſum of the motions before they meet, becauſe they move contrary ways, which is 0; and it is the ſame after they meet. And thus a man may put ſeveral bodies into motion with his hands, which had no motion before; and that in as many ſeveral directions as he will.

PROP. XI.

The motions of bodies included in a given ſpace, are the ſame among themſelves; whether that ſpace is at reſt, or moves uniformly forward in a right line.

For if a body be moving in any right line; and there be any force equally impreſſed, both upon the body and the right line in any direction. And in conſequence of this, they both move uniformly with the ſame velocity; now as there is no force to carry the body out of that line, it muſt ſtill continue in it as before; and as there is no force to alter the motion of the body in the right line; it will (by Ax. 1.) ſtill continue to move in it as before. For the ſame reaſon, the motions of any number of bodies moving in ſeveral directions, will ſtill continue the ſame; and their motions among themſelves will be the ſame, whether that ſpace be at reſt, or moves uniformly forward.

Likewiſe, ſince the relative velocities of bodies, (that is. the difference of the real velocities the ſame way; or their ſum, different ways) remain the ſame, whether that ſpace be at reſt, or it and the bodies move uniformly forward all together. Therefore their mutual impulſes, colliſions, and actions upon one another, being (by Cor. 3. Prop. IX.) as the relative velocities, muſt (by Ax. 3.) remain the ſame in both caſes.

SCHOL.

Before I end this ſection, it may not be amiſs to mention a certain kind of force, called by the foreigners, *vis viva*. This they term *a faculty of acting*; and diſtinguiſh it from the *vis mortua*, which with them ſignifies only a ſolicitation to motion, ſuch as preſſure, gravity, *&c.* concerning this *vis viva* they talk ſo obſcurely, that it is hard to know what they mean by it. But they meaſure its quantity by the number of ſprings which a moving body can bend to the ſame degree of tenſion, or break; whether it be a longer or ſhorter time in bending them. So that the *vis viva* is the total effect of a body in motion, acting till its motion be all ſpent. And according to this, they find, that the force (or *vis viva)* to overcome any number of ſprings, will always be as the body multiplied by the ſquare of the velocity.

FIG. Suppoſe any number of equal and ſimilar ſprings placed at equal diſtances in a right line; and a body be moved in the ſame right line againſt theſe ſprings; then the number of ſprings which that body will break before it ſtop, will be as the ſquare of its velocity; whatever be the law of the reſiſtance of any ſpring in the ſeveral parts of its tenſion. For, from the foregoing Prop. it appears, that the ſwifter the body moves, ſo much the leſs time has any ſpring to act againſt it to deſtroy its motion: And therefore the motion deſtroyed by one ſpring will be as the time of its acting; and by ſeveral ſprings, as the whole time of their acting; and conſequently the reſiſtance is uniform. And ſince the reſiſtance is uniform, the velocity loſt will be as the time, that is as the ſpace directly and velocity reciprocally; whence the ſpace, and therefore the number of ſprings, is as the ſquare of the velocity. And upon this account they meaſure the force of a body in motion, by the ſquare of the velocity. So at laſt the *vis viva* ſeems to be the total ſpace paſſed over, by a body meeting with a given reſiſtance; which ſpace is always as the ſquare of the velocity. And this comes to the ſame thing as the force and time together, in the common mechanics.

Now it ſeems to be a neceſſary property of the *vis viva*, that the reſiſtance is uniform. But there are infinite caſes where this does not happen; and in ſuch caſes, this law of the *vis viva* muſt fail. And ſince it fails in ſo many caſes, and is ſo obſcure itſelf, it ought to be weeded out, and not to paſs for a principle in mechanics.

Likewiſe, if bodies in motion impinge on one another, the conſervation of the *vis viva* can only take place when the bodies are perfectly elaſtic. But as there are no bodies to be found in nature which are ſo; this law will never hold good in the motion of bodies after impulſe; but, in this reſpect, it muſt eternally fail.

This notion of the *vis viva* was firſt introduced by M. *Leibnitz*, who believed that every particle of matter was endued with a living ſoul.

SECT.

SECT. II.

The laws of gravity, the deſcent of heavy Bodies, and the motion of projectiles, in free ſpace.

PROP. XII.

The ſame quantity of force is requiſite to keep a body in any uniform motion, directly upwards, as is required to keep it ſuſpended, or at reſt.

And if a body deſcends uniformly, the ſame force that is ſufficient to hinder its acceleration in deſcending, is equal to the weight of it.

For the force of gravity will act equally on the body in any ſtate whether of motion or reſt. Therefore if a body is projected directly upwards or downwards, with any degree of velocity; it would for ever retain its velocity if it were not for the force of gravity that draws it down, by Ax. 1. If therefore a force equal to its gravity were applied directly upwards; then (by Ax. 9.) theſe two forces deſtroy each other's effects; and it is the ſame thing as if the body was acted on by no force at all. And therefore (by Ax. 1.) it would retain its uniform motion.

Cor. But if a body be moved upwards with an accelerated motion; the force to cauſe that motion will be greater than its weight; and that in proportion to its acceleration (by Ax. 10.).

PROP. XIII.

The velocities of falling bodies are as the times of their falling from reſt.

For by Poſtulate 3. the body is uniformly acted on by gravity, which is its accelerating force downwards; therefore, by Cor. 3. Prop. V. the velocity is as the force and time directly, and the matter reciprocally. But by Ax. 5. the force of gravity is as the quantity of matter; and conſequently, the velocity will be as the time.

Cor. 1. *All bodies falling by their own weight, gain equal velocities in equal times.*

Cor. 2. *Whatever velocity a falling body gains in any time, if it be thrown directly upwards, it will loſe as much in an equal time;* by *Ax.* 12. And therefore.

Cor. 3. *If a body be projected upwards with the velocity it acquired by falling in any time; it will, in the ſame time, loſe all its motion.* Hence alſo,

Cor. 4. *Bodies thrown upwards loſe equal velocities in equal times.*

PROP. XIV.

The ſpaces deſcribed by falling bodies are as the ſquares of the times of their falling from reſt.

For by Poſtul. 3. gravity is a uniformly accelerating force; therefore by Prop. VI. the ſpace deſcribed is as the time and velocity. But by the laſt Prop. the time is as the velocity; and therefore the ſpace deſcribed is as the ſquare of the time.

Cor. 1. *The ſpaces deſcribed by falling bodies are alſo as the ſquares of the velocities; or the velocity is as the ſquare root of the height fallen.*

Cor. 2. *Taking any equal parts of time; then the ſpaces deſcribed by a falling body, in each ſucceſſive part of time, will be as the odd numbers,* 1, 3, 5, 7, 9, 11, *&c.*

For in the times 1, 2, 3, 4, *&c.* the ſpaces deſcribed will be as their ſquares 1, 4, 9, 16, *&c.* And therefore in the differences of the times, or in theſe equal parts of time; the ſpaces deſcribed, will be as the differences of the ſquares, or as 1, 3, 5, 7, *&c.*

Cor. 3. *A body moving with the velocity acquired by falling through any ſpace, will deſcribe twice that ſpace in the time of its fall.* By Cor. 1. Prop. 6.

Cor. 4. If a body be projected upwards with the velocity it acquired in falling, it will, in the ſame time, aſcend to the ſame height it fell from; and deſcribe equal ſpaces in equal times, both in riſing and falling, but in an inverſe order; and will have the ſame velocity at every point of the line deſcribed.

For by Cor. 2. of the laſt Prop. equal velocities will be gained or loſt in eqnal times, (reckoning from the laſt moment of the deſcent). Therefore, ſince at the ſeveral correſpondent points of time, the velocities will be equal, the ſpaces deſcribed in any given time will be equal, and the wholes equal.

Cor. 5. If bodies be projected upwards with any velocities, the heights of their aſcent will be as the ſquares of the velocities, or as the ſquares of the times of their aſcending.

For in deſcending bodies the ſpaces deſcended are the ſquares of the laſt velocities, by Cor. 1. And by Cor. 4. the ſpaces aſcended will be equal to thoſe deſcended.

Cor. 6. If a body is projected upwards with any velocity, with the ſame velocity undiminiſhed, it would deſcribe twice the ſpace of its whole aſcent, in the ſame time. By Cor. 3. and 4.

Cor. 7. Hence alſo all bodies from equal altitudes deſcend to the ſurface of the earth in equal times.

SCHOL.

It is known by experiments, that a heavy body falls $16\frac{1}{12}$ feet in a ſecond of time, and acquires a velocity which will carry it over $32\frac{1}{6}$ feet in a ſecond; which being known, the ſpaces deſcribed in any other times, and the velocities acquired, will be known by the foregoing propoſitions; and the contrary. Theſe propoſitions are exactly true, where there is no reſiſtance to hinder the motion; but becauſe bodies are a little reſiſted by the air, deſcended bodies will be a little longer in falling; and a body projected upwards, will be ſomething longer in deſcending than in aſcending, and falls with a leſs velocity; and conſequently a body projected upwards with the velocity it falls with, will not aſcend quite to the ſame height; but theſe errors are ſo ſmall, that in moſt caſes they may ſafely be neglected.

If the force by which a body is accelerated in falling was directly as the height fallen from; it may be computed (by Cor. 2. Prop. V.) that the velocity acquired will alſo be as the height; or

FIG. or the ſpace deſcribed directly as the velocity. And therefore if bodies were projected upwards, they would in this caſe aſcend to heights, which are as the velocities with which they are projected. This being compared with Cor. 5. of the laſt Prop. it is eaſy to conclude that bodies projected upwards, and acted upon by a force which is neither of a given quantity, nor in proportion to the diſtance of the body from the top of the aſcent, but between them; that theſe bodies will then aſcend to heights which are between the ſimple and duplicate ratio of the velocities.

And as theſe propoſitions lead us to the knowledge of the relation between the velocities and ſpaces deſcribed, from the forces being given. So *vice verſa*, from that relation being given, the forces may be known. Whence if bodies are projected with any velocities into a reſiſting medium, and the ſpaces deſcribed within that body, meaſured; the conſtitution of that body, and the law of its reſiſtance will be found.

PROP. XV.

If a body be projected either parallel to the horizon, or in any oblique direction, it will, by its motion, deſcribe a parabola.

6, 7. Let *AD* be the direction of the motion, *AFG* the curve deſcribed; and let *AB*, *BC*, *CD*, &c. be all equal; draw *AM*, *BF*, *CG*, *DH*, &c. perpendicular to the horizon; and complete the parallelograms, *AF*, *AG*, *AH*, &c. then by Ax. 1. if the body were without gravity, it would move on in the line *AD*; and deſcribe the lines *AB*, *BC*, *CD*, &c. in equal times. Now ſince gravity acts in lines perpendicular to the horizon, it does not affect the motion in direction *AD*; but generates a motion in direction *AM*. So that the body, inſtead of being at *B*, *C*, *D*, &c. will at the ſame points of time be at *F*, *G*, *H*, &c. But in the time of deſcribing *AB*, *AC*, *AD*, the body, by the force of gravity, will deſcend through the ſpaces *BF*, *CG*, *DH*; which are as the ſquares of the times they are deſcribed in, (by Prop. XIV.) that is as the ſquares of the lines, *AB*, *AC*, *AD*. But *AB*, *AC*, *AD* are equal to *KF*, *LG*, *MH*; and *BF*, *CG*, *DH*, equal to *AK*, *AL*, *AM*. Therefore the parts of the axis of the curve, *AK*, *AL*, *AM*, &c. are reſpectively as the ſquares of the ordinates KF^2, LG^2, MH^2, &c. And therefore, by the conic ſections, the curve *AFG* is a parabola.

Cor. 1. *The line of direction AD is a tangent to the curve in A. And the latus rectum to the point A is* $\frac{KF^2}{AK}$ *or* $\frac{AB^2}{BF}$. *And in the oblique projection, the parameter is* $\frac{AO^2}{BF}$, *ſuppoſing AOP perpendicular to AM, and G the vertex.* For then $\frac{AO^2}{BF} = \frac{AP^2}{CG = GP}$. FIG. 7.

Cor. 2. *If the horizontal velocities of projectiles be the ſame, whatever their elevations be, they will deſcribe the ſame parabola.*

For if *AB* be the velocity and direction of the projectile, then *AO* is the horizontal velocity. When the body comes to the vertex *G*, its motion is then parallel to the horizon, which parallel motion remains the ſame as before, that is, equal to *AO*. Therefore it deſcribes the ſame parabola, as a body projected from *G* with the velocity *AO* parallel to the horizon.

Cor. 3. *The velocity of a projectile in any point of the curve, is as the ſecant of the angle of its direction above the horizon.*

For *AO* the horizontal velocity is the ſame at all points of the curve; and the velocity *AB* at *A*, in the curve is the ſecant of the angle of elevation *OAB*.

Cor. 4. *The velocity at any point of the curve is the ſame that is acquired by falling through* ¼ *the parameter belonging to that point; or, which is the ſame, through* ¼ *of the principal latus rectum* + *the abſciſſa to that point.*

For let *IA* be the ſpace fallen through to acquire the velocity in any point as *A*; then the ſpace *AD* deſcribed in the ſame time with that velocity in direction *AV*, will be 2*AI* (by Cor 3. Prop. XIV.) but in the ſame time, by the ſame force of gravity, the body will deſcend through an equal ſpace *DH*, therefore *AD* or $MH = 2DH$ or $2AM$; but the parameter $= \frac{MH^2}{AM} = \frac{4AM^2}{AM}$ or $4AM$. Therefore *AM* or $AI = \frac{1}{4}$ parameter

PROP. XVI.

The horizontal diſtances of projections, made with any velocities, and at any elevations, are as the ſines of the doubled angles of elevation, and the ſquares of the velocities conjunctly.

7. Let v = *velocity of the projectile; meaſured by the ſpace it paſſes through in time* 1″.
f = *deſcent of a body by gravity in the ſame time.*
x = *AE the horizontal diſtance, or amplitude.*

$$\left.\begin{array}{l} s = \text{ſine} \\ c = \text{coſine} \end{array}\right\} \text{of the elevation } VAE.$$

$$\left.\begin{array}{l} A = \text{ſine} \\ B = \text{verſ. ſine} \end{array}\right\} \text{of twice the elevation.}$$

Then by trigonometry, $2sc = A$, and $2ss = B$, when the radius is 1. And in the right angled triangle AEV.

$c : x :: (\text{rad.})\ 1 : \frac{x}{c} = AV$, and

$c : x :: s : \frac{sx}{c} = VE.$

And by Prop. III.

$v : (\text{time})\ 1 :: (AV)\frac{x}{c} : \frac{x}{cv}$ = time of deſcribing AV.

And by Prop XIV.

$f : (\text{time})\ 1 :: \frac{sx}{c}(VE) : \frac{sx}{cf}$ = ſquare of the time of deſcribing VE. Now the times of deſcribing AV, VE, and the curve AGE are all equal. Whence $\frac{sx}{cf} = \frac{xx}{vvcc}$. Therefore $x = \frac{vvsc}{f} = \frac{vvA}{2f}$. And therefore x or AE is as vvA.

Cor. Hence, the altitudes of projections, are as the ſquares of the ſines of elevation, and the ſquares of the velocities; or as the verſed ſines of the doubled angles of elevation, and the ſquares of the velocities.

For if G be the vertex, $GP = \frac{1}{2} CP = \frac{1}{4} VE$, and $VE = \frac{sx}{c} = \frac{vvss}{f}$. Therefore $GP = \frac{vvss}{4f} = \frac{vvB}{8f}$. Therefore GP is as $vvss$ or as $vv B$.

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Cor. 2. *The times of flight of projectiles, are as the velocities, and the ſines of elevation.* FIG. 7.

For the time $=\frac{x}{cv}=\frac{vs}{f}$.

Cor. 3. *The greateſt random or horizontal projection, is at the elevation of* 45 *degrees. And the horizontal diſtances are equal, at elevations equally diſtant above or below* 45°.

SCOL.

Let h be the height of the perpendicular projection with the velocity v; then will $h=\frac{vv}{4f}$. Whence

Horizontal diſtance $=\frac{vvsc}{f}=\frac{vvA}{2f}=4sch=2Ah$.

Altitude of the projection $=\frac{vvss}{4f}=\frac{vvB}{8f}=ssh=\frac{1}{2}Bh$.

Time of flight $=\frac{vs}{f}=2s\sqrt{\frac{h}{f}}$.

PROP. XVII.

The diſtances of projections made on any inclined planes, are in the complicate ratio of the ſines of the angles which the lines of direction make with the plane and zenith, and the the ſquares of the velocities, directly; and the coſines ſquared of the plane's elevation reciprocally.

Let *AE* be the inclined planc, *AV* the direction of the projectile, *SA*, *CP*, *VE* perpendicular to the horizon; *AGE* the path of the projectile, and let 8.

v = *velocity of the projectile in A, meaſured by the ſpace it deſcribes in the times* 1.
f = *ſpace deſcribed by a deſcending body in the ſame time.*
s = *ſine of VAE,*
c = *ſine of VAS,*
z = *ſine of SAE,*
x = *AE the oblique diſtance, or random.*

FIG. 8. Then by plane trigonometry,

$$c : x\ (AE) :: z : AV = \frac{xz}{c}, \text{ and}$$

$$c : x :: s : VE = \frac{sx}{c}.$$

And by Prop. III.

Space v : time 1 :: $(AV)\ \frac{xz}{c} : \frac{xz}{cv}$ = time of deſcribing AB.

And by Prop. XIV.

Space f : time 1 :: $(VE)\frac{sx}{c} : \frac{sx}{fc}$ = ſquare of the time of deſcending through VE.

But the times of deſcribing AV, VE being equal, we have $\frac{sx}{fc} = \frac{xxzz}{ccvv}$, or $\frac{s}{f} = \frac{xzz}{cvv}$; whence $x = \frac{scvv}{fzz}$, and f being a given quantity, x or AE is as $\frac{scvv}{zz}$.

Cor. 1. *The heights above the planes, are as the ſquares of the velocities, the ſquares of the ſines of elevation above the plane, directly; and the ſquares of the coſines of the plane's elevation, reciprocally.*

For if $AP = PE$, then G is the vertex of the parabola, in reſpect of the plane AE. And $GP = \frac{1}{4}VE = \frac{sx}{4c} = \frac{ssvv}{4fzz}$.

Cor. 2. *The times of flight are are as the velocities and ſines of elevation above the plane; and the coſines of the plane's elevation reciprocally.*

For the time is $= \frac{zx}{cv} = \frac{sv}{fz}$.

Cor. 3. *Hence alſo the altitude is as the ſquare of the time of flight.*

For the altitude is $\frac{ssvv}{zz}$, and the time as $\frac{sv}{z}$.

Cor. 4. *The greateſt projection upon an inclined plane, is when the line of direction biſects the angle between the plane and zenith. And the projections are equal at elevations equally diſtant from this line, above and below.*

For upon the ſame plane, *AE* is as sc, and sc is greateſt when $s=c$. And at equal diſtances above and below sc is the ſame. FIG. 8.

Cor. 5. *This Prop. holds true, whether the projections be made up, or down the planes*; or whether both planes be inclined, or one is inclined and the other horizontal.

SCHOL.

Let $h=$ height of the perpendicular projection as before, then $h=\frac{vv}{4f}$. Whence

Length of the projection $=\frac{scvv}{fzz}=\frac{4sch}{zz}$.

Height of the projection $=\frac{ssvv}{4fzz}=\frac{ssh}{zz}$.

Time of flight $=\frac{sv}{fz}=\frac{2s}{z}\sqrt{\frac{h}{f}}$.

And if $d=AE$ the length of the projection, then $\frac{scvv}{fzz}=d$, whence $v=z\sqrt{\frac{df}{sc}}$. And ſupppoſing the utmoſt random of one of our greateſt guns to be 5864 paces; then $v=194$ paces $=324$ yards, ſo that a ball ſhot out of her, moves at the rate of 324 yards in a ſecond. All this ſuppoſes that there is no reſiſtance of the medium. But it may be noted, that by reaſon of the air's reſiſtance, the upper randoms, being more reſiſted, ſcarce go ſo far as the under randoms; and the greateſt random upon a horizontal plane, is therefore at ſomething leſs elevation than 45 degrees.

SECT. III.

The properties of the mechanical powers; the balance, the leaver, the wheel, the pulley, the ſcrue, and the wedge.

PROP. XVIII.

FIG. 9. *If at the ends of a balance AB, which are equally diſtant from the center of motion C, two equal weights be ſuſpended; they will be in equilibrio.*

Here *AB* that repreſents the balance is ſuppoſed to be a right line, in which are the three points *A*, *B*, *C*. Now the weights *A*, *B*, cannot act upon one another any otherwiſe than by means of the balance *AB*, whoſe fixed point is *C*.

Suppoſe then that any force applied at *A* puts the body *A* into motion, and by means of the balance the body *B*; then ſince the brachia of the beam *CA* and *CB* are equal; the arches *Aa*, *Bb*, deſcribed by theſe bodies will be equal. Conſequently the velocities and quantities of matter of *A*, *B*, being equal, their momenta or motions will be equal. And, becauſe *ACB* is a right line, they move in a contrary direction; and therefore by Ax. 9. theſe bodies cannot of themſelves raiſe one the other, but muſt remain in equilibrio.

Cor. Hence equal forces A, B, applied at equal diſtances from the center of motion C, will have the ſame effect in turning the balance.

PROP. XIX.

In any ſtreight leaver, if the power P be to the weight W, as the 10.
diſtance of the weight from the fulcrum C, to the diſtance of the 19.
power from the fulcrum; the power and weight (acting perpen- 20.
dicularly on the leaver) will be in equilibrio. 12.

A leaver is any inflexible beam, ſtaff, or bar, whether of metal or wood, *&c.* that can any way be applied to move bodies. There are four kinds of leavers;

1. A leaver of the firſt kind is, that where the fulcrum is between the weight and the power, (fig. 10.)

2. A leaver of the ſecond kind is, where the weight is between the fulcrum and the power, (fig. 19.)

3. The leaver of the third kind is, where the power *P* is between the weight and the fulcrum, (fig. 20.)

4. The fourth kind is the bended leaver, (fig. 12.)

CASE I.

In the leaver of the firſt kind *WCP*, inſtead of the power *P*, apply a weight *P* to act at the end of *CP*. And let the leaver *WCP* be moved into the poſition *aCb*. Then will the arches *Wa*, *Pb* be as the radii *CW*, *CP*; that is as the velocities of the weight and power. Whence ſince $P : W :: CW ; CP$, therefore $P : W ::$ velocity of W : velocity of P: therefore $P \times$ velocity of $P = W \times$ velocity of W. Conſequently the momenta or motions of *P* and *W* are equal. And ſince they act in contrary directions, therefore by Ax. 9. neither of them can move the other, but they will remain in equilibrio. 10.

CASE II.

The leavers of the ſecond and third kind may be reduced to 19.
the firſt thus; make $Cp = CP$, and inſtead of the power *P*, apply 20.
a weight equal to it at *p*. Then by Caſe 1, the weight *W* and power *p* will keep one another in equilibrio; and (by Cor. Prop. 18.) the weight *p* and power *P* will have the ſame effect in turning the leaver about its center, therefore the power *P* and weight *W* will be in equilibrio.

Cor.

FIG. *Cor.* 1. *In any ſort of leaver whether ſtreight or bended, and whether*
10. *moveable about a ſingle point C, or an axis AB; or whether the*
11. *leaver be fixed to the axis and both together moveable about two centers*
12. *A, B; or whatever form the leavers have; if AB be a right*
13. *line, and from the ends P, W, there be drawn lines to the center C,*
14. *or perpendiculars to the axis AB; and if the power and weight act*
15. *perpendicular to theſe lines, and be always reciprocally as theſe diſtances*
16. *drawn to the center C or axis AB; then they will be in equilibrio.*

17. *Cor.* 2. *In any ſort of leaver WCP, and in whatever directions the*
18. *power and weight act on it; if their quantities be reciprocally as the perpendiculars to their ſeveral lines of direction, let fall from the center of motion, they will be in equilibrio. Or they will be in equilibrio, when the weight multiplied by its diſtance, and the S. angle of its direction, is equal to the power multiplied by its diſtance, and S. < of its direction.* $W \times WC \times S.DWC = P \times PC \times S.EPC.$

For the power and weight will be in equilibrio if they be ſuppoſed to act at *E* and *D*; and by Ax. 14. it is the ſame thing whether they act at *E* and *D*, or at *P* and *W*. Alſo by trigonometry, $WC \times S.W = DC$, and $PC \times S.P = CE$.

Cor. 3. *Hence univerſally, if any force be applied to a leaver, its effect in moving the leaver, will be as that force multiplied by the diſtance of its line of direction from the center of motion. Or the effect is as the force × by its diſtance from the center, and by the ſine of the angle of its direction,* $P \times PC \times S.P$.

Cor. 4. *If two bodies be in equilibrio on the leaver, each weight is reciprocally as its diſtance from the center.*

Cor. 5. *In the ſtreight leaver when the weight and power are in equilibrio, and act perpendicularly on the leaver, or in parallel directions; then of theſe three the power, weight, and preſſure upon the fulcrum, any one of them is as the diſtance of the other two.*

10. For if *CP* repreſent the weight *W*, then *CW* will repreſent the
19. power *P*. And in Fig. 10. *C* ſuſtains both the weights, and
20. therefore the preſſure is *WP*; and Fig. 19, 20. *C* ſuſtains the
difference of the weights, and therefore the preſſure will be *WP*.

PROP. XX.

If ſeveral weights be ſuſpended on a ſtreight leaver AB; and if the ſum of the products of each weight, multiplied by its diſtance from the center of motion C, on one ſide, be equal to the ſum of the like products on the other ſide; then they will be in equilibrio. And the contrary. 21.

For the force of each weight to move the leaver is as the weight multiplied by the diſtance (by Cor. 3. laſt Prop.); and the ſum of the products is as the whole forces; which if they be equal, the forces on both ſides are equal, and the leaver remains at reſt.

PROP. XXI.

If a bended leaver WCP be kept in equilibrio by two powers, acting in the directions PB, WA perpendicular to the ends of the leaver CP, CW; and if the lines of direction be produced till they meet in A, and AC be drawn, and CB parallel to WA. I ſay the power P, the weight or power W, and the force acting againſt the fulcrum C; will be reſpectively as AB, BC, AC; and in theſe very directions. 22.

Draw CB, CF parallel to WA, PA; then the angle $WFC = WAP = CBP$, and the right angled triangles WCF and BCP are ſimilar; whence $CF : CB :: CW : CP ::$ (by Cor. 2. Prop. XIX.) power P : power W. Now ſince (by Ax. 14.) it is the ſame thing to what points of the lines of direction PB, WF, the forces P, W be applied; let us ſuppoſe them both to act at the point of interſection A; then ſince the point A is acted on by two forces which are as CF and CB, or as AB and AF; and both theſe are equivalent to the ſingle force AC (by Cor. 2. Prop. 7.) Therefore the fulcrum C is acted on by the force AC, and in that direction, by Ax. 11.

Cor. 1. Hence the power P, the weight W, and the preſſure the fulcrum C ſuſtains; are reſpectively as WC, PC, and PW. That is, any one is as the diſtance of the other two.

For ſince the angles at P, W are right; CA is the diameter of a circle paſſing through the points A, P, C, W; therefore the

FIG. 22. the angle $WPC = WAC = ACB$, and the angle $CWP = CAP$; therefore the triangles *ABC*, *WCP* are ſimilar; and $AB : BC :: WC : CP$, and $BC : AC :: CP : PW$. Therefore, &c.

Cor. 2. In any leaver WCP, the lines of direction of the powers PW, WF, and of the preſſure on the fulcrum C, all tend to one point A.

For if not, the leaver would not remain in equilibrio.

PROP. XXII.

23. *If AB, CD be two leavers moveable about A and C, and ſome force acts upon the end B of the leaver AB, in a given direction BF; whilſt the leaver AB acts upon CD at B. If BE be drawn perpendicular to CB, and AE parallel to BF: And if theſe leavers keep one another in equilibrio. Then I ſay, the force in direction BF, force againſt DC in direction EB, and the preſſure againſt the center A, are reſpectively as AE, BE, AB.*

For ſince (by Prop. 9.) the leaver *AB* acts upon *BC* at *B*, in the direction *EB* perpendicular to *BC*; and the leaver *CD* re-acts in direction *BE*; and (by Ax 19.) the point *A* is acted on in direction *BA*. Therefore the point *B* is acted on with three forces; *BF* the force applied at *B*, and *BE* the re-action of the leaver *CD*, and *AB* the re-action of the center *A*; and *AE* is parallel to *BF*; therefore (by Prop. VIII.) theſe forces are as *AE*, *BE*, and *AB*.

Cor. If two forces BF, BE acting perpendicular to the leavers AB, DC, keep theſe leavers in equilibrio: The force BE, force BF, and preſſure at A, are reſpectively as radius, Coſ. ABD, and S.ABD.

For then *EAB* is a right-angled triangle; and theſe forces are as *BE*, *AE*, and *AB*; that is, as radius, *S.ABE*, and *S.AEB*; that is, as radius, Coſ. *ABD*, and *S.ABD*.

FIG.

PROP. XXIII.

If AB, BC be two leavers moveable about the centers A and C; and if the circles KBM and DBE be deſcribed with the radii BC, BA; and upon the Circle BM as a baſe, with the generating circle DBE, the epicycloid BNE be deſcribed; and the leaver CB and epicycloid BNE be joined together in that very poſition, ſo as to make but one continued leaver CBNE. And if theſe leavers CBNE, and AB move about the centers C, A; ſo that the end D of the leaver AD be always in the curve of the epicycloid DK; I ſay, that two equal and contrary forces at D and K, acting perpendicular to the radii DA, KC, will always keep theſe leavers in equilibrio. 24.

For let the leavers *AB*, *CBNE* come into the poſition *AD*, *CKDF*; then ſince the epicycloid *KD* is deſcribed by the point *D*, whilſt the arch *DB* rolled upon the equal arch *BK*, therefore the end *D* of the radius *AD* hath moved through an arch *BD* equal to the arch *BK*, through which *K* has moved. Therefore the points *D*, *K*, have equal velocities in any correſpondent places *D*, *K*. Therefore equal weights *H*, *I*, applied to the circles *DB* and *BMG* will have equal quantities of motion; and will therefore keep each other in equilibrio, by Ax. 9.

Cor. 1. *Hence if one leaver AD move uniformly about the center A, the other CKD will alſo move uniformly about its center C. And the arch BD deſcribed by D will be equal to the arch BK deſcribed by K.*

Cor. 2. *It is the ſame thing whether the leaver AB act againſt the convex or concave ſide of the leaver CKD, provided the end D be always in the curve KD.*

Cor. 3. *After a like manner if BE be an epicycloid deſcribed within the circle BRM, by the generating circle BD. And the leaver CBE be compounded of the right line CB and the epicycloid BE; then the leavers CBE and AB, by equal forces acting at B, will keep one another in equilibrio in any poſition, as CKF and AD.* 25.

For when *AB* is come to *AD*, and *CBE* to *CKDF*; then the arch *BK* = arch *BD*. Whence the weight or forces acting at the diſtances *CB*, *AB*, have equal velocities; and therefore will ſuſtain one another.

Cor.

FIG. 26. *Cor. 4. If BC be infinite, or (which is the ſame thing) if BK be a right line perpendicular to AB; then BE or KDF will be the common cycloid. Therefore, whilſt the point D moves uniformly about the center A, the point K will move uniformly along the right line BK, and with equal velocities and forces: The point D in the mean time acting upon the cycloidal tooth KD. And any equal oppoſite forces will ſuſtain one another.*

27. *In like manner, if BA be infinite, or BD a right line perpendicular to BC; then BE or DK will be an epicycloid generated by the tangent DB revolving on the circle BK. And the velocities of K in the right line BK, and of D in the right line BD, will be always equal: And equal forces will be ſuſtained at B, in all poſitions of the leaver CKD.*

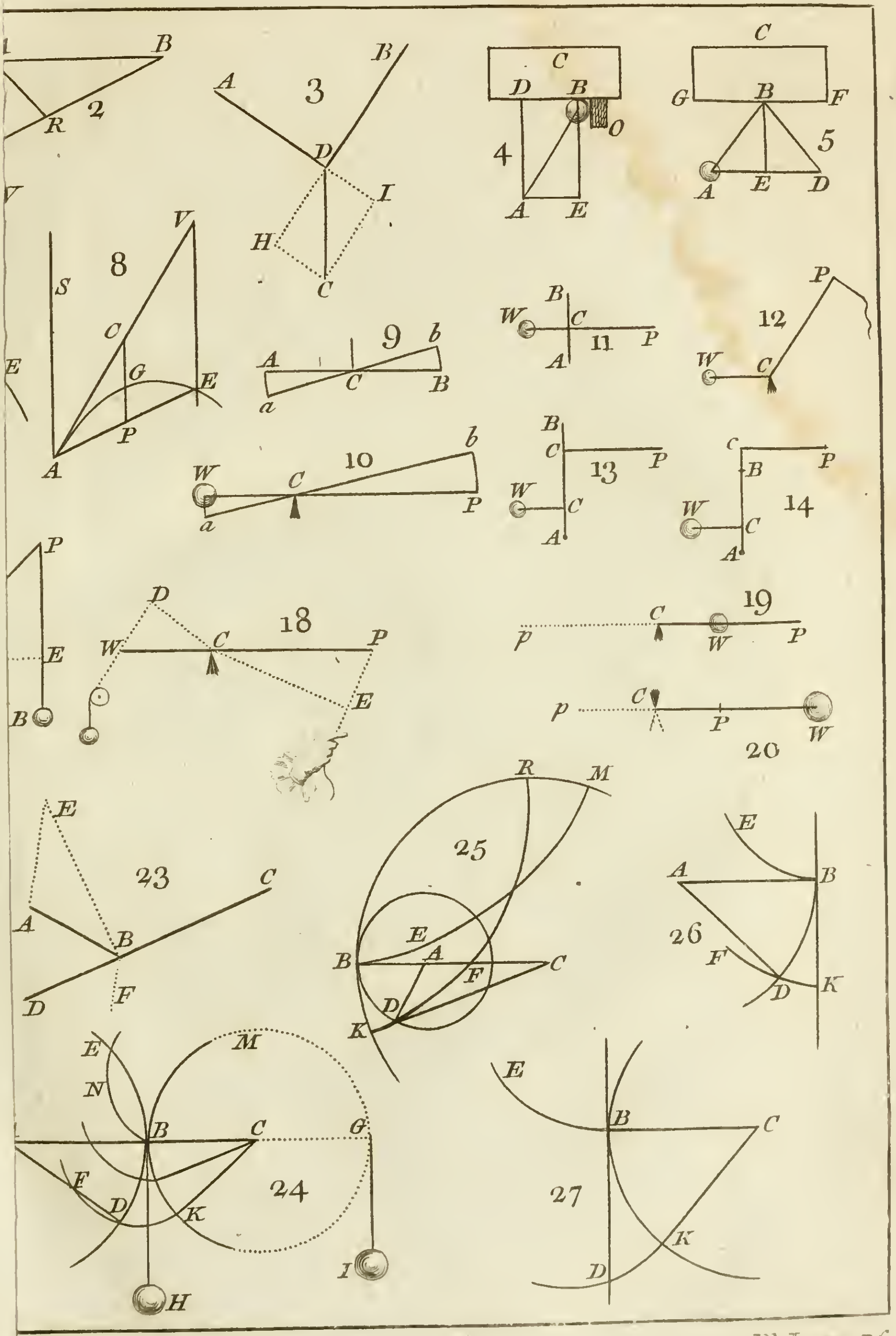
28. *Cor. 5. If the figure of the tooth ef, at the end of the leaver AB, be given; and the epicycloid BE be deſcribed as before. And if the leavers AB, CB, be made to revolve about their centers A, C; ſo that the point B always move in the epicycloid BE or KD. And if the tract of the extreme points of the tooth be marked out upon the plane of the cycloidal tooth, as at fgggge, or at fnnnne. And if the part fgDdKf be cut away, if the tooth be to act on the concave ſide of the epicycloid; or fneDdKf, if on the convex ſide. Then if the leavers revolve ſo that the tooth move along the curve gg or nn; the points D and K of the leavers AB, CD, will move with equal velocities, in the arches BD, BK, as before.* For the fixed point *B* in the tooth will ſtill deſcribe the epicycloid.

29. *Cor. 6. If the two epicycloids BE, BO be deſcribed upon BM, BL with the generating circles BD, BK; and the leavers AB, CB, revolve about the centers A, C; ſo that the point B or D of the leaver AB move along the epicycloid BE or KDS. Then the point B or K, of the leaver CB, will at the ſame time move along the epicycloid BO or DKT; and the points D, K, will deſcribe the equal arches BD, BK. And therefore it is the ſame thing, on which leaver the cycloidal tooth be placed, or whether on one or both.*

For the epicycloid *DKT* generated on *DB*, will paſs through *K*, if $BD = BK$. Alſo the epicycloid *KDS*, generated upon *KB*, will paſs through *D*, when $BK = BD$.

SCHOL.

24.
25. The leavers *AB*, *CB* are ſuppoſed only to act upon one another, below the line *AC*; for was the action ſuppoſed to be continued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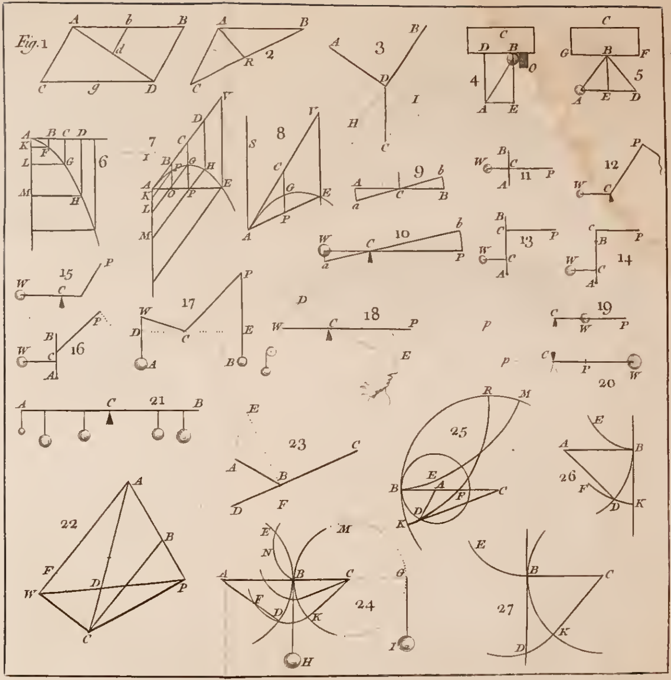


Mechanics.

Pl I. pa. 36

continued above the line AC, the point B would no longer act FIG.
on the ſame, but on a different epicycloid; and the equality of 24.
motion would hold no longer. 25.

PROP. XXIV.

30. *In the wheel and axel, if the power P be to the weight W, as the diameter of the axel EF where the weight acts, to the diameter of the wheel AB, where the power acts; then the power and weight will be in equilibrio. And the contrary.*

For let AB be the wheel, CD the axel; and ſuppoſe the wheel and axel to turn once round; then it is plain the power P will have deſcended a ſpace equal to the circumference of the wheel; and the weight W will have riſen a height equal to the circumference of the axis. Therefore, velocity of P, to velocity of W :: as circumference of the wheel, to circumference of the axis :: or as diameter of the wheel, to diameter of the axis :: that is (by ſuppoſition) as W, to P. Therefore the motions of P and W are equal; and have equal forces to move each other; and therefore (by Ax. 9.) will remain in equilibrio.

This Prop. will appear otherwiſe. For the wheel and axel may be reduced to a leaver of the firſt kind: For the fulcrum will be in the middle of the axis CD. Therefore drawing lines from the middle of the axis to the power and weight, parallel to the horizon; and the radius of the wheel will be the diſtance of the power, and the radius of the axel the diſtance of the weight. And as their radii are reciprocally as the weight and power, therefore (by Prop. XIX.) they will be in equilibrio. And thus the wheel and axel is no more but a perpetual leaver.

Cor. 1. *If the rope have any ſenſible thickneſs; then if the power P : weight :: diameter of the axel + the diameter of the rope : diameter of the wheel where the power acts; they will be in equilibrio.*

For the weight really hangs half the thickneſs of the rope, beyond the axel.

31. *Cor.* 2. *If the direction of the power is not a tangent to the wheel; ſuppoſe it to act at D or d, and let CA be perpendicular to the line of direction; then if P : W :: CB : CA, then they will be in equilibrio, by Cor.* 2. *Prop.* XIX.

Cor.

FIG. 32. *Cor. 3. If the wheel and axel, one or both have teeth; then if the power P acting on the teeth at B, be to the weight W acting on the teeth at A; as the diameter of the axel at A, to the diameter of the wheel at B; then the wheel is in equilibrio.*

Cor. 4. And it is the ſame thing if inſtead of a wheel there be only ſpokes fixed in the axis, whoſe length is equal to the radius of the wheel: And any other equal force be applied for a power, inſtead of the weight P.

Cor. 5. The force of the weight is increaſed when one or more ſpires of the rope is folded about the axel. For that, in effect, augments the diameter of the axel.

Cor. 6. It matters not how low the weight hangs. For whilſt the axel remains the ſame, the reſiſtance of the weight remains the ſame, ſetting aſide the weight of the rope.

PROP. XXV.

33. *Let NBD, MBK be two toothed wheels in the ſame plane, and if the teeth of the wheel BM be the epicycloids Be, kd, KD, deſcribed on the baſe KBM, with the generating circle BN, and theſe teeth all equidiſtant; and if B, d, D the ends of the teeth of the wheel NBD be alſo equidiſtant, and theſe diſtances Bd, dD equal to Bk, kK: Then, I ſay, the points of the teeth B, d, D, will all act together, on the cycloidal teeth BE, kd, KD, as the wheels turn round. And any points D, K, will move through equal arches BD, BK in equal times.*

Draw the radii *AD*, *Ad*, *CK*, and *Ck*; then *AD* and *CKD* may be conſidered as two leavers moving about *A*, *C*, and acting on one another in *D*: And the ſame of *Ad*, *Ckd*, acting at *d*. But by the motion of the wheels *BD*, *BK*, ſuppoſe *D* always to be in the epicycloid *KD*; then (Cor. 1. Prop. XXIII.) will $BD = BK$, and ſince $Dd = Kk$, therefore *Bd* will be $= Bk$, and conſequently (by Cor. 1. Prop. XXIII.) the point *d* will be in the epicycloid *kd*. And thus, if there be never ſo many teeth *B*, *d*, *D*, *&c.* they will always be in the curves of the epicycloids *BE*, *kd*, *KD*, *&c.* therefore the working teeth either act all at once upon one another, or they act not at all. And as the velocities of any points are equal in the two wheels *BD*, *BK*, when only one tooth acts upon one, they will ſtill be equal, if never ſo many act together. FIG. 33.

Cor. 1. *Hence equal weights or forces applied to the circumferences of theſe two wheels, as at B, and acting one againſt the other, will keep theſe wheels in equilibrio. Likewiſe, it is the ſame thing whether the wheel AB drive the wheel CB, its teeth acting upon the concave ſide of the cycloids; or the wheel CB drive AB, the convex ſide of the cycloid acting againſt the teeth of AB.*

34. *Cor.* 2. *Hence, inſtead of the points B, or the infinitely ſmall teeth of the wheel ABD; if any ſort of a tooth rs be placed at B; and if the wheels be made to move about ſo that the given point B may deſcribe the epicycloid BE or KD, whilſt the track of the extreme points of the tooth is marked out as KeD, KfD; and the ſpace KeDf be cut away; and the ſame be done for all the other teeth, being equidiſtant and of the ſame form and bigneſs. Then if one of theſe wheels is ſuppoſed to drive the other, by theſe teeth running in the ſpaces DfKe; I ſay, the circumferences of theſe wheels will move with equal velocities, and all the working teeth will act together.* This is evident, becauſe the points *B*, *D*, will by this motion deſcribe the epicycloids as before.

35. *Cor.* 3. *If the epicycloid BV be deſcribed on the baſe KBH, with the generating circle BD; and a portion of the epicycloid be placed at equal diſtances B, L, K, for teeth; then the teeth of the wheel A acting againſt the cycloidal teeth, will make the motion equal in the two wheels. Where we may take as great a portion of the cycloid as we will; and the ſides BO, LI, which act not, may be of any figure, not to hinder the motion of the teeth of A. And it is the ſame thing what part of the tooth LO, the tooth G acts againſt.*

Cor. 4. *But the teeth ought not to act upon one another before they arrive at the line AC, which joins their centers And though the ſide BO of the tooth may be of any form; yet it is better to make them both ſides alike, which will ſerve to make the wheels turn backwards. Alſo a part as pqr may be cut away on the back of every tooth, to make way for thoſe of A. And the more teeth that work together, the better; at leaſt one tooth ſhould always begin before the other hath done working: The teeth ought to be diſpoſed in ſuch manner as not to trouble or hinder one another, before they begin to work; and that there be a convenient length, depth and*

FIG. 35. *thickneſs given them, that they may more eaſily diſengage themſelves; as well as for ſtrength.*

PROP. XXVI.

36. *In a combination of wheels with teeth, if the power P be to the weight W; as the product of the diameters of all the axels, pinions, or trundles; to the product of the diameters of all the wheels, they will be in equilibrio.*

For by Prop. XXIV.

The power P acting at A : Force on B :: diam. B : diam. A, and force on B or C : force on D :: diam. D : diam. C, and force on D or E : weight W at F :: diam. F : diam. E.

Therefore, ex equo.

Power P : weight W :: product of the diameters B, D, F: to the product of the diameters A, C, E.

38. Cor. 1. *In a combination of wheels going by cords, if the power P be to the weight W, as the product of all the diameters of the axels, B, D, F, to the product of all the diameters of the wheels, A, C, E; they will be in equilibrio.*

For the cords ſupply the place of teeth.

36. Cor. 2. *In any combination of wheels with teeth, if the power P be to the weight W; as the diameter of the axel F where the weight acts, multiplied into the product of the teeth in each pinion or ſpindle, is to the diameter of the wheel A, where the power acts, multiplied by the product of the teeth in each, of the wheels (that the pinions act againſt); the weight and power will be in equilibrio.*

For the number of teeth in each wheel and pinion that act againſt one another, are as the circumferences or as the diameters of that wheel and pinion.

Co. 3. *And hence alſo, if the power be to the weight, in a ratio compounded of the diameter of the axel F, where the weight acts to the diameter of the wheel A, where the power acts, and the ratio of the number of teeth in the firſt axel, (B), reckoning from the power; to the number of teeth in the ſecond wheel (C), and of the number of teeth in the ſecond axel (D), to the number in the third wheel (E); and ſo on till the laſt; then they will be in equilibrio.*

Cor.

FIG. 36. *Cor.* 4. *In a combination of wheels, the number of revolutions of the wheel F where the weight acts, to the number of revolutions of the wheel A where the power acts, in the ſame time; is as the product of the teeth in the pinions, to the product of the teeth in the wheels which act in them; or as the product of the diameters of the pinions, to the product of the diameters of the wheels.*

SCHOL.

Wheels with oblique teeth come under the ſame rules; but as they are related to the ſcrew, we refer you thither for a farther account thereof.

In wheels whoſe teeth work together, they ſhould not encounter before they come to the line joining their centers; becauſe the rubbing is greater on that ſide; but being paſt the line, the teeth ſlip eaſily along one another, in making their eſcape, ſo that the friction is very inconſiderable.

PROP. XXVII.

39. 40. *If a power ſuſtain a weight by means of a rope going over a fixed pulley; then the power is equal to the weight. But if the pulley be moveable together with the weight, and the other end of the rope fixed; then the power will be but half the weight.*

For ſuppoſe a horizontal line AB drawn through the center of the pulley C; then that line will repreſent a leaver, and (in Fig. 39.) where the pulley is fixed, the center C being kept immoveable, repreſents the fulcrum; whilſt the weight acts at B, and the power at A. And becauſe $BC=CA$, therefore (by Prop. XIX.) the power P is equal to the weight W.

And (in Fig. 40.) the fixed point B is the fulcrum, and the weight acts at C, and the power at A; and ſince BC is half AB, therefore (by Prop. XIX.) the power at P is half the weight W.

41. *Cor. Hence all fixed pulleys are equivalent to leavers of the firſt kind. And they add no new force to the power, but only ſerve to change the direction, and facilitate the motion of the rope: But a moveable pulley doubles the force. And if a rope go over ſeveral pulleys, A, B, C, whoſe blocks are all fixed; the power is neither increaſed nor diminiſhed.*

PROP. XXVIII.

42. *In a combination of pulleys all drawn by one running rope; if the power P be to the weight W; as* 1 *to the number of parts of the rope at the moveable block A; they will be in equilibrio.*

For (by Ax. 18.) all the parts of the rope *m*, *o*, *n*, *r*, *s*, *t*, *v*, are equally ſtretched; and the weight *W* is ſuſtained by the number of ropes that act againſt the moveable block; and the rope *v* or the power *P* acts with the force of one or unit. Therefore the power is to the weight, as 1 to the number of ropes pulling at the moveable block *A*.

Cor. Hence the power is to the force by which the immoveable block B is drawn; as 1 *to the number of ropes acting againſt that immoveable block.*

PROP. XXIX.

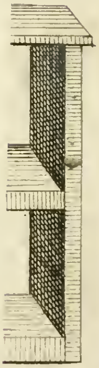
37. *In the ſcrew, if the power P be to the weight W, as the height of one thread (reckoned according to the length of the ſcrew) to the circumference deſcribed by one revolution of the power; then they will be in equilibrio.*

For the weight *W* raiſes the height of one thread, whilſt the power deſcribes the circumference whoſe radius is *PC*. Therefore the velocities of the power and weight are reciprocally as their quantities: Therefore their motions are equal, and they are in equilibrio.

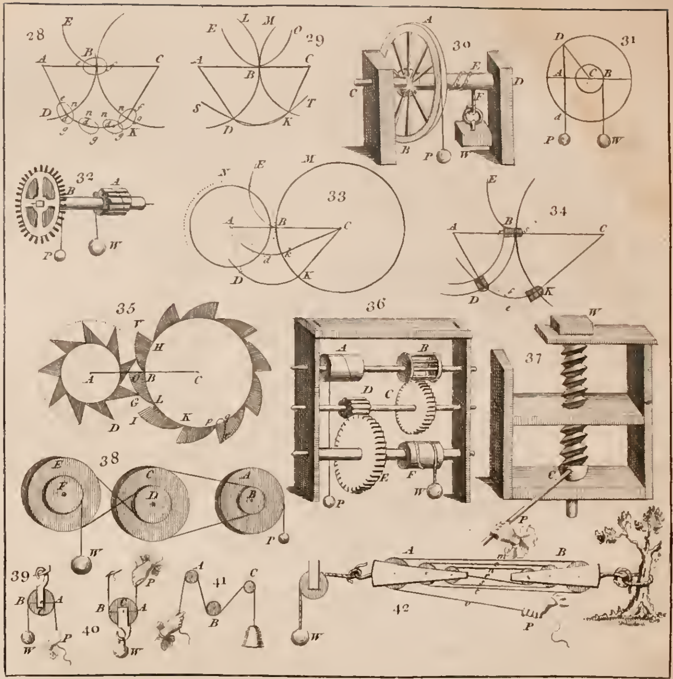
43. *Cor.* 1. *In the endleſs or perpetual ſcrew AB, having one worm, leaf, or tooth, which drives the teeth of the wheel CD. If you take the diſtance of two threads in the ſcrew AB, according to the length of the axis AB; or the diſtance of two teeth in the wheel CD, in direction of the circumference. And if a weight W act at the circumference of the wheel CD. Then if the power P be to the weight W, as that diſtance (of the teeth or threads) to the length deſcribed by the power P, in one revolution; then they are in equilibrio.*

For in one revolution of *P*, the wheel *DC* with the weight *W*, has moved the diſtance of one tooth.

31



I.II. *pa.*42.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Pl.II. pa. 42.

FIG. 40.

Cor. 2. *And univerſally, if there be ſeveral worms or ſpiral leaves, upon the axis AB, and the weight G hangs upon the axel EF. Then if the power P, is to the weight G :: as the radius of the axel $EF \times$ number of worms in AB, to $AP \times$ number of teeth in CD. Then the power and weight are in equilibrio. For by Cor. 3. Prop.* XXVI. *if n be the number of worms, then $P : G :: n \times \frac{1}{2} EF : AP \times$ teeth in CD.*

Cor. 3. *And by reaſon of the obliquity of the teeth, the force acting perpendicular to the teeth, the lateral force perpendicular to the wheel, and the direct force in the plane of the wheel; will be reſpectively, as radius, the ſine, and coſine of the obliquity of the teeth.*

44.

For let GD be the ſide of a tooth acted on; GE parallel to the axis of the wheel, and DE perpendicular to it, or in the plane of the wheel. Now if GD repreſent the force acting perpendicular to the tooth. Then DE, GE will be the forces acting in the directions GE, DE, (by Cor. 1. Prop. VIII.) but if GD be radius, DE is the ſine of the obliquity, and GE the coſine.

Cor. 4. *In the common ſcrew the leſs the diſtances of the threads are, and the longer the handle is, the eaſier any given weight is moved.*

Cor. 5. *What is here demonſtrated, will hold equally true, if the wheel CD act upon another wheel with oblique teeth, inſtead of the worm AB.*

SCHOL.

The force of the ſcrew reſembles the force that drives a body up an inclined plane; the force acting parallel to the baſe of the plane.

All things here laid down relating to the perpetual ſcrew, do ſuppoſe that the axis of the worm ſpindle lies in the plane of the wheel it works in, and that their axles are perpendicular to each other; but if they are in oblique poſition, and the teeth of one or both alſo oblique, they cannot work without loſs of power; a part being loſt proportional to the obliquity.

If any worm ſpindle contains one leaf or worm, then a ſpindle of twice the diameter will require two worms, and one of thrice the diameter, three worms, &c. to work in the ſame wheel; and the power is beſt eſtimated by the riſe or fall of a tooth of CD (Fig. 43.) for a revolution of the power P.

FIG.

PROP. XXX.

45. *Let EFG be the back or baſe of a wedge in form of an iſoceles triangle; then if the power acting perpendicular to the back FG, is to the force or reſiſtance acting againſt either ſide, in a direction perpendicular to that ſide; as the back of the wedge FG, to either of the ſides EF, EG: Then the wedge is in equilibrio, or, which is the ſame thing, the power is to the whole reſiſtance againſt both ſides, as the back FG, to the ſum of the ſides EF, EG.*

For draw the axis *ED* perpendicular to the baſe *FG*; and *CA*, *CB* perpendicular to the ſides *EF*, *EG*; then *DC* is the direction of the power. And (by Prop. 9.) the impediment to be removed, acts againſt the wedge in the directions *AC*, *BC*; and therefore (by Cor. 1. Prop. VIII.) the power, and the actions of the impediment, are as *FG*, *FE*, *EG* reſpectively, when they are in equilibrio.

Cor. 1. *The power acting perpendicular to the baſe, is to the force acting againſt either ſide, in a direction parallel to the baſe FG, or perpendicular to the axis DE; as the baſe FG, to the height ED: When the wedge is in equilibrio, or the power is to the whole force againſt both ſides (in direction parallel to FG) as the back FG, to twice the height DE.*

For the force *EG* may be divided into the two *ED*, *DG*, (by Cor. 3. Prop. VII.) Then ſince (by this Prop.) *EG* is the force acting in direction *CB*; *ED* will be the force acting in direction *DG*.

Cor. 2. *The ſharper the wedge, or the more acute its angle, the eaſier it will divide any thing or overcome any reſiſtance.*

SECT.

SECT. IV.

The deſcent of bodies upon inclined planes, and in curve ſurfaces. Alſo the motion of pendulums.

PROP. XXXI.

If a heavy body W, be ſuſtained upon an inclined plane AC, by a power acting in a direction parallel to that plane. Then FIG. 46.

The weight of the body,	*the length AC,*
The power that ſuſtains it,	*the height CB,*
And its preſſure againſt the plane:	*and the baſe AB,*
Are reſpectively, as	*of the plane.*

Draw *BD* perpendicular to *AC*; then the force of gravity tends perpendicular to the horizon, or parallel to *CB*; and the direction of the power is parallel to *DC*; and the preſſure againſt the plane is (by Prop. IX.) parallel to *DB*. And therefore their quantities are reſpectively as the three lines *CB*, *CD*, *BD*, (by Prop. VIII.) that is, by ſimilar triangles, as *AC*, *CB*, and *AB*.

Cor. 1. *The weight, power, and preſſure on the plane, are reſpectively, as radius, the ſine and coſine of the plane's elevation.*

For the ſides of a triangle are as the ſines of the oppoſite angles.

Cor. 2. *The relative weight of a body, to make it run down an inclined plane, is as the height directly, and length reciprocally.* $\frac{BC}{AC}$; *or it is as the ſine of the plane's elevation.*

Cor. 3. *If a cylinder be ſuſtained upon an inclined plane, by a power drawing one end of a rope parallel to the plane, whilſt the other end is fixed. This power is to the weight of the cylinder, as half the height to the length of the plane.* 47.

For half the relative weight of the cylinder is ſuſtained by the other end of the rope which is fixed.

FIG.

SCHOL.

46. If it be required to find the poſition of the plane *AC*, whoſe height *BC* is given, ſo that the given weight *W* may be raiſed through the length of the plane *AC*, in the leaſt time poſſible, by any given power *P*, acting in the direction *DC*. Make $AC = \frac{2W}{P} \times BC$, and you have your deſire.

PROP. XXXII.

46. *If a heavy body W be ſuſtained upon an inclined plane AC, by a power acting parallel to the horizon. Then,*

The weight of the body,	*the baſe AB,*
The power that ſuſtains it,	*the height CB,*
The preſſure againſt the plane,	*and the length AC,*
Are reſpectively as	*of the plane.*

For the body is ſuſtained by three forces, the power, the gravity, and re-action of the plane; the weight is perpendicular to *AB*, the power is perpendicular to *CB*, and the preſſure is perpendicular to *AC*. Therefore (by Cor 1. Prop. VIII.) theſe forces are as *AB*, *CB*, *AC*.

Cor. Hence the preſſure on the plane, the power, and the weight, are reſpectively, as radius, the ſine and coſine of the plane's elevation.

PROP. XXXIII.

48.
49. *If a heavy body W be ſuſtained upon an inclined plane AC, by a power P acting in any given direction WP. And if BED be let fall perpendicular on WP. Then,*

Power P,	*DB,*
Weight of the body W,	*AB,*
Preſſure upon the plane,	*AD.*
Will be reſpectively, as	

For ſince BD is perpendicular to the direction of the power, FIG.
AB to the direction of gravity, and AD to the direction of the 48.
preſſure on the plane. Therefore (by Cor. 1. Prop. VIII.) theſe 49.
forces will be reſpectively as BD, AB, AD, when they are in equilibrio.

Cor. 1. *The power, weight, and preſſure againſt the plane, are reſpectively as the ſine of the planes elevation, coſine of the angle of traction* CWP, *and the coſine of the angle of direction of the power above the horizon.*

The angle of traction is the angle that the direction of the power makes with the plane. And in the triangle ABD, the ſides are as the ſines of the oppoſite angles, where $< D =$ complement of DWP.

Cor. 2. *Hence whether the line of direction of the power be elevated above or depreſſed below the plane; if the angles of traction be equal, equal powers will ſuſtain the weight; but the preſſure is greater when the line of direction runs below the plane.*

Cor. 3. *The power* P *is leaſt when the line of direction is parallel to the plane; and infinite when perpendicular to it; and equal to the weight, when perpendicular to the horizon.*

Cor. 4. *If a weight upon an inclined plane be in equilibrio with* 50.
another hanging freely, their perpendicular velocities will be reciprocally as their quantities of matter.

For let the weight at W be made to deſcend to A, and draw Wr perpendicular to AE, and Wt, Dv to AB; then the weight P will have aſcended a height $= Ar$, which is its perpendicular aſcent; and Wt is the perpendicular deſcent of W. The figures $Arwt$ and $AEDv$ are ſimilar, as are alſo the triangles AEB, DvB. Whence $Wt : Ar :: Dv : AE :: DB : AB =$ (by this Prop.) $P : W$.

Cor. 5. *And therefore if any two bodies be in equilibrio upon two inclined planes, their perpendicular velocities will be reciprocally as their quantities of matter.*

FIG.

PROP. XXXIV.

51. *The ſpace which a body (deſcending from reſt) deſcribes upon an inclined plane, is to the ſpace which a body falling perpendicularly, deſcribes in the ſame time, as the height of the plane CB, to its length AC.*

The force wherewith a body endeavours to deſcend upon an inclined plane, is equal to the power that ſuſtains it; and (by Prop. XXXI.) that power is to the weight of the body as *CB* to *CA*. Therefore the body is urged upon the plane, by an uniformly accelerating force, which is to the force of gravity as *CB* to *CA*. But (by Prop V.) the motion generated in the ſame time, and in the ſame body, is as the force, that is (ſince the body is given) the velocity is as the force. And (by Prop. III.) the ſpaces uniformly deſcribed with the laſt velocities will be as theſe velocities; and (by Cor. 1. Prop. VI.) theſe ſpaces are double the ſpaces deſcribed by the accelerating forces. Therefore the ſpaces deſcribed on the plane and in the perpendicular, are as the laſt velocities, or as the forces, that is as *CB* to *CA*.

Cor. 1. *Hence if BD be let fall perpendicular to AC; then in the time a body falls through the height CB; another body, deſcending along the inclined plane, will run through the ſpace CD.*
For theſe ſpaces are as *CA* to *CB*, that is, as *CB* to *CD*, by ſimilar triangles.

Cor. 2. *The velocity acquired upon an inclined plane, is to the velocity acquired in the ſame time by falling perpendicularly, as CB to CA, or as CD to CB.*

Cor. 3. *The ſpace deſcribed by a body moving down any plane in a given time, is as the ſine of the plane's elevation.*
For if *CB* be given, *CD* is as the ſine of *CBD* or *CAB*.

Cor. 4. *The ſpaces deſcribed by a body deſcending on any given plane, are as the ſquares of the times.*

PROP.

PROP. XXXV.

The time of a body's deſcending through the plane CD, is to the time of falling through the perpendicular height CE, as the length of the plane DC, to the height CE. 51.

For *DE*, *AB* being perpendicular to *CB*, and *BD* to *AC*. The time of deſcending through *CD* or the perpendicular *CB* : time of deſcending through *CE* :: (by Prop. XIV.) $\sqrt{CB} : \sqrt{CE} :: CD : CE$.

Cor. If the body be made to move back again with the velocity acquired in deſcending, it will aſcend to the ſame height on the plane, and in the ſame time.

For it will be uniformly retarded in aſcending; and in all points will have the ſame velocity in aſcending as deſcending.

SCHOL.

Since the force by which bodies deſcend down an inclined plane, is a uniformly accelerative force; therefore whatever is demonſtrated of falling bodies in Sect. II. holds equally true, in regard to the motion of bodies upon an inclined plane; ſubſtituting the relative weight upon the plane, inſtead of the abſolute weight of the body.

Hence therefore a body projected on an inclined plane, will deſcribe a parabola. And if the velocity of projection upon the plane, be to the velocity of a projectile in the air; as the relative gravity on the plane, to the abſolute gravity. And both projected at the ſame obliquity; the ſame parabola will be deſcribed in both caſes.

PROP. XXXVI.

A body acquires the ſame velocity in deſcending down an inclined plane CD, as by falling perpendicularly through the height of that plane CE. 51.

For draw *DB* perpendicular to *CD*, and the bodies will deſcend through *CD*, *CB* in the ſame time; then (by Cor. 2.

FIG. Prop. XXXIV.) velocity in D : velocity in B :: $CE : CD$, and (Cor. 1. Prop. XIV.) velocity in B : velocity in E :: $\surd CB$:
51. $\surd CE$ :: $CD : CE$. Therefore velocity in D : velocity in E :: $CE : CE$, and therefore the velocities in D and E are equal.

Cor. 1. *A body acquires the ſame velocity in falling from any height, whether it falls perpendicularly, or down an inclined plane of equal height.*

Cor. 2. *Hence the velocities acquired by heavy bodies falling from the ſame height, to the ſame horizontal right line, on any planes whatever, are equal among themſelves.*

Cor. 3. *If the velocities be equal at any two equal altitudes D, E; they will be equal at any other two equal altitudes A, B: And acquire equal increaſes of velocity, in paſſing through EB, DA of equal heights.*

Cor. 4. *The velocities acquired by deſcending down any planes whatever, are as the ſquare roots of the heights.*

PROP. XXXVII.

52. *In a circle whoſe diameter CB is perpendicular to the horizon, a body will deſcend through any cord CD or DB, in the ſame time as it will deſcend perpendicularly through the diameter CB.*

For the angle at D is right, therefore (by Cor. 1. Prop. XXXIV.) the time of deſcending through CD will be equal to the time of deſcending perpendicularly through CB. Draw CE parallel to DB, then will CE be equal to DB; and a body will deſcend through the cords CE, DB in the ſame time. But the time of deſcending through CE is the ſame as falling through the diameter. Therefore the time of deſcending through any cord CD, DB, is the ſame as falling through the diameter CB.

Cor. 1. *The times of deſcending through all the cords of a circle, drawn from either point C or B, are equal among themſelves.*

Cor. 2. *The velocity acquired by deſcending through any cord CD, or DB, is as the length of the cord.*

For draw DF perpendicular to CB, then $CD = \sqrt{CB \times CF}$, FIG. and $DB = \sqrt{CB \times BF}$; and (by Prop. XXXVI.) a body acquires the ſame velocity in deſcending through CD, as in falling through CF, but this (by Cor. 1. Prop. XIV.) is as $\sqrt{CF}$, that is as CD. Alſo a body acquires the ſame velocity through DB as FB, and that is as $\sqrt{BF}$, or as DB. 52.

Cor. 3. *But a body will deſcend ſooner through the ſmall arch of a circle, than through its cord TB.*

For if BG, TG be two tangents, then the relative gravity at T in the arch and cord, will (by Cor. 1. Prop. XXXI.) be as the ſines of the angles TGO, TBO, or as BT and TG, or BG, that is nearly as two to one when the arch is very ſmall. And the accelerative force in the circle being double to that in the cord; therefore the velocity will be greater in the arch, and the time of deſcription ſhorter, though their lengths are nearly the ſame.

PROP. XXXVIII.

If a body deſcends freely along any curve ſurface, and another body deſcends from the ſame height in a perpendicular right line; their velocities will be equal at all equal altitudes. 53.

Let a body deſcend from A towards C perpendicular to the horizon BC; and another deſcend through the curve ſurface AKB. Divide AC into an infinite number of equal parts, at the points D, E, F, *&c.* to which draw lines parallel to BC, interſecting the curve in I, K, G, *&c.* then the curve line AKB will be divided into an infinite number of parts, IK, KG, *&c.* which may be taken for right lines; or the curve ſurface into an infinite number of planes, joining at I, K, G, *&c.*

Now if the velocities be ſuppoſed to be equal in any correſpondent points as I and D, then (by Cor. 3. Prop. XXXVI.) they will be equal in K and E, after the deſcent through IK; and being equal in K and E, they will alſo be equal in G and F, after the deſcent through KG, and ſo on. Therefore ſince the motion begins in A, they will acquire equal velocities in deſcending through the firſt plane, and likewiſe through the 2d, 3d, 4th, *&c.* And therefore the velocities will be equal in all correſpondent points I and D, K and E, G and F, *&c.* and at B and C.

FIG. 53. *Cor.* 1. *Therefore if a body be ſuſpended by a ſtring, and by oſcillating deſcribes any curve AB; or if it is any way forced to move in any poliſhed, and perfectly ſmooth ſurface AB; whilſt another body aſcends or deſcends in a right line. Then if their velocities be equal at any one equal altitude; they will be equal at all other equal altitudes.*

For the ſame thing is effected by the ſtring of the pendulous body, as by the ſmooth ſurface of a poliſhed body.

Cor. 2. *Hence a body oſcillating in any curve line whatever, acquires the ſame velocity in the curve; as if it had fallen perpendicularly from the ſame height. And therefore the velocity in any point of the curve, is as the ſquare root of the height deſcended.*

Cor. 3. *And a body after its deſcent through any curve, will aſcend to the ſame height in a ſimilar and equal curve, or even in any curve whatever. And the velocities will be equal at all equal altitudes. And the aſcent and deſcent will be in the ſame time, if the curves are the ſame.*

For the forces that generated the motion in deſcending, will equally deſtroy it in aſcending, and therefore they will loſe equal velocities by aſcending equal heights. And if the curves are ſimilar and equal, every particle of the curve will be deſcribed with the ſame velocity, and therefore in the ſame time, whether aſcending or deſcending.

Cor. 4. *This Prop. is equally true, whether the curve AKB be in one plane perpendicular to the horizon, or in ſeveral planes IK, KG, &c. winding about in nature of a ſpiral.*

PROP. XXXIX.

54. *The times of deſcent through two ſimilar parts of ſimilar curves, are in the ſubduplicate ratio of their lengths, ab, AB.*

Divide both curves into an equal number of infinitely ſmall parts, ſimilar to each other; and let *bc*, *BC*, be two of them, ſimilarly poſited; and draw *rb*, *RB*, perpendicular to *ab*, *AH*. By Prop. III. the ſpace deſcribed is as the time and velocity, and the time of deſcribing any ſpace, is as the ſpace directly and velocity reciprocally. By Cor. 2. Prop. XXXVIII. the velocities in

FIG. 54. in b and B are as $\sqrt{rb}$ and $\sqrt{RB}$, that is becauſe arb, ARB are ſimilar, as $\sqrt{ab}$ and $\sqrt{AB}$. Therefore the time of deſcribing bc : to time of deſcribing BC :: $\frac{bc}{\sqrt{ab}} : \frac{BC}{\sqrt{AB}} :: \frac{ab}{\sqrt{ab}} : \frac{AB}{\sqrt{AB}} :: \sqrt{ab} : \sqrt{AB} :: \sqrt{ad} : \sqrt{AD}$, becauſe the curves are ſimilarly divided. Whence, by compoſition, the whole time of deſcribing ab : whole time of deſcribing AB :: is in the ſame given ratio of $\sqrt{ab} : \sqrt{AB}$, or $\sqrt{ad} : \sqrt{AD}$.

Cor. 1. *Hence if two pendulums deſcribe ſimilar arches, the times of their vibrations, are as the ſquare roots of their lengths: Or the lengths as the ſquares of the times of vibration.*

For let hd, Hd, be the lengths of the pendulums; then becauſe the figures are ſimilar, it is $ad : AD :: hd : HD$.

Cor. 2. *If a pendulum vibrates in a circle, the velocity in the loweſt point, is as the cord of the arch it deſcribed in deſcending.*

For (by Cor. 2. Prop. XXXVIII.) it acquires the ſame velocity in the arch as in the cord; and (by Cor. 2. Prop. XXXVII.) the velocity in the cord, is as the cord.

Cor. 3. *The lengths of pendulums vibrating in ſimilar arches, are reciprocally proportional to the ſquares of the number of their vibrations, in a given time.*

PROP. XL.

55. *If a pendulum vibrates in a cycloid, the time of one vibration, is to the time of a body's falling perpendicularly through half the length of the pendulum, as the circumference of a circle to the diameter.*

Let ADa be the cycloid, FD its axis, FGD the generating circle. Let the body deſcend from H, and in vibrating deſcribe the arch Hdh. Divide HD into innumerable ſmall parts, and let Bb be one of them. Through H, B, b, draw HMh, BL, bl, perpendicular to the axis FD. About the diameter MD deſcribe the ſemi-circle MLD; and from its center Q, draw QL, alſo draw LP parallel to MD, and DE, DG, GE.

The triangles CDG, GDE are ſimilar, and $CD \times DE = GD^2$. Alſo the triangles QLN and lLP are ſimilar, and $NL : PL :: QL : Ll$, and $2Nl : Nn :: MD : Ll$. And ſince by the nature of the cycloid, the tangent in B is parallel to the arch GD, therefore Gg is equal and parallel to Bb.

Now ſuppoſe a body to deſcend from E through the inclined plane ED, ſince this is a motion uniformly accelerated, therefore (by

FIG. 55. (by Cor. 1. Prop. VI.) it would, in the time of its fall, deſcribe $2ED$, with the velocity acquired in D. And ſince (by Cor. Prop. III.) the times are as the ſpaces directly, and velocities reciprocally; and (by Cor. 2. Prop. XXXVIII.) the velocities are as the ſquare roots of the heights; therefore it will be, as time of deſcribing ED : time in Cc :: $\frac{2ED}{\sqrt{MD}} : \frac{C}{\sqrt{MN}}$ :: $\frac{2MD}{\sqrt{MD}}$ or $2\sqrt{MD} : \frac{Nn}{\sqrt{MN}}$ :: $2\sqrt{MD \times MN} : Nn$; by ſimilar triangles.

Again, when the velocity is given, the time is as the ſpace deſcribed. Therefore it will be, as time in Cc : time in Bb :: $Cc : Bb$ or Gg :: $CD : GD$ or $\sqrt{CD \times DE}$ :: $\sqrt{CD} : \sqrt{DE}$ :: $\sqrt{DN} : \sqrt{DM}$; by ſimilar triangles. Therefore ex equo, time in ED : time in Bb :: $2\sqrt{MD \times MN \times DN} : Nn\sqrt{DM}$ :: $2\sqrt{MN \times DN}$ or $2NL : Nn$ :: $MD : Ll$. Therefore, by compoſition it is, as time in ED : time in the arch Hb :: MD : arch Ml. And as the time in ED : whole time in HD :: MD : ſemi-circumference MLD.

And ſince the time of deſcending through HD is equal to the time of deſcending through Db: and (by Prop. XXXVII.) the time of deſcending through ED is equal to the time in the diameter FD. And $2FD$ is $= DV$, the length of the pendulum (being the radius of curvature in D); therefore as the time of falling through half the length of the pendulum FD : time in HDb, or time of one vibration :: diameter MD : circumference $2MLD$.

Cor. 1. *Hence all vibrations great and ſmall, are performed in equal times. And in unequal arches the velocities generated, and the parts deſcribed, and thoſe to be deſcribed, in the ſame time, will always be as the whole arches; and in any arch HD, the accelerative force at any point B, will be as the length BD from the bottom.*

For the deſcent through HD is always the ſame, wherever the point H is taken. Alſo (by the nature of the cycloid) the tangent at B is parallel to GD, and $BD = 2DG$. And (by Cor. 2. Prop. XXXI.) the relative weight on GD (which is the accelerating force along GD, or the tangent at B) is as $\frac{ND}{GD}$, that is as $\frac{GD}{FD}$, or $\frac{2GD}{FD}$, or as $\frac{BD}{FD}$, or as BD, becauſe FD is given. Whence the accelerating force being always as the diſtance from the bottom, therefore, in any two arches, the ve-

FIG. 55. locities generated every moment, and the parts continually deſcribed, will be as theſe forces; that is, as the whole arches. And conſequently the ſpaces deſcribed, and the velocities generated, in any time, will be as the whole arches; and therefore the parts to be deſcribed will alſo be as the wholes.

Cor. 2. *The time of deſcent in HB, to the time of deſcent in HD, is as the arch ML, to the ſemi-circumference MLD.*

Cor. 3. *The velocity of the pendulum in any point B, is as* $\sqrt{DH^2-DB^2}$, *or* $\sqrt{HB\times BDh}$.

For (by Cor. 1. Prop. XIV.) the ſquare of the velocity in B is as MN, that is as $MD-ND$, or $\frac{ED^2-GD^2}{DF}$ or DH^2-DB^2; becauſe $DH=2DE$, and $DB=2GD$, by the nature of the cycloid; and DF is given.

56. *Cor.* 4. *If the length of the pendulum VD be made double the axis FD; and ARV, arV, be two ſemi-cycloids equal to AHD, and ſo placed that the vertex (as D) be at A and a. Then the pendulum VH vibrating between the cycloidal cheeks ARV, arV; the point H will deſcribe the cycloid AHDha; and the time of its vibration will be* 3.1416 × *time of falling through FD, half the length of the pendulum.*

All this follows from the nature of the cycloid.

Cor. 5. *Hence alſo it appears from experiments on pendulums, that at the ſurface of the earth, a heavy body will deſcend through a ſpace of* 16 $\frac{1}{12}$ *Engliſh feet nearly, in one ſecond of time.*

For it is found by obſervations upon clocks, that a pendulum 39, 13 inches long, vibrates once in a ſecond; therefore $\frac{1}{3\cdot1416}$ = time of a body's falling through FD or $\frac{39\cdot13}{2}$ inches. And conſequently (by Prop. XIV.) the ſpace fallen through in one ſecond will be $\frac{39\cdot13}{2}\times\overline{3\cdot1416}^2=193{,}096$ inches $=16.0913$ feet. Yet a pendulum vibrating freely will be ſomething longer in vibrating than a clock, becauſe the palate wheel of the clock acting againſt it, takes off ſomething from its aſcent, and makes it return ſooner, or ſhortens the time.

Cor. 6. *Hence alſo if pendulums of the ſame quantity of matter and any lengths, be acted on by different forces of gravity, their lengths will be as the forces of gravity, and the ſquares of the times of vibration.*

For the times of vibration are in a given ratio to the times of deſcent through half the lengths of the pendulums. And (by Cor. 3. Prop. V.) if the matter be given, the velocity generated

FIG. 56. nerated in deſcending bodies is as the force and time; and (by Prop. VI.) the ſpace deſcended is as the velocity and time, that is as the force and ſquare of the time. Therefore half the length of the pendulum, is as the force, and ſquare of the time of deſcending half its length; whence the length is as the force and ſquare of the time of vibration.

Cor. 7. *From the motion of pendulums it alſo follows, that in any one place, the quantity of matter in any body is proportional to its weight.* For it is certain from experience that pendulums of equal length, whatever quantities of matter they contain, vibrate in the ſame time. Therefore they will deſcend through half the length of the pendulum in the ſame time; and conſequently would acquire equal velocities in the ſame time. Therefore (by Prop. V.) the velocity and time being given, the quantity of matter is as the force of gravity.

Cor. 8. *Hence it alſo follows, that there are vacuities or empty ſpaces in bodies.* For ſince (by Cor. 7.) the quantity of matter is as the weight of the body, if it were true that there is an abſolute *plenum*, all bodies of the ſame bulk muſt be of equal weight; which is contrary to all experience.

PROP. XLI.

57. *If a pendulum AT oſcillates in a circle TRQ, and in the mean time be acted on in the ſeveral points T, by a force tending perpendicular to the horizon, which is to the uniform force of gravity; as the arch TR, is to the ſine TN: The times of all vibrations will be equal, whether great or leſs.*

For from any point T draw TZ perpendicular to the horizon, and TY a tangent to the circle in T; and let AT expreſs the uniform force of gravity, TZ the variable force at T; draw ZY perpendicular to TY. Then the force TZ will be reſolved into the two TY, YZ. Of which YZ, acting in direction AT, does not at all change the motion of the body. But the force TY directly accelerates its motion in the circle TR. The triangles ATN, ZTY are ſimilar, and $TZ : TA :: TY : TN$; but (by ſuppoſition) $TZ : TA ::$ arch $TR : TN$; therefore $TY =$ arch TR: that is, the force TY is as the arch to be deſcribed TR. Therefore if AT, At, be let fall together from the points T, t; the velocities generated in equal times, will be as the forces TY, ty; that is, as the arches TR, tR, to be deſcribed. But the parts deſcribed at the beginning of the motion, are as the velocities; that is, as the wholes to be deſcribed at the beginning; and therefore the parts which remain to be deſcribed, and the ſubſequent accelerations proportional to theſe parts, are alſo as the wholes, *&c.* Therefore the velocities generated, and the parts deſcribed with theſe velocities, and the parts to be deſcribed, are always as the wholes. And therefore the parts to be deſcribed, being every where as the velocities they are deſcribed with, will be deſcribed in equal times and vaniſh together; that is, the two bodies oſcillating will arrive at the perpendicular *AR* together. FIG. 57.

Cor. 1. *Hence that the vibrations in a circle may be iſocronal; the force* TZ *muſt be* $= \frac{TR}{TN} \times$ *gravity.*

Cor. 2. *Hence if a pendulum vibrates by the force of gravity only, the times of vibration, in very ſmall different arches, will be very nearly equal.*

For in ſmall arches the ratio of the arch to the cord is nearly a ratio of equality.

Cor. 3. *But the time of vibration in larger arches, is greater than the time in leſs arches of a circle.*

For the gravity at *T* being leſs than the iſocronal force, the body will be longer in deſcribing that arch.

Cor. 4. *Hence alſo if a pendulum vibrates in the ſmall arch of a circle, the time of one vibration is to the time of a body's falling through twice the length of the pendulum, as half the circumference of a circle to the diameter.*

For *AR* is the radius of curvature of a cycloid, whoſe axis is $\frac{1}{2}$ *AR*. Therefore the circle and cycloid coincide at *R*, and the ſmall arches of both will be deſcribed in the ſame time; that is, as expreſſed by Prop. XL. only here we take twice the length of the pendulum and half the circumference, which comes to the ſame thing, by Cor. 1. Prop. XXXIX.

SCHOL.

In theſe propoſitions, the vibrating body is ſuppoſed to be very ſmall, and is therefore conſidered only as a point. But if it be of any determinate bigneſs, the point to which the length of the pendulum is meaſured, is not in the middle or center of gravity of the body; but in another place, and is called the center of oſcillation, as will appear in the VI. Section.

FIG. 57. It has been proved, that the ſame pendulum is longer in vibrating in a large arch of a circle than in a ſmall one. And it may be computed, that if a pendulum vibrates ſeconds in an extremely ſmall arch; and C be the length of inches of the cord of any arch A; then $3\frac{1}{2}CC$ will be the ſeconds loſt in 24 hours by vibrating in the arch $2A$.

And if a pendulum vibrates ſeconds in an arch $2a$, and c be the cord of a, or of half the whole arch. Then $3\frac{1}{2}\times\overline{CC-cc}$ will be the ſeconds loſt in 24 hours, by vibrating in the arch, the cord of whoſe half is C.

Alſo if the bob of ſuch a pendulum can be ſcrewed up or down; and you put $n=$number of threads of the ſcrew contained in an inch, $y=$time in minutes that the clock gains or loſes in 24 hours. Then it follows, by the theory of pendulums, that $\frac{2}{37}ny$ will be the number of threads or revolutions of the nut, that the bob is to be let down or raiſed up, to beat ſeconds.

* * *

SECT. V.

Of the center of gravity and its properties.

PROP. XLII.

If a line be drawn from the center of gravity of a body perpendicular to the horizon; and this perpendicular falls within the baſe upon which the body reſts, the body will ſtand; but if it falls without the baſe, it will fall down.

CASE I.

Let *C* be the center of gravity, *CD* perpendicular to the horizon, falling within the baſe *BEFG*; draw *RC*, and ſuppoſe the whole body ſuſpended at the point *C*; then (by Def. 12.), the body will be in equilibrio, and remain at reſt upon *DC*. Now take away *DC*, and ſuppoſe the body to be ſupported only upon the line *RC*, moveable about *R*; then (by Ax. 8.) the body *AB*, together with the line *RC* will endeavour to deſcend from the poſition *RC* towards *D*. Alſo, for the ſame reaſon, the body and the line *CS* will endeavour to deſcend from the poſition *CS* towards *D*; but as theſe two motions oppoſe each other, the body will be ſuſtained by the points *R*, *S*, and therefore it will ſtand. And the ſame is true of every two oppoſite points *R*, *S*. FIG. 58.

CASE II.

But if *CD* fall without the baſe, then the line *RC* and the body at *C* will endeavour to deſcend towards *D*; alſo the body *C* and line *CS* will endeavour to deſcend towards *D* likewiſe; and as this motion does not oppoſe the other, there will be nothing to ſupport the body; therefore it muſt neceſſarily fall towards *D*. 59.

FIG. 59. *Cor.* 1. *Hence it follows, that if the center of gravity of a body be ſupported, the whole body is ſupported. And the center of gravity of the body muſt be eſteemed the place of the body. And if it be ſuſtained by any leaver or beam, its place is at the point where the beam is cut by a line drawn from the center of gravity perpendicular to the horizon.*

Cor. 2. *All the gravity of a body, or the force it endeavours to deſcend with, is collected into the center of gravity; and therefore whatever ſuſtains the center of gravity, ſuſtains the whole weight. And the deſcent of a body muſt be eſtimated by the deſcent of its center of gravity.*

Cor. 3. *Hence alſo the larger the baſe is, upon which a body ſtands, and the further within it the center of gravity lies, the firmer the body will ſtand, and the more difficult to be removed. On the contrary, the leſs the baſe, or the leſs the center of gravity falls within it, ſo much the eaſier it is to be moved out of its place.*

60. *Cor.* 4. *If a body be laid upon a plane GF, and one end F gradually raiſed up, the body will ſlide down the plane, if the perpendicular CD fall within the baſe; but if it fall without, it will roll down.*

PROP. XLIII.

61. *The common center of gravity C of two bodies A, B, is in the right line joining their centers of gravity; and the diſtance of either body from the common center of gravity, is reciprocally as the quantity of matter in it.*

Let A, B be the centers of gravity of A and B, and ſuppoſe AB to be an inflexible right line, or leaver; and C the fulcrum. Then if C be the center of gravity of the bodies A, B; thoſe bodies (by def. 12.) will be in equilibrio. And conſequently (by Cor. 4. Prop. XIX.) $AC : CB :: B : A$.

Cor. 1. *If there be never ſo many bodies, the common center of gravity of them all, is in the right line drawn from the center of gravity of any one, to the common center of gravity of all the reſt; and it divides*

divides this line into two parts, reciprocally as that body to the ſum of all the reſt of the bodies. FIG. 61.

For let D be another body, and let B and A be placed in C, then will $C : D :: DE : CE$. And ſo on for more bodies.

Cor. 2. *If ſeveral bodies A, B, D, E, F, be in equilibrio upon a ſtreight leaver AF, then the fulcrum C is at the common center of gravity of all thoſe bodies.* 62.

PROP. XLIV.

If there be ſeveral bodies, A, B, D, E, F; and if any plane PQ be drawn perpendicular to the horizon; the ſum of the products of each body multiplied by its diſtance from that plane, if they are all on one ſide; or their difference, if on contrary ſides; is equal to the ſum of all the bodies multiplied by the diſtance of their common center of gravity from that plane. 63.

Draw lines perpendicular and parallel to the plane PQ as in the fig. and let C be the center of gravity. Then (by Cor. 3. Prop. XIX.) the force of all the bodies to move the plane PQ about R, will be $mD\times D+oE\times E+rF\times F-Ak\times A-Bl\times B$. That is $\overline{dC+RC}\times D+\overline{eC+RC}\times E+\overline{RC-fC}\times F-\overline{aC-RC}\times A-\overline{bC-RC}\times B$, or $dC\times D+eC\times E-fC\times F-aC\times A-bC\times B+RC\times\overline{D+E+F+A+B}$. But becauſe C is the center of gravity of the bodies, therefore (By Prop. XX.) $dC\times D+eC\times E=fC\times F+aC\times A+bC\times B$; therefore we have $mD\times D+oE\times E+rF\times F-Ak\times A-Bl\times B=RC\times\overline{A+B+D+E+F}$.

Cor. 1. *This Prop. is equally true for any plane whatever.* For ſuppoſe the plane and the bodies to be put into any oblique poſition, all the diſtances will remain the ſame as before.

Cor. 2. *If any plane be drawn through the common center of gravity C, of any number of bodies A, B, D, &c. and each body be multiplied by the diſtance of its center of gravity from that plane; the ſum of the products on each ſide are equal: $A\times aC+B\times bC+F\times fC=D\times dC+E\times eC$.*

For the diſtance of a body muſt be eſtimated by the diſtance of its center of gravity.

Cor.

FIG. 63. *Cor.* 3. *Hence alſo the ſum (or difference) of the products of each particle of a body multiplied by its diſtance from any plane whatever, is equal to the whole body multiplied by the diſtance of its center of gravity from that plane. And if the plane paſs through the center of gravity, the ſums of the products on each ſide are equal.*

Cor. 4. *The ſum of the forces of a ſyſtem of bodies is the very ſame, as if all the bodies were collected into their common center of gravity, and exerted their ſeveral forces there.*

For the ſum of all the forces are $mD\times D+oE\times E$, &c. or $RC\times\overline{A+B+D+E+F}$.

Cor. 5. *And the ſame is true of any forces whatever, with regard to the center of gravity of thoſe forces. And therefore if ſeveral forces act in parallel directions; the ſum of all theſe forces will be equivalent to one ſingle force; and their common center of gravity, the place where it acts.*

80. *Cor.* 6. *If a circle be deſcribed about the center of gravity G, of a ſyſtem of bodies A, B, C; and any point S be taken at pleaſure in the circumference; then* $SA^2\times A+SB^2\times B+SC^2\times C$, *is a given quantity. And the ſame holds true for the ſurface of a ſphere, and the bodies not all in one plane.*

For draw *SG*, on which let fall the perpendiculars *Aa*, *Bb*, *Cc*. Then (by Eucl. II. 12, 13.) $SA^2\times A+SB^2\times B+SC^2\times C=\overline{SG^2+GA^2+2SG\times Ga}\times A+\overline{SG^2+GB^2-2SG\times Gb}\times B+\overline{SG^2+GC^2+2SG\times Gc}\times C$. But (by Cor. 2.) $Ga\times A-Gb\times B+Gc\times C=0$, and all the reſt are given quantities.

PROP. XLV.

64. *If there be ſeveral forces in one plane, acting againſt one another in the point C, whoſe quantities and directions are CA, CB, CD, CE, CF; and if they keep one another in equilibrio: I ſay C is the common center of gravity of all the points A, B, C, D, E. And anyone of them as EC being produced, will paſs through the center of gravity G of all the reſt.*

Since all the forces are in equilibrio, the ſum of the forces acting againſt EC will deſtroy its effects, and act againſt it in the ſame line of direction. FIG. 64.

Upon EC let fall the perpendiculars Aa, Bb, Dd, Ff. Then any force AC is divided into two Aa, aC. Now as the point C is in equilibrio, all the perpendicular forces Aa, Bb, on one ſide, are equal to all thoſe Dd, Ff, on the other, by Ax. 11. And if the body 1 be ſuppoſed to be ſuſpended at A, B, D, E, F. Then ſince $Aa\times 1+Bb\times 1=Dd\times 1+Ff\times 1$; the center of gravity of the bodies A, B, C, D, (and alſo of all the bodies) is in the line EC. Again it follows from the equilibrium of the forces, that $EC+aC=Cb+Cd+Cf$, by Ax. 11. And therefore if the body 1 be ſuſpended at the points E, a, d, b, f; C is their center of gravity. That is, C is the center of gravity of E, A, B, D, F.

Cor. 1. *If G be the center of gravity of A, B, D, F; then $EC=CG\times$ number of points A, B, D, F.*

For $EC=Cb+Cd+Cf-Ca=CG\times\overline{A+B+D+F}$, or $4CG$, by Prop. XLIV.

Cor. 2. The ſum of all the perpendiculars on one ſide, Aa, Bb $=$ ſum Dd, Ff, on the other ſide of EC. And the ſum of their diſtances CE, Ca, on one ſide $=$ ſum Cd, Cb, Cf, on the other ſide of C.

PROP. XLVI.

If a body be acted on by ſeveral forces A, B, C, D, E, in the parallel directions Aa, Bb, &c. and kept in equilibrio; and if any plane RN be drawn from any point R: The ſum of the forces on each ſide are equal, $A+D=B+C+E$; and the ſum of the products on the other ſide, $Ra\times A+Rd\times D=Rb\times B+Rc\times C-Re\times E$; and the contrary: Where any product lying the contrary way from R, muſt be taken negative. 65.

For ſuppoſe RN to be the plane, acted on by theſe forces; then (by Cor. 5. Prop. XLIV.) the effect of the forces A and D acting at a and d, is the ſame as if they both acted at O, their center

FIG. 65. center of gravity. And the effect of B, C, E, acting at b, c, e, is the ſame as if they all acted at their center of gravity; which, becauſe the body is unmoved, is the ſame point O. And therefore, becauſe of the equilibrium; $A+D=B+C+E$, for the quantity of force.

In reſpect to their places, becauſe O is the center of gravity of A and D, as well as of B, C, and E; therefor (by Prop XLIV) $Ra\times A+Rd\times D=RO\times\overline{A+D}=RO\times\overline{B+C+E}=Rb\times B+Rc\times C-Re\times E$. And on the contrary, if theſe forces be equal, the body will be in equilibrio, by Ax. 9.

66. *Cor.* 1. *If a body FGIH be at reſt whilſt it is acted upon by ſeveral forces, in the ſame plane, whoſe quantities and directions are* pA, qB, rC, sD, tE, *cutting any line RN drawn in the body at* p, q, *&c. and the perpendiculars* Aa, Bb, *&c. are drawn; then I ſay,* 1. *The ſum of the perpendicular forces on each ſide are equal,* $Aa+Dd=Bb+Cc+Ee$. 2. *The ſums of the contrary forces in direction of the line RN are equal,* $pa+qb=rc+sd+te$. 3. *The ſum of the rectangles on each ſide, from any point R, are equal,* $Rp\times Aa+Rs\times Dd=Rq\times Bb+Rr\times Cc+Rt\times Ee$. *But where the points lie the contrary way from R, the rectangles muſt be negative. And when all theſe are equal, the body is at reſt.*

For ſince it is the ſame thing whether any force A act, at A at F, or at p, we will ſuppoſe it to act at p; then if the oblique force pA be divided into the two pa, aA; and the ſame for the reſt: then the ſum of all the forces pa muſt be equal to the ſum of all the contrary forces cr, by Ax. 11. The reſt follows from this Prop.

Cor. 2. *And if a body be kept in equilibrio by ſeveral forces acting at different points, and in different directions, either in the ſame plane, or in different planes; it will ſtill be in equilibrio, by the ſame forces, acting from any one point, and in directions reſpectively parallel to the former.*

For, in the ſame plane, the forces parallel and perpendicular to RN, will remain the ſame as before. And when the directions of any of the forces are out of this plane, all theſe extraneous forces may be reduced to others, one part acting in the plane, the other perpendicular to it; and both theſe remain the ſame in quantity as before. And ſince the forces acting in the plane, kept one another in equilibrio at firſt, they will do the ſame ſtill. And as the parts perpendicular to this plane, alſo kept one another in equilibrio at firſt, they will do

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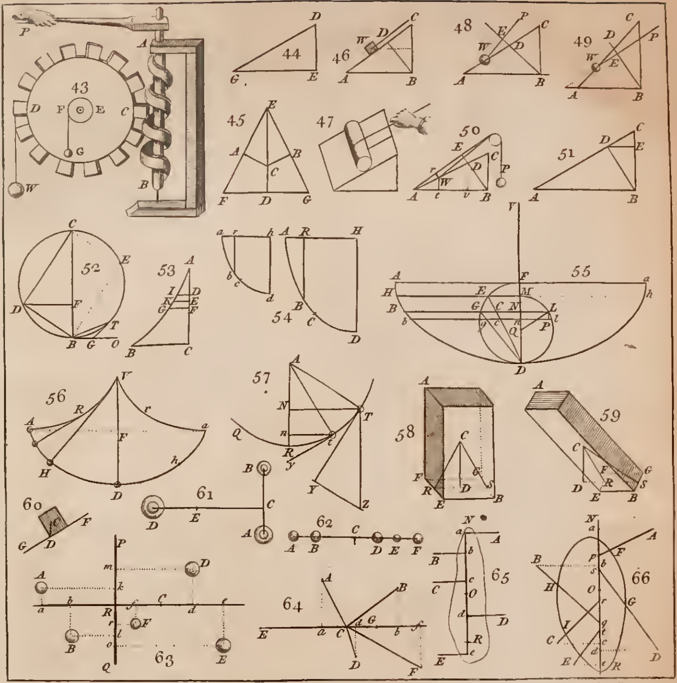
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do the ſame when applied to their common center of gravity, FIG.
or to any other point. 66.

Cor. 3. *If ſeveral forces acting after any manner keep a body unmoved; and any plane whatever be drawn; and the vagrant forces be all reduced to that plane; then all the perpendicular forces on one ſide, are equal to thoſe on the other; and their centers of gravity fall in the ſame point. When this does not happen in all planes, the body will be moved ſome way or other.*

PROP. XLVII.

To find the center of gravity of a ſyſtem of bodies, A, B, C.

Draw any plane *ST*, and from the centers of gravity of all the bodies, draw perpendiculars to this plane, *Aa*, *Bb*, *Cc*; then (by Cor. 3. Prop. XIX.) the forces of *A*, *B*, *C*, at the diſtances *Aa*, *Bb*, *Cc*, from the plane, will be $A\times Aa$, $B\times Bb$, $C\times Cc$. Let *G* be the center of gravity, then the ſum of the forces $A\times Aa+B\times Bb+C\times Cc$ muſt be $=\overline{A+B+C}\times Gg$, the power of all the bodies ſituated in *G* (by Prop. XLIV.) whence the diſtance of the center of gravity from the plane, that is $Gg=\dfrac{Aa\times A+Bb\times B+Cc\times C}{A+B+C}$: where if an of the bodies be ſituate on the other ſide of the plane, the correſpondent rectangles will be negative. 67.

And if the diſtance be in like manner found from the plane *TV*, perpendicular to *ST*, the point *G* will be determined by making the parallelogram *TG* with the reſpective diſtances from thoſe planes.

Cor. 1. *Let b be any body, p any particle in it, d its diſtance from a given plane; then the diſtance of its center of gravity from that plane is* $=\dfrac{\textit{Sum of all the } dp^s}{b}$.

68.

Cor. 2. *To find the center of gravity of an irregular plane figure. Suſpend it by the ſtring AEB, at E; and draw the plumb line ECF. Then ſuſpend it by another point of the ſtring as D, and draw another plumb line through E, to interſect CF; and the point of interſection is the center of gravity.*

FIG. 68. *Cor.* 3. *To find the center of gravity of a flexible body: lay it upon a board whoſe center of gravity is known; lay the center of gravity of the board upon the edge of a priſm; and lay the body upon it, and remove it back or forwards, till it be in equilibrio upon the board.*

SCHOL.

The centers of gravity of ſeveral planes and ſolids have been determined to be as follows.

69. 1. If two lines be drawn from two angles of a *triangle*, to the middle of the oppoſite ſides, the point of interſection is the center of gravity. Therefore the diſtance of the center of gravity from the vertex, is $\frac{2}{3}$ of the line biſecting the oppoſite ſide.

2. In a *Trapezium ABCD*, the center of gravity is found by dividing it into triangles. Find *E*, *G*, the centers of gravity of the triangles *ADB*, *CDB*; and *F*, *H*, the centers of gravity of *ABC*, *ADC*. Then draw *EG*, *FH*, to interſect in *O*, the center of gravity of the Trapezium.

3. The center of gravity of a *right line*, *parallelogram*, *cylinder*, and *priſm*, is in the middle.

4. For the *arch of a circle*, as $\frac{1}{2}$ arch : ſine of $\frac{1}{2}$ arch :: radius : diſtance of its center of gravity from the center.

5. For the *ſector of a circle*, as arch : cord :: $\frac{2}{3}$ radius : diſtance of its center of gravity from the center.

6. For the *parabolic ſpace*, the diſtance of the center of gravity from the vertex is $\frac{3}{5}$ of the axis.

7. In the *cone and pyramid*, the diſtance of the center of gravity from the vertex is $\frac{3}{4}$ of the axis.

8. In a *paraboloid*, the diſtance of the center of gravity from the vertex is $\frac{2}{3}$ of the axis.

9. For the *ſegment of a ſphere*, let $r =$ radius, $x =$ height of the ſegment; then the diſtance of the center of gravity from the vertex is $\frac{8r-3x}{12r-4x}x$.

PROP. XLVIII.

If two or more bodies move uniformly in any given directions, their common center of gravity will either be at reſt, or move uniformly in a right line.

Caſe 1. Let one body ſtand ſtill, and the other move directly to or from it in a right line. Then ſince the center of gravity divides the diſtance, in a given ratio; and the diſtance increaſes uniformly, therefore that center moves uniformly. Now ſuppoſe the other body likewiſe to move in the ſame right line, and any quantity of ſpace to move along with it; then ſince the body is relatively at reſt in this ſpace, the center of gravity, in regard to that ſpace, moves uniformly; to which adding or ſubtracting the uniform motion of that ſpace; the center of gravity will ſtill move uniformly. FIG. 69.

Caſe 2. Let the bodies move in one plane, in the directions DE, AB. Produce their lines of direction till they meet in D. And when one body is in D and E, let the other be in A and B reſpectively. Let H be their center of gravity, when in D and A, and K when in E and B, and draw HK, and make $BP = AD$, and draw EP, and KL parallel to AB. Then DE is to AB or DP in the given ratio of the motion of the bodies; and ſince the $\angle EDP$ is given, therefore all the angles of the triangle EDP are given, and DP will be to PE in a given ratio. But by ſimilar triangles PE is to PL in the given ratio of BE to BK, by the property of the center of gravity: therefore DP is to PL in a given ratio. And all the angles in the triangle DPL are given, and therefore the angle PDL. Therefore the point L is always in the line DL given in poſition. And by the nature of the center of gravity, $DA : DH :: EB : EK :: PB$ or $DA : LK$. Therefore $DH = LK$, whence $DHKL$ is a parallelogram, and HK parallel to DL, and therefore the angle BHK is given; and the center of gravity K is always in the right line HK given by poſition. And becauſe all the angles of the triangles DPL, and DLE are given; therefore the lines DP, DE, DL, that is, AB, DE, HK are in a given ratio; and conſequently the point K moves uniformly along the right line HK. And the demonſtration is in the ſame manner, if one of the bodies B moves from B towards A. 70.

Caſe 3. Let the paths of the bodies AB, DE be in different planes. Through the path AB draw a plane Bde parallel to the path DE, and through DE draw the plane $DdeE$ perpendicular to Bde, produce AB to d, and let Dd, Ee be perpendicular to de. Then the planes DdA, EeB will be perpendicular to the plane edB. Let one body be in A and B, when the other is in D and E reſpectively. Now if the body at D were to move in de, then 71.

FIG. 71. by Caſe 2d, the center of gravity would move uniformly along ſome right line HK; through HK erect the plane $HKkh$ perpendicular to HBK. Then by ſimilar triangles, and the nature of the center of gravity, $Ah : hD :: (AH : Hd :: BK : Ke ::) Bk : kE$. And hk is the path of the center of gravity of the bodies moving in AB, DE. Likewiſe $Dd : Hh :: Ad : AH :: hE : BK :: eE$ or $Dd : Kk$; therefore $Hh = Kk$, and kk is equal and parallel to Hk; therefore the center of gravity of the bodies (moving in AB, DE) moves uniformly through the right line hk.

Caſe 4. The common center of gravity of two bodies, and a third is either at reſt, or moves uniformly in a right line; for theſe two may be put into the place of their center of gravity, which before moved uniformly; and then the center of gravity of the three will move uniformly. Likewiſe the common center of gravity of three bodies and a fourth, will move uniformly in a right line; and ſo on.

PROP. XLIX.

The common center of gravity of two or more bodies, does not change its ſtate of motion or reſt, by any actions of the bodies among themſelves, or by any forces they exert upon one another.

Suppoſe any ſpace in which the bodies are incloſed, to move uniformly along with the center of gravity of the bodies, before the actions of the bodies upon one another; then the center of gravity is at reſt in that ſpace. Now, if two bodies mutually act upon one another, ſince their diſtances from their center of gravity are reciprocally as the bodies; and as action and reaction are equal, the bodies will approach or recede from that center by ſpaces which are in the ſame ratio; therefore the center of gravity will ſtill remain at reſt. And in a ſyſtem of ſeveral bodies, becauſe the common center of gravity of any two acting mutually upon each other, is at reſt: and the actions of all the bodies being the ſum of the actions of every two, it is evident the center of gravity of all the bodies remains the ſame, as if they did not act at all upon one another; and therefore is at reſt in this ſpace, or moves uniformly forward along with it.

Cor. 1. *Hence if a body be projected into free ſpace, if it have any circular motion, this motion will be performed uniformly about an axis paſſing through the center of gravity.* FIG. 71.

For if every particle of the body retained the diſtinct motion firſt impreſſed on it; the common center of gravity of the whole would move in a right line, by the laſt Prop. And ſince the coheſion of the parts of the body retains the particles in one maſs, therefore (by this Prop.) the motion of the center of gravity is not altered, Which it would be if the axis of circular motion did not paſs through the center of gravity, but through ſome other point.

Cor. 2. *And if a body be hurled into the air, its center of gravity will either move in a right line, or deſcribe a parabola; whilſt that body revolves about an axis paſſing through the center of gravity, if it have any circular motion.*

PROP. L.

The ſum of the motions of ſeveral bodies in any given direction, is the ſame as the motion of all the bodies in the ſame direction, moved with the velocity of their common center of gravity.

Let the bodies, A, B, move round the center of gravity C at reſt, to the places a, b; draw BCA, bCa. Then ſince $A : B :: BC : AC :: bc : ac$; therefore the triangles ACa, BCb, are ſimilar, and $< bBc = CAa$, therefore Bb is parallel to Aa, and the bodies move in contrary directions. Alſo ſince $Aa : Bb :: AC : CB :: B : A$, or $Aa \times A = Bb \times B$. Therefore the motions of A, B, in contrary directions are equal, or their motion the ſame way is 0. Now let the ſpace and bodies moving in it, be moved in any direction with any velocity v; it is manifeſt, the motion of each body in that direction will be greater than before, by the quantity of matter × velocity. Therefore the ſum of the motions is now $vA + vB$ or $v \times \overline{A+B}$, that is equal to the ſum of the bodies × velocity of the center of gravity. 72.

After the ſame manner, the motion of three bodies is the ſame as the motion of two of them, moved with the velocity of their common center of gravity, together with the motion of the third;

FIG. 72. third; that is (by what has been ſhown) equal to the ſum of all the three, moved with the velocity of the center of gravity of all the three. And ſo for more bodies.

Cor. The center of gravity of a body muſt be taken for the place of the body. And the motion of any body, or of any ſyſtem of bodies, muſt be eſtimated by the motion of the center of gravity.

PROP. LI.

If two weights on any machine keep one another in equilibrio, if they be any how raiſed or moved by help of the machine, the center of gravity of the weight and power, will always be in the ſame horizontal right line.

For in the leaver, the center of gravity is at the fulcrum, and therefore it neither aſcends nor deſcends. In the wheel and axel, and in the pulley or any combination of pulleys, the weight and power approach or recede from each other, by ſpaces which are reciprocally as the bodies; and therefore their center of gravity is at reſt. And upon any inclined plane, the perpendicular velocities of the power and weight (by Cor. 4. Prop. XXXIII.) are reciprocally as their quantities: and the diſtance of the center of gravity from each, being in the ſame ratio, is alſo at reſt. And univerſally in any combination of theſe, or any machine whatever, where the equilibrium continues; the aſcent and deſcent of the power and weight being reciprocally as their quantities; the center of gravity neither aſcends nor deſcends.

PROP. LII.

73. *If a heavy body AB be ſuſpended by two ropes AC, BD; a right line perpendicular to the horizon, paſſing through the interſection F, of the ropes, will alſo paſs through the center of gravity G, of the body.*

For continue the lines *AC*, *BD* to *F*; then it is the ſame thing whether the lines that ſuſtain the body, act at *C* and *D*, or

FIG. 73. or at *F*, in the ſame directions; ſuppoſe therefore that the body *AFB* is ſuſpended at *F*; then ſince (by Ax. 7.) the body will deſcend as low as it can get; and (by Cor. Prop. L.) the center of gravity muſt be taken for the place of the body; therefore the center of gravity *G* will be in the line *FG* perpendicular to the horizon. And it is the ſame thing if *AC*, *BD*, interſect in a point *F* below the body; for the body cannot be ſupported except the center of gravity *G* be in the perpendicular *GF*.

Cor. Hence, if GN *be drawn parallel to* AC, *the weight of the body, the forces acting at* C *and* D, *are reſpectively as* FG, GN, *and* FN; *or as the ſines of the angles* AFB, GFB, *and* GFA.

Cor. 2. *The lines* AC, DB *and* FG, *are all in one plane perpendicular to the horizon.*

Cor. 3. *If the center of gravity fall not in the line* FG, *the body will not reſt till it fall in that line.*

PROP. LIII.

74. *If any body whatever, as* BC, *or any beam loaded with a weight, be ſupported by two planes* AB, CD, *at* C *and* B; *and from the points* C, B, *the lines* CF, BF, *be drawn perpendicular to theſe planes; and from the interſection* F, *the line* FH *be drawn perpendicular to the horizon, it will paſs through the center of gravity* G, *of the body.*

For ſince the body is ſuſtained by the planes at *B*, *C*, and theſe planes re-act againſt the body in the perpendicular directions *BF*, *CF*; therefore it is the ſame thing as if the body was ſuſtained by the two ropes *BF*, *CF*; and conſequently (by Prop. laſt) *FH* will paſs through *G* the center of gravity of the whole weight.

Cor. 1. *If* EG *be drawn parallel to* CF, *then the whole weight, the preſſure upon the planes* CD, AB; *are reſpectively as* FG, EG, EF; *and in theſe very directions; or as the ſines of the angles* BFC, BFG, *and* CFG.

Cor. 2. *If the line* FG *drawn (from the interſection of the perpendiculars* FC, FB) *perpendicular to the horizon, does not paſs through the center of gravity; the body will not be ſuſtained, but will move till the center of gravity fall in that line.*

FIG. 74. *Cor.* 3. *Hence if the poſition of one plane CD be given, and the poſition of the body CB, and its center of gravity G. The poſition of the other plane AB may be found, by which the body will be ſupported; by drawing CF perpendicular to CD, and GF perpendicular to the horizon; and from F drawing FB; then BA perpendicular to it, is the other plane.*

SCHOL.

Some people have objected againſt the truth of the two laſt Propoſitions, as well as ſome others, though demonſtrably proved. But this ariſes only from their own ignorance of the principles. They that have a mind may ſee this very Propoſition demonſtrated five or ſix different ways in Prop. XXIX. of the ſmall Treatiſe of Mechanics, publiſhed in Vol. VII. of the Cyclomatheſis.

PROP. LIV.

75. *If a heavy body HD, whoſe center of gravity is G, be ſuſtained by three forces A, B, C, in one plane, acting in directions AH, BI, CD. And if FGP be drawn perpendicular to the horizon, and CD produced to cut it in P; and if AH, BI produced, interſect in O; then if OP be drawn; and if EP, OF, be drawn parallel to AO, PC; then I ſay the weight of the body, the three forces A, B, C, are reſpectively as FP, EP, EO, OF.*

Becauſe the line *OP* is unmoved, the point *O* is ſuſtained by three forces in directions *OP*, *OA*, *OB*; which therefore, are as the lines *OP*, *EP*, *OE*. Alſo the point *P* is ſuſtained by three forces in the directions *PO*, *PC*, *GP*; which therefore, are as the lines *OP*, *OF*, *FP*: of which that in direction *FP* is the weight of the body, at *G* the center of gravity. And the forces at *O* and *P*, in the direction *OP* and *PO*, are equal and contrary.

Cor. Hence if any other force inſtead of the weight act at G, in direction GP; then the forces at P, A, B, C, will be reſpectively as FP, EP, EO, OF.

SCHOL.

If one of the forces be given, all the reſt may be found, if they act two and two at different points *O*, *P*. But if five forces act in one plane, two of them muſt be given.

PROP. LV.

If EBDF be any priſmatic ſolid erected upon a plane AD; and if it be cut by any plane AGH. I ſay the ſurface, or ſolid GBDH, cut off by this plane, is reſpectively equal to the ſurface or ſolid EBDF, whoſe altitude is CI, the line paſſing through the center of gravity of the baſe, and parallel to the axis of the ſolid. 76.

I ſhall not demonſtrate this geometrically by meaſuring, but mechanically by weighing them. Suppoſe the periphery, or the baſe, *BD*, to be divided into an infinite number of equal parts, by planes perpendicular to the horizon, and parallel to the axis of the ſolid, and to one another. And imagine *AD* to be a leaver, and let each particle be placed on *AD* where its plane cuts it. Then ſince the force of any particle to move the leaver *AD*, is as that particle multiplied by its diſtance from *A*, (by Cor. 3. Prop. XIX.) Therefore the forces of the equal particles at *B*, *C*, *D*, *&c.* will be as *AB*, *AC*, *AD*, *&c.* and the ſum of all, as the ſum of theſe lines. And becauſe *C* is the center of gravity of all the particles. Therefore the ſum of all, *AB*, *AC*, *AD*, *&c.* = ſum of as many times *AC*; that is, (becauſe the parts of the baſe are given) = *AC* × baſe. But *GB*, *IC*, *HD*, *&c.* are as *AB*, *AC*, *AD*. Therefore all the *GB*'s, *IC*'s, *HD*'s, *&c.* = whole baſe × *IC*. That is the whole ſurface or ſolid *GBDH* = whole ſurface or ſolid *BEFD*.

Cor. 1. *If a line right or curve, or any plain figure whether right-lined or curve-lined, revolve about an axis in the plane of the figure; the ſurface or ſolid generated, is reſpectively equal to the ſurface or ſolid, whoſe baſe is the line or figure given, and height equal to the arch deſcribed by the center of gravity.*

Let *BDdb* be the figure generated. On the baſe *BCD* erect the ſurface or ſolid *BDFE*, and let *C* be the center of gravity. Since the arches *Bb*, *Cc*, *Dd*, are as the radii, *AB*, *AC*, *AD*, that is as *BG*, *CI*, *DH*: therefore if *CI* = *Cc*, then will all the lines *BG*, *CI*, *DH*, *&c.* = all the arches *Bb*, *Cc*, *Dd*, *&c.* that is, the ſurface or ſolid *BDdb* = *BDHG*, that is (by this Prop.) = *BDFE*. 77.

Cor. 2. *Alſo if a curve revolves about any right line drawn through its center of gravity: the ſurfaces generated (either by a partial or total revolution) on oppoſite ſides of the line, will be equal.*

FIG. 77. For by Cor. 2. Prop. XLIV. each part of the curve multiplied by the diſtance of its center of gravity from this line, muſt be equal on both ſides. And by Cor. 2. each ſurface generated, is equal to the curve multiplied by the arch deſcribed at that diſtance; and theſe arches (being ſimilar) are as theſe diſtances. Whence each ſurface is as the curve multiplied by the diſtance of its center of gravity: and therefore they are equal.

SECT. VI

Of the centers of percuſſion, oſcillation, and gyration.

PROP. LVI.

FIG. 78.

Let there be any ſyſtem of bodies A, B, C, conſidered without weight, and moveable about an axis paſſing through S; and if any force f can generate the abſolute motion m in a given time; if the ſame force act at P, perpendicular to PS; the motion generated in the ſyſtem, in the ſame time, revolving about the axis at S, will be $\frac{A\times SA+B\times SB+C\times SC}{A\times SA^2+B\times SB^2+C\times SC^2}\times SP\times m$.

For ſuppoſe PS perpendicular to the axis at S, and to the line of direction PQ. And SA, SB, SC, perpendicular to the axis at S. And ſuppoſe the force f divided into the parts p, q, r, acting ſeparately at P to move A, B, C. Then (by Cor. 3. Prop. XIX.) the bodies A, B, C will be acted on reſpectively with the forces $\frac{SP}{SA}p$, $\frac{SP}{SB}q$, $\frac{SP}{SC}r$.

Since the angular motion of the whole ſyſtem is the ſame, the velocities of A, B, C, are as SA, SB, SC; and their motions as $A\times SA$, $B\times SB$, $C\times SC$; and theſe motions are as their generating forces $\frac{SP}{SA}p$, $\frac{SP}{SB}q$, $\frac{SP}{SC}r$. Whence p, q, r, are as $\frac{A\times SA^2}{SP}$, $\frac{B\times SB^2}{SP}$, $\frac{C\times SC^2}{SP}$: put the ſum of theſe $=s$, and ſince $p+q+r=f$. Therefore $s:f::\frac{A\times SA^2}{SP}:\frac{f\times A\times SA^2}{s\times SP}=p$. And $\frac{SP}{SA}p=\frac{f\times A\times SA}{s}=$ force acting at A. Then $f:m::\frac{f\times A\times SA}{s}:\frac{m\times A\times SA}{s}=$ motion of A. After the ſame manner

FIG. 78. $\frac{m\times B\times SB}{s}$, $\frac{m\times C\times SC}{s}$ are the motions of B and C. Therefore the whole motion generated in the ſyſtem is $\frac{A\times SA+B\times SB+C\times SC}{s}\times m$.

Cor. 1. *If you make* $SO = \frac{A\times SA^2+B\times SB^2+C\times SC^2}{A\times SA+B\times SB+C\times SC}$, *then if all the bodies be placed in O, the motion generated in the ſyſtem, will be the ſame as before, as to the quantity of motion, or the ſum of all the abſolute motions; but the angular velocity will be different.*

For the motion generated in theſe two caſes, will be $\frac{A\times SA+B\times SB+C\times SC}{A\times SA^2+B\times SB^2+C\times SC^2}\times SP\times m$, and $\frac{\overline{A+B+C}\times SO}{\overline{A+B+C}\times SO^2}\times SP\times m$; and if theſe be ſuppoſed to be equal, there comes out $SO = \frac{A\times SA^2+B\times SB^2+C\times SC^2}{A\times SA+B\times SB+C\times SC}$.

Cor. 2. *The angular velocity of any ſyſtem A, B, C, generated in a given time, by any force f, acting at P, perpendicular to PS, is as* $\frac{SP\times f}{A\times SA^2+B\times SB^2+C\times SC^2}$.

For the angular velocity of the whole ſyſtem is the ſame as of one of the bodies A. But the abſolute motion of A is $= \frac{m\times A\times SA}{s}$, and the abſolute velocity of $A = \frac{m\times SA}{s}$; but the angular velocity is as the abſolute velocity directly, and the radius or diſtance reciprocally; therefore the angular velocity of A, and conſequently of the whole ſyſtem, is as $\frac{m}{s}$ or $\frac{m\times SP}{A\times SA^2+, \&c.}$ that is (becauſe m is as the force f), as $\frac{f\times SP}{A\times SA^2+B\times SB^2, \&c.}$

Cor. 3. *Hence there will be the ſame angular velocity generated in the ſyſtem, and with the ſame force, as there would be in a ſingle body placed at P, and whoſe quantity of matter is* $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP^2}$.

For let $P =$ that body, then (by Cor. 2.) ſince f and SP are given; the angular velocities of the ſyſtem and body P, will

be to one another, as $\frac{1}{A\times SA^2+B\times SB^2\&c.}$ to $\frac{1}{P\times SP^2}$. Which be- FIG. 78.
ing ſuppoſed equal, we ſhall have $P=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP^2}$.

Cor. 4. *The angular motion of any ſyſtem, generated by a uniform force, will be a motion uniformly accelerated.*

PROP. LVII.

To find the center of percuſſion of a ſyſtem of bodies, or the point, which ſtriking an immoveable object; the ſyſtem ſhall incline to neither ſide, but reſt as it were in equilibrio.

Through the center of gravity *G* of the ſyſtem, draw a plane 79.
perpendicular to the axis of motion in *S*. And if the bodies are not all ſituated in that plane, draw lines perpendicular to it from the bodies, and let *A*, *B*, *C*, be the places of theſe bodies in the plane. Draw *SGO*, and let *O* be the center of percuſſion. Draw *Af*, *Bg*, *Ch*, perpendicular to *SO*, and *Aad* to *SA*, and make $ad=SA$, and draw *ea* perpendicular, and *de* parallel to *SO*. Then *aA* will be the direction of *A*'s motion, as it revolves about *S*. And the ſyſtem being ſtopt at *O*, the body *A* will urge the point *a* forward, with a force proportional to its matter and velocity; that is as $A\times SA$ or $A\times ad$. And the force wherewith *A* acts at *a* in direction *ea*, is $A\times ea$ or $A\times Sf$. And the force of *A* to turn the ſyſtem about *O*, is $A\times Sf\times aO$ (by Cor. 3. Prop. XIX.) $=A\times Sf\times\overline{SO-Sa}=A\times Sf\times SO-A\times SA^2$. Likewiſe the forces of *B* and *C* to turn the ſyſtem about *O*, is as $B\times Sg\times SO-B\times SB^2$, and $C\times Sh\times SO-C\times SC^2$. And ſince the forces on the contrary ſides of *O* deſtroy one another; therefore $A\times Sf\times SO-A\times SA^2+B\times Sg\times SO-B\times SB^2+C\times Sh\times SO-C\times SC^2=0$. Therefore $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2\&c}{A\times Sf+B\times Sg+C\times Sh\ \&c}$, the diſtance of the center of percuſſion, from the axis of motion. Where note, if any points *f*, *g*, *h*, fall on the contrary ſide of *S*; the correſpondent rectangles muſt be negative, $-A\times Sf$, $-B\times Sg$, &c.

Cor. 1. *If G be the center of gravity of a ſyſtem of bodies A, B, C; the diſtance of the center of percuſſion from the axis of motion, that is,* $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2\&c}{SG\times\overline{A+B+C\&c.}}$.

For by Prop. XLIV. $A\times Sf+B\times Sg+C\times Sh=\overline{A+B+C}\times SG$.

Cor. 2. *The diſtance of the center of percuſſion from the center of gravity G, is* $GO=\frac{GA^2\times A+GB^2\times B+GC^2\times C}{SG\times\overline{A+B+C}}$.

For $A\times SA^2+B\times SB^2+C\times SC^2=A\times\overline{SG^2+GA^2-2SG\times Gf}+B\times\overline{SG^2+GB^2+2SG\times Gg}+C\times\overline{SG^2+CG^2+2SG\times Gh}$, by Eucl.

FIG. 79. II. 12 and 13. But (by Cor. 2. Prop. XLIV.) $-A\times Gf+B\times Gg+C\times Gb=0$; therefore $A\times SA^2+B\times SB^2+C\times SC^2=\overline{A+B+C}\times SG^2+A\times GA^2+B\times GB^2+C\times GC^2$. Whence (by Cor. 1.) SO or $SG+GO=\frac{SG^2\times\overline{A+B+C}}{SG\times\overline{A+B+C}}+\frac{A\times GA^2+B\times GB^2+C\times GC^2}{SG\times\overline{A+B+C}}$.

Cor. 3. *Hence* $SG\times GO=$ *the given quantity* $\frac{A\times GA^2+B\times GB^2+C\times GC^2}{A+B+C}$; *and therefore GO is reciprocally as SG.*

For each of the bodies A, B, C, and their diſtances from G. are given.

Cor. 4. *Hence alſo if SG be given, GO will be given alſo. And therefore if the plane of the motion remain the ſame, in reſpect to the bodies, and the diſtance SG remain the ſame; the diſtance of O from G will remain the ſame alſo.*

Cor. 5. *The percuſſion or quantity of the ſtroke at O, by the motion of the ſyſtem, is the ſame as it would be at G; ſuppoſing all the bodies placed in G, and the angular velocity the ſame. For the ſum of the motions of A, B, C, in the ſyſtem, acting againſt O, is as* $A\times SA\times\frac{SA}{SO}+B\times SB\times\frac{SB}{SO}+C\times SC\times\frac{SC}{SO}=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SO}=SG\times\overline{A+B+C}$ (by this Prop.) *but* $SG\times\overline{A+B+C}$, *denotes the motion of* $A+B+C$, *acting againſt G, or an obſtacle placed there.*

Cor. 6. *In a body or ſyſtem of bodies, oſcillating about a center S, if V be the velocity of O, the center of percuſſion; the ſhock or quantity of the ſtroke at any point P, againſt an obſtacle there, is* $\frac{SG}{SP}\times V\times$ *ſum of the bodies, or of the whole ſyſtem, and therefore are reciprocally as SP. For the velocity of A being denoted by SA, its quantity of motion is* $A\times SA$. *But by the property of the leaver ASP, its quantity of motion againſt P is* $A\times SA\times\frac{SA}{SP}$, or $\frac{A\times SA^2}{SP}$. *In like manner the motions of B, C againſt P are* $\frac{B\times BC^2}{SP}$, $\frac{C\times SC^2}{SP}$, *therefore the ſum of all, or the whole ſhock againſt P, is* $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP}$, *and that is reciprocally as SP, all the reſt being given quantities. But by this* Prop. $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP}=\frac{SG}{SP}\times SO\times\overline{A+B+C}$; *where SO denotes the velocity of O, the ſame as V.*

Cor. 7. *If OT be drawn perpendicular to SO, then OT will be the locus of all the centers of percuſſion.*

For the direction of O is in the line OT; and therefore it is the ſame thing which point of the line OT ſtrikes an obſtacle.

PROP. LVIII.

To find the center of oſcillation of a ſyſtem of bodies, or ſuch a point, in which a body being placed, will vibrate in the ſame time, and with the ſame angular velocity as the whole body.

Let the axis of motion be at S, perpendicular to which draw the plane in which the center of gravity G moves; draw SGO, and let O be the center of oſcillation; draw the horizontal line Sr, and from the bodies A, B, C, draw Aa, Bb, Cc, perpendicular to SO; and alſo Ae, Bn, Cd, Gg, Or, perpendicular to Sr. 81.

Put $s = A \times SA^2 + B \times SB^2 + C \times SC^2$. Then (by Cor. 2. Prop. LVI.) the angular velocity which A, B, C, generate in the ſyſtem by their weight, is $\frac{-Se \times A}{s}$, $\frac{Sn \times B}{s}$, $\frac{Sd \times C}{s}$; and the whole angular velocity generated by them all is $\frac{-Se \times A + Sn \times B + Sd \times C}{s}$. Likewiſe the angular velocity which any particle p, ſituated in O, generates in the ſyſtem, by its weight, is $\frac{Sr \times p}{p \times SO^2}$ or $\frac{Sr}{SO^2}$ or $\frac{Sg}{SG \times SO}$, becauſe of the ſimilar triangle SgG, SrO. But their vibrations, and every part of them, are performed alike; therefore their angular velocities muſt be every where equal; that is $\frac{-Se \times A + Sn \times B + Sd \times C}{s} = \frac{Sg}{SG \times SO}$; whence by reduction $SO = \frac{Sg}{SG} \times \frac{s}{-Se \times A + Sn \times B + Sd \times C}$. But (by Prop. XLIV.) $-Se \times A + Sn \times B + Sd \times C = Sg \times \overline{A + B + C}$. Therefore the diſtance of the center of oſcillation from the axis of motion, $SO = \frac{s}{SG \times \overline{A + B + C}} = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2 \text{ \&c.}}{SG \times A + B + C \text{ \&c}}$ $= \frac{A \times SA^2 + B \times SB^2 + C \times SC^2 \text{ \&c}}{A \times Sa + B \times Sb + C \times Sc \text{ \&c}}$. Where $A \times Sa$, $B \times Sb$, *&c.* muſt be negative, when a, b, *&c.* lie on the contrary ſide of S. And ſince all theſe quantities are the ſame at all elevations of the axis SO; therefore the point O is rightly found; and the ſyſtem has ſuch a point as is required. Likewiſe it appears by Cor. 1. of the laſt Prop. that the center of oſcillation is the ſame with the center of percuſſion.

FIG. 81. *Cor.* 1. *If p be any particle of a body, d its diſtance from S, the axis of motion; G, O, the centers of gravity and oſcillation. Then the diſtance of the center of oſcillation of the body, from the axis of motion,* $SO = \dfrac{\textit{ſum of all the } p\times dd}{SG\times \textit{body}}$.

Cor. 2. *If the bodies A, B, C, be large, and therefore the center of oſcillation of each, not in the center of gravity. Let d, e, f, be the reſpective diſtances of their centers of gravity, and p, q, r, of their centers of oſcillation, from S. Then will the diſtance of the center of oſcillation from S, the axis of motion,* $SO = \dfrac{dpA+eqB\times frC}{A\times Sa+B\times Sb+C\times Sc} = \dfrac{dpA+eqB+frC}{SG\times\overline{A+B+C}}$.

For let a, b, c, be any particles in A, B, C; and x, y, z, their diſtances from S reſpectively. Then by this Prop. $SO = \dfrac{\text{ſum } x^2a + \text{ſum } yyb + \text{ſum } zzc}{SG\times\overline{A+B+C}}$. But $\dfrac{\text{ſum } xxa}{dA} = p$, or ſum $xxa = dpA$, and ſum $yyb = ebB$, and ſum $zzc = frC$; and $SG\times\overline{A+B+C} = Sa\times A+Sb\times B+Sc\times C$.

Cor. 3. *To find the center of oſcillation of an irregular body, ſuſpend it at the given point, and hang up a ſingle pendulum of ſuch a length, that making them both vibrate, they may keep time together. Then the length of this pendulum is equal to the diſtance of the center of ſuſpenſion from the center of oſcillation of the body.*

Cor. 4. *What has been demonſtrated in the laſt Prop. and Cor.* 1, 2, 3, 4. *for the center of percuſſion; holds equally true for the center of oſcillation.*

SCHOL.

I ſhall juſt take notice, that if the diſtance of the axis of ſuſpenſion from the center of gravity SG, be made equal to $\sqrt{\dfrac{GA^2\times A+GB^2\times B+GC^2\times C}{A+B+C}}$; the body will oſcillate in the leaſt time poſſible.

In very ſmall bodies, or any bodies oſcillating at a great diſtance from the axis of motion, the center of oſcillation or percuſſion is in or very near the center of gravity. And the reaſon why the center of oſcillation or percuſſion is not always in the center of gravity; is becauſe the body in vibrating is made to turn about a center. But if it be ſo contrived as always to move parallel to itſelf, without any circular motion, the centers of gravity, of oſcillation, and percuſſion will be the ſame. FIG. 81.

The diſtance of the centers of oſcillation and percuſſion, from the axis of motion, as calculated by Cor. 1. is as follows. Where the axis of motion is at the vertex, and in the plane of the figure.

1. In a right line, ſmall parallelogram, and cylinder, $\frac{2}{3}$ the axis of the figure.

$$\left.\begin{array}{l}\text{2. In a triangle, }\frac{3}{4}\text{ the axis}\\ \text{3. In a plane of a circle, }\frac{5}{4}\text{ the radius}\\ \text{4. In the parabola, }\frac{5}{7}\text{ the axis}\\ \text{5. Pyramid and cone, }\frac{4}{5}\text{ axis}\end{array}\right\}\text{ nearly.}$$

6. In a ſphere, $r =$ radius, $d =$ diſtance of the axis of motion from its center. Then the diſtance of the center of oſcillation from the axis of motion, is $d + \frac{2rr}{5d}$.

PROP. LIX.

82. *To find the center of gyration of a ſyſtem of bodies, or ſuch a point O, as that a given force, acting at a certain place, will in the ſame time generate the ſame angular velocity in the ſyſtem, about an axis SR, as if the whole ſyſtem was placed in O.*

Draw the plane PQS perpendicular to the axis of rotation SR; and let SA, SB, SC be the neareſt diſtances of the bodies A, B, C from the axis SR; and let the force f act at P, in direction PQ perpendicular to PS. Then (by Cor. 2. Prop. LVI.) the angular velocity generated in the ſyſtem by the force f, will be as $\frac{SP \times f}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, and in the ſyſtem placed in O, it will be $\frac{SP \times f}{\overline{A+B+C} \times SO^2}$; and if theſe velocities be made equal, we ſhall have $SO^2 = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A+B+C}$. Whence the diſtance of the center of gyration O from the axis of motion at S, that is $SO = \sqrt{\frac{A \times AS^2 + B \times SB^2 + C \times SC^2}{A+B+C}}$.

83. *Cor.* 1. *Let b = quantity of matter in any body $ABRCS$, p any particle, $d = ap$, its diſtance from the axis of rotation SR: then the ſquare of the diſtance of the center of gyration, from the axis of motion, that is $SO^2 = \frac{\text{ſum of all the } ddp}{b}$.*

FIG. 82. *Cor.* 2. *If any part of the ſyſtem be ſuppoſed to be placed in the center of gyration of that particular part; the center of gyration of the whole ſyſtem will continue the ſame as before.*

For by this Prop. the ſame degree of force which moved this part of the ſyſtem before, along with the reſt, will move it now without any alteration. And therefore if each part of the ſyſtem be collected into its proper center of gyration, the center of gyration of the whole will continue the ſame.

Cor. 3. *If a circle be deſcribed from G, the center of gravity of the ſyſtem; and the axis of rotation be made to paſs through any point S in its periphery; the diſtance of the center of gyration from that point will always be the ſame.*

For (by Cor. 6. Prop. XLIV.) the quantity $A\times SA^2+B\times SB^2+C\times SC^2$ will be given.

Cor. 4. *The diſtance of the center of gyration from the axis of motion, is a mean proportional between the diſtances of the centers of gravity and percuſſion from that axis.*

It follows from this and the laſt Prop.

Cor. 5. *The momentum or quantity of motion of the whole ſyſtem, acting againſt an obſtacle at O the center of gyration, is the ſame as if all the bodies were placed in O, the angular velocity remaining the ſame. For the momenta or quantities of motion are as the forces.*

SCHOL.

It is the ſame thing on whatever ſide of the axis of rotation *SR*, the point *O* or center of gyration be taken, provided it be at its proper diſtance.

By a computation from Cor. 1. the diſtance of the center of gyration from the axis of rotation, in the following bodies will be,

1. In a *right line* or ſmall *cylinder* (revolving about the end) $SO = \text{length} \times \sqrt{\tfrac{1}{3}}$.

2. The *plane* of a *circle*, or *cylinder* (revolving about the axis) $SO = \text{radius} \times \sqrt{\tfrac{1}{2}}$.

3. The *periphery* of a *circle* (about the diameter) $SO = \text{radius} \times \sqrt{\tfrac{1}{2}}$.

4. The *plane* of a *circle* (about the diameter) $SO = \tfrac{1}{2}\,\text{radius}$.

5. The *ſurface* of a *ſphere* (about the diameter) $SO =$ radius $\times\sqrt{\tfrac{2}{3}}$. FIG. 82.

6. A *globe* (revolving about the diameter) $SO =$ radius $\times\sqrt{\tfrac{2}{5}}$.

7. In a *cone* (about the axis) $SO =$ radius $\times\sqrt{\tfrac{3}{10}}$.

If the periphery of a circle revolve about an axis in the center, perpendicular to its plane, it is the ſame thing as if all the matter was collected into any one point in that periphery. And the plane of a circle of double the mater of this periphery, and the ſame diameter, will in an equal time acquire the ſame agular velocity.

If the matter of any gyrating body were actually to be placed in its center of gyration, it ought either to be diſpoſed of in the circumference of a circle, whoſe radius is *SO*, or elſe into two points *O*, diametrically oppoſite, equal and equi diſtant from *S*. For by this means the center of motion *S*, will be in the center of gravity. And the body will revolve without any lateral force towards any ſide.

PROP. LX.

If EF be any body at reſt in free ſpace, G its center of gravity, the points S, O, one the center of ſuſpenſion, the other of percuſſion; and if a moving body B ſtrike directly againſt the point O, the motion generated in the body EF by the ſtroke ſhall be ſuch; that in the time that the body makes one revolution about its center of gravity G, the center of gravity will move forward a ſpace, equal to the circumference of a circle, whoſe radius is SG. 84.

Let the body vibrate about the point *S*, and in a very ſmall time, from the poſition *SGO*, come into the poſition *Sgd*. Now the arches *Gg*, *Od*, will be as the velocities of the points *G*, *O*, vibrating about *S*; therefore when it comes into the poſition *SGO*, if it were diſengaged from the point *S*; the center of gravity *G* would ſtill move forward with the ſame velocity *Gg*; and the body, inſtead of revolving about *S*, would (by Cor. 1 Prop. XLIX.) revolve about *G* with the ſame angular motion as before. Therefore if *rGo* be drawn parallel to *Sgd*, *Gg* will repreſent the velocity of *G*, and *Oo* the velocity of *O* about *G*. And becauſe *Oo*, *Gg* are very ſmall ſimilar arches, therefore their circumferences will be deſcribed in equal times; that is, in the time that *O*, or the body itſelf, makes one revolution

about

FIG. 84. about G; the point G will advance forward a ſpace, equal to the circumference of a circle, whoſe radius is SG.

Now this is the motion acquired by revolving about S. But (by Prop. LVII.) if a body ſo revolving, ſtrikes an immoveable object at O, both the progreſſive and circular motion will be deſtroyed; and the body will be at reſt. It is evident on the contrary, that if a moving body ſtrike the body at reſt in the point O, with the ſame force; the ſame motion will be reſtored again: and is the ſame as above deſcribed.

Cor. 1. *At the beginning of the motion, and alſo after every revolution of the body, when the line SGO comes into its original poſition, ſo as to be perpendicular to the line of direction OB, the point S will be at reſt for a moment.*

For in this poſition, it will be (by this Prop.) as velocity of O about G : velocity of G :: $OG : GS$. And by compoſition, vel. O about G + vel. G that is abſolute vel. O : abſolute vel. G :: $OG + GS$ or $OS : GS$. Therefore ſince the abſolute velocities of O and G are directly as their diſtances from S; it follows that the point S is at reſt.

Cor. 2. *Let a body* $A = \frac{SG}{SO} \times$ body EF. *And if V be the velocity which the body A would receive by the direct ſtroke of B: then I ſay, the abſolute velocity of the body EF (or of its center of gravity G), which it receives by B impinging at O, will be* $\frac{SG}{SO}V$.

For let p be any particle of the body EF, and Sp its diſtance from S. Then (by Cor. 3. Prop. LVI.) if a body = $\frac{\text{ſum of all the } Sp^2 \times p}{SO^2}$ be placed in O, it will receive the ſame angular velocity, by the ſtroke, about S at reſt; as the body EF when ſtruck in O. But (by Cor. 1. Prop. LVIII.) ſum $Sp^2 \times p = SO \times SG \times$ body EF; whence the body $\frac{SO \times SG \times \text{body } EF}{SO^2}$ or A, placed in O, receives the ſame angular velocity about S, as the point O of the body EF. But vel. of O or A : vel. G :: $SO : SG$. For at the beginning of the motion, S is at reſt, by Cor. 1.

Cor. 3. *The velocity loſt in B by the ſtroke, will be* $\frac{\text{body } EF}{\text{body } B} \times \frac{SG}{SO}V$.

For the ſum of the motions of all the bodies, after the ſtroke, is the ſame as the motion of *B* before it, by Prop. X. FIG. 84.

SCHOL.

The point *S* is by ſome called the *ſpontaneous center of rotation*; becauſe the body (or ſyſtem of bodies) at the beginning of the motion, moves as it were of its own accord, or without any compulſion, about the center *S* at reſt.

PROP. LXI.

Let DE be any body, C its center of gravity; and if from the center C, the circle BFS be deſcribed; and if about BFS as an axis, a cord ASBFS, be wound, and the end fixed at A. And if O be the center of oſcillation, in reſpect to the center of ſuſpenſion S. Then if the body deſcend by a rotation round the axis BFS, by unwinding the cord ASBF, &c. then I ſay the ſpace deſcended by the whirling body DE, is to the ſpace deſcended in the ſame time, by a body falling freely, as SC to SO. 85.

Through the point of contact *S* and the center of gravity *C*, draw the horizontal line *SCO*. Then (by Prop. LVIII.) the angular velocity of the body about the point of ſuſpenſion *S*, at the beginning of the motion, will be the ſame as if the whole body was placed in *O*. But if a body was placed in *O*, its velocity generated at the beginning, will be the ſame as of a body falling freely. Therefore drawing *Sco* infinitely near *SCO*, and the ſmall arches *Oo*, *Cc*: then the velocity of *O*, is to the velocity of the center of gravity *C*, as *Oo* to *Cc*, or as *SO* to *SC*; that is, the velocity generated by a body falling freely, is to the velocity of the deſcending body *DE*, as *SO* to *SC*. Now ſince the points *S* and *O* are always in the horizontal line *SCO*, and the radius *SC* is given, and likewiſe (by Cor. 4. Prop. LVII.) the diſtance *CO*. Therefore the velocities of *O* and *C* in any times will always be as *SO* to *SC*: that is, the velocity of a body deſcending freely, is to the velocity of the whirling body *DE*, always in the ratio of *SO* to *SC*. And therefore (by Prop. VI.) the ſpaces deſcribed are in the ſame ratio.

Cor. 1. *The weight of the body DE, is to the tenſion of the cord AS; as SO to CO.*

For

FIG. 85. For let the body be ſupported at O, then ſince B is the center of gravity, therefore (by Cor. Prop L. and Cor. 5. Prop. XIX.) the weight DE is to the preſſure at S, as SO to CO. Now if the point O be let go, the force acting at O will generate a motion about S, whilſt the preſſure at S, and conſequently the tenſion of the cord, is neither increaſed nor decreaſed, but remains the ſame as before.

Cor. 2. *If a circular body as BFS runs down an inclined plane, whilſt the thread ASB unfolds; or if a round body roll down an inclined plane, and by its friction be hindered from ſliding: The ſpace it deſcribes in any time, is to the ſpace deſcribed by a body ſliding down freely without friction, as SC to SO.*

For the forces that generate their motions are both decreaſed, in the ſame ratio, that is as the abſolute gravity to the relative gravity upon the plane; therefore the ſpaces deſcribed will remain in the ſame ratio of SC to SO. And in the rolling body, the friction ſupplies the place of the cord, the ſame as if it had teeth.

Cor. 3. *This motion of the body DE by rotation, is a motion uniformly accelerated. And the tenſion of the cord is always the ſame, through the whole deſcent.*

SCHOL.

86. Let W = weight of a body, S = ſpace deſcribed by a body falling freely. Then the ſpaces deſcribed by rotation or whirling, in the following bodies, as SBF, in the ſame time are,

1. In the circumference of a circle, SBF, or ſurface of a cylinder, ſpace $= \frac{1}{2} S$; tenſion of the ſtring $= \frac{1}{2} W$.

2. In the circumference of a circle, SBF, without weight, and the weight be in the center C; ſpace $= S$, tenſion of the ſtring $= 0$.

3. In the plane of a circle SBF, or a cylinder; ſpace $= \frac{2}{3} S$, and the tenſion of the ſtring $AS = \frac{1}{3} W$.

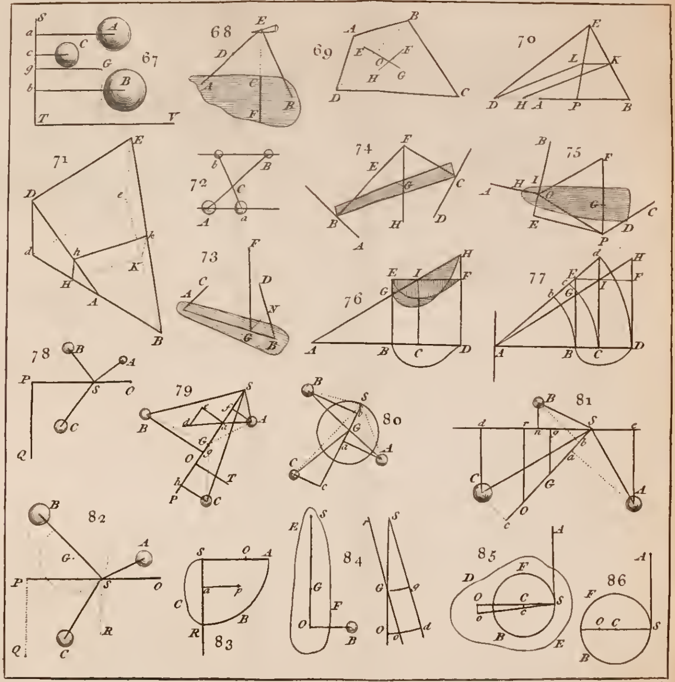
4. In the ſurface of a ſphere SBF; ſpace $= \frac{3}{5} S$, and the tenſion of the ſtring $AS = \frac{2}{5} W$.

5. In a ſphere SBF; ſpace $= \frac{5}{7} S$, and tenſion of the thread $= \frac{2}{7} W$.

SECT.

IG.

87.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Pl. IV. *pa.* 86.

SECT. VIII.

The quantity and direction of the preſſure of beams of timber, by their weights; and the forces neceſſary to ſuſtain them.

FIG. 87.

PROP. LXII.

If a beam of timber be ſupported at C and B, lying upon the wall ACE, with one end. And if G be the center of gravity of the whole weight ſuſtained: and the line FGH be drawn perpendicular to the horizon, and CF and BH to CB, and BF drawn; I ſay,

The weight of the whole body } *FH*
Preſſure at the top C } *BH*
Thruſt or preſſure at the baſe B } *FB,*
are reſpectively as } *and in theſe ſeveral directions.*

If the beam ſupport any weight, the beam and weight muſt be conſidered as one body, whoſe center of gravity is *G*. Then the end *C* is ſupported by the plane *BCE*; and (by cor 3. Prop. LIII.) the other end *B* may be ſuppoſed to be ſuſtained by a plane perpendicular to *BF*; therefore (by Cor. 1. Prop. LIII.) the weight and forces at *C* and *B*, are reſpectively as *FH*, *BH*, and *BF*.

Cor. 1. *Produce FB towards Q, then BQ is the direction of the preſſure at B. And the preſſures at B in directions BQ, FD, DB, are as FB, FD, DB.*

Cor. 2. *Draw Dr perpendicular to BC, and draw CD. Then the weight, preſſure at the top, direct preſſure at bottom, and horizontal preſſure at bottom, are reſpectively as CB, BD, DC, and Dr.*

FIG. 87. For ſince the angles *BCF*, *BDF* are right; a circle deſcribed upon the diameter *BF*, will paſs through *C*, *D*. Therefore $< BCD = BFD$ ſtanding on the ſame arch *BD*. And becauſe the $< GBH$ and $< s$ at *D* are right, $BHF = CBD$; therefore the triangles *FHB* and *CBD* are ſimilar, and the figure *BHDF* ſimilar to the figure, *DBrC*, whence $FH : BH : BF : BD ::$ are as $CD : BD : DC :$ and *Dr*.

Cor. 3. *All this holds true for any force inſtead of gravity, acting in direction GD.*

PROP. LXIII.

88. *If BC be any beam, bearing any weight, G the center of gravity of the whole. And if it lean againſt the perpendicular wall CA. and be ſupported in that poſition; draw BA, CF parallel, and FGD perpendicular to the horizon; and draw FB, then*

The whole weight	*FD*
Preſſure at the top C	*BD*
Thruſt or preſſure at the bottom B	*FB,*
are reſpectively as	*and in the ſame directions.*

For the end *C* is ſuſtained by the plane *AC*; and if the end *B* be ſuppoſed to be ſuſtained by a plane perpendicular to *FB*. Then (by Cor. 1. Prop. LIII.) the weight, and preſſure at top and bottom, are as *DF*, *DB*, *FB*. If you ſuppoſe the end *B* is not ſuſtained by a plane perpendicular to *FB*, the body will not be ſupported at all; by Cor. 2. Prop. LIII.

Cor. 1. *If FB be produced to Q, then BQ is the direction of the preſſure at B. And the perpendicular preſſure at B (FD) is equal to the weight; and the horizontal preſſure at B (BD), is equal to the preſſure againſt C.*

PROP. LXIV.

89. *If a heavy beam, or one bearing a weight, be ſuſtained at C, and moveable about a point C; whilſt the other end B lies upon the wall BE. And if HGF be drawn through the center of gravity G,* *per-*

perpendicular to the horizon, and BF, CH perpendicular to BC, and CF be drawn; then FIG. 89.

The whole weight	*HF*
Preſſure at B	*HC*
Force acting at C,	*CF,*
are reſpectively as	*and in theſe directions.*

For the end *B* is ſuſtained by the plane *CB*, and (by Cor. 2. Prop. LIII.) the end *C* may be ſuppoſed to be ſuſtained by a plane perpendicular to *FC*, or by a cord in direction *CF*. Then ſince *HC* is parallel to *BF*, the weight, force at *C*, preſſure at *B*, are reſpectively as *HF*, *CF*, *HC*; by Cor. 1. Prop. LII. or Cor. 1. Prop. LIII.

Cor. But if inſtead of lying upon the inclined plane at B, the end B laid upon the horizontal plane AB, then the weight, the preſſure at B and C, are reſpectively as BC, GC, and BG; and in this caſe there is no lateral preſſure.

For *BF* will be perpendicular to *BA*, and parallel to *HF*, and conſequently *CF* is alſo parallel to *HF*, therefore (by Cor. 5. Prop. XIX.) the forces at *C*, *G*, *B*, are as *BG*, *BC*, and *CG*.

PROP. LXV.

If a heavy beam BC, whoſe center of gravity is G, be ſupported upon two poſts BA, CD; and be moveable about the points, A, B, C, D. And if AB, DC, produced, meet in any point H, of the line GF, drawn perpendicular to the horizon. And if from any point F, in the line GF, FE be drawn parallel to AB; I ſay, 90.

The whole weight	*HF*
Preſſure at C	*HE*
Thruſt or preſſure at B,	*EF,*
are reſpectively as	*and in theſe directions.*

For the points *A*, *B*, *C*, *D*, being in a plane perpendicular to the horizon, the body may be ſuppoſed to be ſupported by two planes at *B*, *C*, perpendicular to *AB*, *DC*; or by two ropes *BH*, *CH*. And in either caſe, the weight in direction *HG*, the preſſure at *B*, *C*, in directions *HB*, *HC*, are as *HF*, *EF*, and *HE*.

FIG. 90. *Cor.* *Hence, whether a body be ſuſtained by two ropes BH, CH, or by two poſts AB, CD, or by two planes perpendicular to BA, CD: The body then can only be at reſt, when the plumb line HGF paſſes through G, the center of gravity of the whole weight ſuſtained. Or, which is the ſame thing, when AB, DC interſect in the plumb line HGF paſſing through the center of gravity.*

SCHOL.

By the conſtruction of theſe four laſt propoſitions, there is formed the *triangle of preſſure*, repreſenting the ſeveral forces. In which, *the line of gravity* (or plumb line paſſing through the center of gravity) always repreſents the abſolute weight, and the other ſides the correſponding preſſures.

PROP. LXVI.

91. *If ſeveral beams AB, BC, CD, &c. be joined together at B, C, D, &c. and moveable about the points A, B, C, &c. be placed in a vertical plane, the points A, F, being fixt, and through B, C, D drawing ri, sm, tp perpendicular to the horizon. And if ſeveral weights be laid on the angles B, C, D, &c. ſo that the weight on any angle C may be as* $\frac{S.BCD}{S.mCB \times S.mCD}$. *Then all the beams will be kept in equilibrio by theſe weights.*

Produce DC to r. Then (by Cor. 2. Prop. VIII.) $S.<ABC : S.<ABr ::$ weight B : force in direction $BC = \frac{B \times S.ABr}{S.ABC}$; and $S.BCD : S.DCs ::$ weight : force in direction $CB = \frac{C \times S.DCs}{S.BCD}$; which, to preſerve the equilibrium, muſt be equal to the force in direction BC, that is $\frac{B \times S.ABr}{S.ABC} = \frac{C \times S.DCs}{S.BCD}$; whence $B : C :: \frac{S.ABC}{S.ABr} : \frac{S.BCD}{S.DCs}$. And by the ſame way of reaſoning, $C : D :: \frac{S.BCD}{S.BCs} : \frac{S.CDE}{S.EDt}$. Therefore, ex equo, weight B : weight $D ::$

$$\frac{S.ABC}{S.ABr \times S.BCs} : \frac{S.CDE}{S.DCs \times S.EDt} :: \frac{S.ABC}{S.ABi \times S.CBi} : \frac{S.CDE}{S.CDp \times S.EDp}.$$

Cor. 1. *Produce CD, ſo that Dw may be equal to Cr, and draw wx parallel to Dp, cutting DE in x. Then the weight C, the forces in directions CB and CD, are as rB, CB and Cr reſpectively. And weight C is to the weight D, as Br to wx.* FIG. 91.

Cor. 2. *The force or thruſt at C, in direction CB, or at B, in direction BC, is as the ſecant of the elevation of the line BC above the horizon.*

For, force in direction *CB* : force in direction *CD* :: *CB* : *Cr* :: *S. CrB* or *rCm* or *sCD* : *S. rBC* :: coſ. elevation of *CD* : coſ. elevation of *CB* :: ſec. elevation of *CB* : ſec. elevation *CD*; becauſe the ſecants are reciprocally as the coſines.

Cor. 3. *Draw Cp, Dm parallel to DE, CB, then the weights on C and D to preſerve the equilibrium, will be as Cm to Dp. And therefore if all the weights are given, and the poſition of two lines CD, DE; then the poſitions of all the reſt CB, BA, &c. will be ſucceſſively found.*

For let the force in direction *CD* or *DC* be *CD*, then *Cp* is the force in direction *DE*, and *Dm*, in direction *CB*. And *Dp* or the weight *D*, is the force compounded of *DC*, *Cp*; and *Cm* or the weight *C* is the force compounded of *CD*, *Dm*, by Cor. 2. Prop. VII.

Cor. 4. *If the weights lie not on the angles B, C, D, &c. let the places of their centers of gravity be at g, h, k, l. And let g, h, k, l, alſo expreſs their weights. And take the weight* $B = \frac{Ag}{AB}g + \frac{hC}{BC}h$, $C = \frac{Bh}{BC}h + \frac{kD}{CD}k$, $D = \frac{Ck}{CD}k + \frac{lE}{DE}l$, *&c. then B, C, D, &c. will be the weights lying upon the reſpective angles.*

This is evident by Cor. 5. Prop. XIX.

Cor. 5. *If the weights were to act upwards, in the directions mC, pD, &c. or, which is the ſame thing, if the figure A, B, C, D, E, F, was turned upſide down, and the weights remain the ſame, and the points A, F be fixt as before. All the angles at B, C, D, &c. and conſequently the whole figure will remain the ſame as before; and that whether the lines AB, BC, CD, &c. be flexible or inflexible cords or timbers.*

This will eaſily appear by the demonſtration of the Prop. For the ratio of the forces at any angle *C*, will be the ſame, whether they act towards the point *C*, or from it; by Prop. VIII. that is, it will be the ſame thing whether the weight at any

FIG. 91. angle C, acts in direction Cm or Cs. And as the forces were ſuppoſed before to thruſt againſt C, the ſame forces now do pull from it.

SCHOL.

92. If $DABF$ be a ſemi-circle, whoſe diameter is DF. Draw AG perpendicular to DF. Then the force or weight at any place A, to preſerve the equilibrium, will be reciprocally as AG^3, or directly as the cube of the ſecant of the arch BA.

Likewiſe it follows from Cor. 5. that if any cords of equal lengths be ſtretched to the ſame degree of curvature, the ſtretching forces will be as the weights of the cords.

SECT. VIII.

The ſtrength of beams of timber in all poſitions, and their ſtreſs by any weights acting upon them, or by any forces applied to them.

PROP. LXVII.

FIG. 93.

The lateral ſtrength of any piece of timber in any place, whoſe ſection is a rectangle, is directly as the breadth and ſquare of the depth.

Let *BD* be any beam, placed horizontally, and fixt at the end *BC*. And let *AFG* be the perpendicular ſection. Divide the depth *AF* into an infinite number of equal parts at *a*, *b*, *c*, *d*, *&c.* whoſe number is *AF* or *n*; through which, ſuppoſe lines drawn parallel to *FG*. And let any force be applied at *P* in direction *DP*, to break the beam at *AF*; then ſince the ſtrength of the timber is nothing but the force by which the parts of the timber at *a*, *b*, *c*, *&c.* cohere together: the breaking the timber is nothing but overcoming this force, and ſeparating the parts at *a*, *b*, *c*. Suppoſe $1 =$ force of coheſion of any of the parts *Aa*, *ab*, *bc*, *&c.* and imagine *QAa*, *QAb*, *QAc*, *&c.* ſo many bended leavers whoſe fulcrum is at *A*. And let us ſee what will be the ſum of all the forces applied at *Q* to break the timber at *A*. Now (by Cor. 1. Prop. XIX) the power applied at *Q* to equal or overcome the reſiſtances at *A*, *a*, *b*, *c*, *&c.* will be $\frac{0}{AQ}$, $\frac{Aa}{AQ}$, $\frac{Ab}{AQ}$, $\frac{Ac}{AQ}$, $\frac{Ad}{AQ}$, *&c.* to $\frac{AF}{AQ}$; that is, as $\frac{0}{AQ}$, $\frac{1}{AQ}$, $\frac{2}{AQ}$, $\frac{3}{AQ}$, . . . $\frac{n}{AQ}$. Therefore the effect of all the forces applied to *Q*, or the whole ſtrength of the beam at *A*, will be $\frac{0+1+2+3\ \cdot\ \cdot\ \cdot\ +n}{AQ}$ or $\frac{nn}{2AQ}$, that is, becauſe *AQ* is given, as *nn* or AF^2. Now if the breadth

FG

FIG. 93. *FG* be increaſed in any proportion; it is evident the ſtrength of every part *Aa*, *ab*, *&c.* will be increaſed in the ſame proportion, and therefore the abſolute lateral ſtrength will be as $AF^2 \times FG$.

Cor. 1. *In ſquare timber, the lateral ſtrength is as the cube of the breadth or depth.*

Cor. 2. *And in general, the lateral ſtrength of any pieces of timber, whoſe ſections are ſimilar figures, are as the cubes of the ſimilar ſides of the ſections.*

Cor. 3. *And in any pieces of timber, whoſe ſections are ſuch figures, that the correſpondent ordinates, parallel to the horizon, are proportional, the ſtrengths are as the breadths and ſquares of the depths, or as the ſections multiplied by the depths.*

289. Cor. 4. *The ſtrength of cylindrical pieces, or of any ſimilar pieces of timber, being forced or twiſted round the axis, will alſo be as the cubes of the diameters.*

For let $AD = r$, circumference of the ſection $DEFG = c$, $Ap = x$, then the circumference $pqr = \frac{cx}{r}$, and if the coheſion of a particle at p be $= 1$. Then the force applied at Q to overcome it, will be $\frac{x}{AQ}$; and the force applied at Q to overcome the coheſion of all the parts, in the circumference pqr, will be $\frac{x}{AQ} \times \frac{cx}{r}$, or $\frac{c}{r \times AQ} \times xx$; and the total force at Q to overcome the coheſion of all the particles in the whole ſection $DEFG$ is $= \frac{c}{r \times AQ} \times$ ſum of all the $xx = \frac{c}{r \times AQ} \times 1^2 + 2^2 + 3^2 + 4^2 \ldots$ to $r^2 = {}^{*}\frac{c}{r \times AQ} \times \frac{r^3}{3}$. Therefore, becauſe AQ is given, and the ratio $\frac{c}{r}$; and this force is the ſtrength of the beam; therefore the ſtrength is as r^3 or AD^3.

SCHOL.

93. What is here ſaid of timber, is true of any homogeneous bodies whatever ſort of matter they are of. But the abſolute

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* *Ward*'s Math. Guide, Part V. Lem. 3.

ſtrength of any beam, lever, rope, &c. when drawn in direction of its length, will be as the ſection of it. For every part does in this caſe bear an equal ſtretch, and the ſum of all the parts is equal to the whole, and that is as the ſection. FIG. 93.

PROP. LXVIII.

The lateral ſtrength of a tube, or hollow cane AB, to that of a ſolid one CD, is as the ſection of the tube (excluding the hollow), to the ſection of the ſolid cane, and the whole diameter of the tube to the diameter of the ſolid cane, nearly. 94.

For (by Cor. 2. of the laſt Prop.) the ſtrength of the ſolid cylinder BF is AF^3, and the ſtrength of the inner ſolid cylinder, whoſe fulcrum is at G, is EG^3, and whoſe fulcrum is at F, is greater than EG^3, and leſs than EF^3, and is nearly $\overline{EG + \frac{1}{2}GF}|^3 = \overline{AF - \frac{3}{2}AE}|^3$, that is $AF^3 - \frac{9}{2}AF^2 \times AE$ nearly. Therefore the ſtrength of the tube $AFIGE$, is the difference of the ſtrength of theſe cylinders, that is, $AF^3 - AF^3 + \frac{9}{2}AF^2 \times AE$ or $\frac{9}{2}AF^2 \times AE$. Likewiſe the ſtrength of the ſolid cylinder DCH, is CH^3. Therefore the ſtrength of the tube FB : ſtrength of the cylinder HD $:: \frac{9}{2}AF^2 \times AE : CH^3$. But the ſection of the tube is as $AF^2 - EG^2$ or $AF^2 - \overline{AF - 2AE}|^2 = 4AF \times AE$ nearly. Whence, ſtrength of the tube FB : ſtrength of the cylinder HD $:: \frac{9}{2}AF \times AE : CH \times CH^2 :: 4AF \times AE \times 1\frac{1}{8}AF : CH^2 \times CH ::$ ſection of the tube $\times 1\frac{1}{8}$ its diameter : ſection of the cylinder × its diameter :: ſection tube × diameter : ſection cylinder × diameter, nearly.

Otherwiſe,

Let the area of the ring $AEGF$ be diſpoſed into another ring, whoſe diameter is leſs than AF; then ſince every particle of it acts at a leſs diſtance from the fulcrum F, its ſtrength will be leſs in proportion; that is, its ſtrength will be as the diameter of the ring. And when the ring is ſo far diminiſhed as to become an entire circle, the proportion of their ſtrength will not differ far from the proportion of their diameters. Let the diameter of that circle be R; then, ſtrength of the ring or tube : ſtrength of an equal circle $:: AF : R$. And the ſtrength of R : to that of $CH :: R^3 : CH^3$. Therefore, ex equo, ſtrength of the tube BF : ſtrength of the cylinder $HD :: AF \times R^3 :$
$R \times CH^3$

FIG. 94. $R \times CH^3 :: AF \times R^2 : CH \times CH^2 :: AF \times$ area ring : $CH \times$ area of the circle CH.

Cor. Hence the ſtrength of different tubes are as their ſections, and diameters, nearly.

PROP. LXIX.

95. *If any force be applied laterally to a leaver or beam, the ſtreſs upon any place is directly as the force and its diſtance from that place.*

For ſuppoſe *PAF* to be a bended leaver. It is evident the greater the power at *P*, the greater force is applied at *F* to ſeparate the parts of the wood. Alſo the greater the diſtance *AP*, the greater power has any given force applied at *P*, to overcome the coheſion of the wood at *F*. And therefore the whole ſtreſs depends on both.

96. *Cor. 1. If two equal weights lie upon the middle of two beams, or upon any other ſimilar places, the ſtreſs in theſe planes, will be as the lengths of the beams.*

For if *C* be the middle point, then *A* bears half the weight; therefore the ſtreſs at *C* is as $AC \times \frac{1}{2}$ weight. And becauſe half the weight, or the force acting at *A* is given; therefore the ſtreſs is as *AC* or half *AB*, and therefore as *AB*. And if *C* be in any other ſimilar ſituation in both beams, the ſame thing will follow.

Cor. 2. If two beams bear two weights proportional to their lengths, and in a like ſituation; the ſtreſs upon each will be as the ſquare of its length.

Cor. 3. And if two beams bear two weights reciprocally as their lengths, in a ſimilar ſituation, the ſtreſs where the weights lie, is equal in both.

PROP. LXX.

Let AB be any beam of a given length, ſupported at A and B, and any given weight either ſuſpended at any point C, or equally diffuſed through the whole length of the beam AB; I ſay, in either caſe, the ſtreſs of the beam in C, is as the rectangle AC×CB. 97.

CASE I.

Let the given weight be repreſented by the given length of the leaver *AB*. Then (by Cor. 5. Prop. XIX.) the weight at *A*, and the re-action equal thereto, will be *CB*. And by the laſt Prop. the ſtreſs at *C* will be as the force acting at *A* × diſtance *AC*, that is $AC \times CB$.

CASE II.

Let *AB* be divided into an infinite number *n* of equal parts, each $=1$. Then as *AB* repreſents the whole weight, 1 will be the weight ſupported upon 1 part of the beam, let it reſt at *p*; then $\frac{Ap}{AB} =$ its preſſure on *B*. Therefore (by the laſt Prop.) the ſtreſs at *p* is $\frac{Ap \times pB}{AB}$; and the ſtreſs at *C* is $\frac{Ap \times BC}{AB}$, ariſing from the weight at *p*. Conſequently the ſtreſs at *C* ariſing from the ſum of all the weights between *A* and *C*, will be $\frac{0+1+2+3 \ldots AC}{AB} \times BC$; that is, (becauſe *AC* is the number of them) $\frac{AC^2 \times BC}{2AB}$. And by a like reaſoning, the ſtreſs at *C*, ariſing from the whole weight between *B* and *C*, will be $\frac{CB^2 \times AC}{2AB}$. Conſequently the whole ſtreſs at *C* is

$$\frac{AC^2 \times BC + CB^2 \times AC}{2AB} = \frac{AC+CB}{2AB} \times AC \times CB = \frac{AC \times CB}{2}.$$

Cor. 1. *The greateſt ſtreſs of a beam is in the middle, the weight being either ſuſpended there; or equally diſpoſed over the whole length of the beam.*

FIG. 97. *Cor.* 2. *The ſtreſs of the beam at any point p, by a weight applied to any other point C, is as $Ap \times CB$.*

For $AC \times CB$ is the ſtreſs at *C*, and (by Prop. laſt) $Ap \times CB$ will be the ſtreſs at *p*.

Cor. 3. *The ſtreſs of the beam at any point C, by a weight ſuſpended there is double the ſtreſs of the beam at the ſame point C, by the ſame weight preſſing uniformly on all parts of the beam.*

For by Caſe I. the ſtreſs at *C* is $AC \times CB$; and by Caſe II. the ſtreſs at *C* is $\frac{AC \times CB}{2}$.

Cor. 4. *The ſtreſs of a beam at any point C, by a weight ſuſpended there, is double to the ſtreſs at C, when the ſame weight is uniformly diſperſed on all the parts of AC.*

Cor. 5. *If a weight preſs equally on all the parts of pC, the ſtreſs at C by that preſſure, is to the ſtreſs at C, when ſuſpended at C :: as $Ap+AC$ to $2AC$.*

For (by Cor. 2.) the ſtreſs at *C*, by the weight 1 lying at *p*, is $Ap \times CB$, and at *C*, is $AC \times CB$. Therefore the whole ſtreſs at *C*, by the whole weight on all the points of *pC*, is the ſum of all the $Ap \times CB = Ap + \overline{Ap+1} + \overline{Ap+2} \ldots AC : \times CB = \frac{Ap+AC}{2} \times pC \times CB$. But the ſtreſs by the whole weight at *C*, is $AC \times CB \times pC$, and the former is to the latter, as $\frac{Ap+AC}{2}$ to *AC*.

Cor. 6. *If a weight preſs equally on all the parts of Ap, the ſtreſs at any point C by that weight, is to the ſtreſs at C if ſuſpended there :: as Ap to 2AC.*

For the ſtreſs at *C* by all the weight on *Ap*, is $0+1+2 \ldots Ap \times CB = \frac{Ap^2}{2} \times CB$. And the ſtreſs by the weight *Ap* at *C* is $AC \times CB \times Ap$.

Cor. 7. *The ſtreſs at p by a weight at C, is equal to the ſtreſs at C, by the ſame weight at p.*

PROP.

FIG. 98.

PROP. LXXI.

If CD be a prominent beam, fixt horizontally at the end C, as in a wall; and if a weight proportional to the length of the beam, be diſperſed uniformly on all the parts of the beam. The ſtreſs at any point F, will be as DF^2, *the ſquare of the diſtance from the extremity.*

For let *FD* be divided into an infinite number of equal parts at *p*, *q*, *r*, *s*, &c. and let each be $=1$, and ſuſtain the weight 1. Then (by Prop. LXIX.) the ſtreſs at *F*, by the weights at *F*, *p*, *q*, *r*, &c. will be 1×0, $1\times Fp$, $1\times Fq$, &c. or as 0, 1, 2, 3, &c. reſpectively: therefore the whole ſtreſs at *C* will be $0+1+2+3 \ldots FD = \frac{FD^2}{2}$.

Cor. 1. *Hence the ſtreſs at F, by any weight ſuſpended at D, will be double the ſtreſs at the ſame point F, when the ſame weight preſſes uniformly on all the parts between F and D.*

For (by Prop. LXIX.) the ſtreſs at *F* by the weight *DF*, is $DF\times DF$, or FD^2.

Cor. 2. *The ſtreſs at the end BC, by the weight P, is the ſame as the ſtreſs upon the middle of a beam of twice the length DC, with twice the weight P laid on its middle, this beam being ſupported at both ends.*

For the ſtreſs now at *C*, is the ſame as if *DC* was continued to the ſame length beyond *C*, and a weight equal to *P* ſuſpended at the end; and then the fulcrum *C* will be acted on with twice the weight *P*. And this is the ſame as if the beam was turned upſide down, and twice the weight *P* laid on the middle *C*.

PROP. LXXII.

If there be two beams ſtanding a ſlope, and bearing two weights upon them, either in the middle, or in any given ſituation, or equally diffuſed over the whole length of the beams; the ſtreſs upon them will be directly as the weights, and the lengths, and the coſines of elevation. 99. 100.

For

FIG. 99. 100. For (by Cor. 1. Prop. XXXI.) the weight is to the preſſure upon the plane, as radius to the coſ. elevation. Therefore the preſſure is as coſ. elevation × weight : and this is the force acting againſt the beam. Therefore (by Prop. LXVIII.) the ſtreſs will be as its length and this force; that is, as the length, the weight, and coſ. elevation.

Cor. 1. *If the weights and length of the beams be the ſame, the ſtreſs will be as the coſine of elevation, and therefore greateſt when it lies horizontal.*

Cor. 2. *If the beams lie horizontal, or at any equal inclinations, and the weight be as the length, then the ſtreſs is as the ſquare of the length.*

99. Cor. 3. *If the weights are equal, on the horizonal beam AB, and the inclined one AC, and BC be perpendicular to AB : then the ſtreſs will be equal upon both.*

For the length × coſ. elevation is the ſame in both, or $AC \times$ coſ. $A = AB \times$ radius.

Cor. 4. *But if the weights on the ſame beams be as their lengths, then the ſtreſs will alſo be as their lengths, AB and AC.*

Cor. 5. *And univerally, the ſtreſs upon any point of a ſloping beam, is as the rectangle of the ſegments, and the weight, and coſ. inclination directly, and the length of the beam reciprocally.*

97. For in the horizontal beam AB, if the weight W lie upon C, the preſſure at A will be $\frac{CB}{AB} \times W$. And (by Prop. LXIX.) this preſſure or force × diſtance AC, will be as the ſtreſs at C; that is, $\frac{AC \times CB}{AB} \times W$ is as the ſtreſs at C. And if the beam lie a ſlope, the ſtreſs (by Cor. 1. Prop XXXI.) will then vary in proportion to the coſ. elevation.

PROP. LXXIII.

If any beam of timber be to ſupport any weight, or preſſure, or force, acting laterally upon it, the breadth multiplied by the ſquare of the

the depth, or in ſimilar ſections, the cube of the diameter, in every place, ought to be proportional to the length multiplied by the weight or force acting on it, or as the ſtreſs in that place. And the ſame is true of ſeveral different pieces of timber compared together. FIG.

For every ſeveral piece of timber, as well as every part of the ſame timber or beam, ought to have its ſtrength proportioned to the weight, force or preſſure it is to ſuſtain. And therefore the *ſtrength* ought to be univerſally as the *ſtreſs* upon it. But (by Prop. LXVII.) the ſtrength is as the breadth × ſquare of the depth. And (by Prop. LXIX.) the ſtreſs is as the weight or force × by the diſtance it acts at. And therefore theſe muſt be in an invariable ratio.

102.

Cor. 1. *If AEB be a prominent beam fixt at the end AE, and ſuſtaining a weight at the other end B. And if the ſections in all places be ſimilar figures, and CD be the diameter in any place C, then CB will be every where as* CD^3. *And if ACB be a right line, EDB will be a cubic parabola. Therefore* $\frac{2}{5}$ *of ſuch a beam may be cut away without any diminution of the ſtrength.*

But if the beam be bounded by two parallel planes, perpendicular to the horizon, then CB will be as CD^2, *and then EDB will be the common parabola. Whence a third part of a beam may be thus cut away.*

102.

Cor. 2. *But if a weight preſs uniformly on every part of AB, and the ſections in all points as C, be ſimilar; then* BC^2 *will be every where as* CD^3, *and EDB a ſemi-cubical parabola.* 101.

But if the beam be bounded by parallel planes, perpendicular to the horizon, then BC will be as CD, and EDB a right line. Here half a beam may be cut away without loſing any ſtrength.

103.

Cor. 3. *If AB be a beam ſupported at both ends, and if it bear a weight in any variable point C, or uniformly on all the parts of it. And if all the ſections be ſimilar figures, and CD be the diameter in that place C, then will* CD^3 *be every where as* $AC \times CB$.

But if it be bounded by two parallel planes, perpendicular to the horizon, then will CD^2 *be every where as* $AC \times CB$, *and therefore the curve ADB is an ellipſis, ſuppoſing AB a right line.*

104.

Cor. 4. *But if a weight be placed at any given point P, and all ſections are ſimilar figures, and if CD be any diameter, then will BC be as* CD^3, *and AQ and BQ are two cubic parabolas.*

But

FIG. 104. *But if the beam be bounded by two parallel planes, perpendicular to the horizon, then* BC *is as* CD^2, *and* AQ *and* BQ *are two common parabolas.*

Cor. 5. *All circular plates, whether great or little, being of the ſame matter and thickneſs, and ſupported all round on the edges, will bear equal weights. The ſame is true of ſquare plates, or any ſimilar ones.*

290. For let AD, ad, be two ſquares, and let them firſt, be only ſupported at the ends AB, CD, and ab, cd. Then by this Prop. (ſince the thickneſs is given) $AB : AC \times$ weight on $AD :: ab : ac \times$ weight on ad, and weight on AD : weight on ad :: $\frac{AB}{AC} : \frac{ab}{ac}$, but $\frac{AB}{AC} = \frac{ab}{ac}$, therefore weight on $AD =$ weight on ad, when the plates are only ſupported by AB, CD, and ab, cd. And for the ſame reaſon the weights will be equal, when only ſupported by the ſides AC, BD, and ac, bd. And conſequently the weights will ſtill be equal; when the plates are ſupported by all four ſides, in which caſe, twice the weight will be ſupported; and the ſame holds equally true of all ſimilar figures. For,

291. Let AD, ad, be two hollow circles, draw the inſcribed ſquares $ABCD$, $abcd$, theſe ſquares are ſupported upon the four ſides AB, BD, DC, CA, and ab, bd, dc, ca, by the continuity of the plates; therefore the weights will be equal, as was proved before. And that the continuity of the plates will equally ſupport the weights in both circles is plain, becauſe the ſtrength of both ſegments AB, ab, are equal, the length being as the breadth.

Cor. 6. *Hence, the weight a ſquare plate will bear* :
To the weight which a bar of the ſame matter and thickneſs will bear : :
As twice the length of the bar :
To its breadth.

And a circular plate is but very little weaker.

S C H O L.

All theſe things appear from the foregoing propoſitions; but it is here ſuppoſed that the timber is homogeneous and of the ſame goodneſs, otherwiſe a proper allowance muſt be made for the defect. And what is here ſaid of pieces of timber, holds equally true of any other ſolid bodies, ſuch as pieces of metal, ſtone, &c. And if pieces of timber or metal be cut into the figures mentioned in the foregoing corollaries, all the parts will be diſpoſed to break together. And if a ſpring is to be made, its ſhape ought to be as in Cor. 1. and then every part will bear a ſtreſs proportional to its ſtrength. FIG. 291.

PROP. LXXIV.

If a weight A be ſupported upon the end of a crooked piece of timber ABD, and from the ends, a line AB be drawn perpendicular to the horizon, and from the angle B, the line BC perpendicular to AD; the ſtreſs at B will be as the perpendicular BC. 105.

For as the weight *A* acts not in direction *AB*, but in direction *AD*, therefore it is the ſame as if it were applied at the point *C*; but a force applied at *C*, has a greater power to break the timber at *B*, in proportion as the leaver *BC* is longer. This force therefore, or the ſtreſs at *B*, is as *BC*.

Cor. 1. *Hence, if any two forces acting from or againſt one another, at the ends A, F, of any crooked beam ABDEF, and keep one another in equilibrio; and the line AF, or the direction of the forces being drawn, the ſtreſs at any point, is as the perpendicular upon AF. So the ſtreſs at b is bc; at B, BC, at D, DI; at E, EK; and at G, H, nothing.* 106.

Cor. 2. *Hence alſo, that the ſtrength in any part b, may be proportional to the ſtreſs there, the breadth multiplied by the ſquare of the depth, muſt be as the perpendicular bc, reckoning that the depth, which is in the plane paſſing through AF.*

PROP. LXXV.

Having the length AB and weight W, of a cylinder or priſm, that can juſt ſupport the weight P at the end, to find the length of another beam FG, ſimilar to the former, and of the ſame matter, that will juſt break with its own weight, or only ſupport itſelf. 107. 108.

Since

FIG. 107. 108. Since the weights of ſimilar ſolids of the ſame matter, are as the cubes of the lengths, it will be, $AB^3 : W :: FG^3 : \frac{FG^3}{AB^3}W =$ the weight of the beam GH. Then by Cor. 2. Prop. LXVII. the ſtrength of the beam AB is AC^3; and of FG, is FH^3. And by Prop. LXIX. the ſtreſs at A is $\overline{\tfrac{1}{2}W+P}\times AB$. And the ſtreſs at F $\frac{FG^3}{2AB^3}W\times FG$. And ſince the beams are both ſuppoſed to break with theſe weights; therefore the ſtrength muſt be as the ſtreſs; that is, $\overline{\tfrac{1}{2}W+P}\times AB : \frac{FG^4}{2AB^3}W$ $(:: AC^3 : FH^3) :: AB^3 : FG^3$. Whence $\frac{FG^4\times W}{2} = FG^3\times AB\times\overline{\tfrac{1}{2}W+P}$. Or $FG\times W = AB\times\overline{W+2P}$. Whence $W : W+2P :: AB : FG$, the length required.

Cor. 1. *If* $eW=P$. *Then* $FG=AB\times\overline{1+2e}$.

Cor. 2. *Hence there is one and only one beam, that will juſt break by its own weight, or juſt ſuſtain itſelf.*

Cor. 3. *The ſame Prop. will likewiſe hold good, in regard to two beams ſupported at both ends and breaking in the middle, by Cor.* 3. *Prop.* LXX.

Cor. 4. *If the beam FG break by its own weight, a beam of twice the length of FG, and ſupporting at both ends, will alſo break by its own weight; or if one ſuſtain itſelf, the other will.*

For the ſtreſs is the ſame in both of them, by Cor. 3. Prop. LXX. and Cor. 1. Prop. LXXI. each of them being equal to the ſtreſs of a beam, twice the length of FG, and ſuſpended in the middle.

PROP. LXXVI.

109. 110. *If any weight be laid on the beam AB, as at C, or any force applied to it at C; the beam will be bent through a ſpace CD proportional to the weight or force applied at C. And the reſiſtance of the beam will be as the ſpace it is bent through nearly.*

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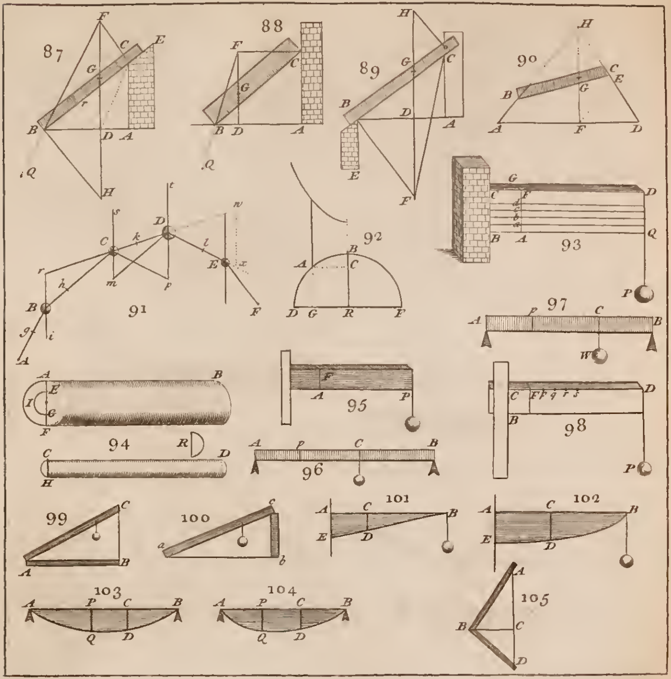
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Pl.V. pa 104.

In order to find the law of reſiſtance of beams of timber or ſuch like bodies, againſt any weights laid upon them, or ſtraining them, I took a piece of wood plained ſquare, and ſupporting it at both ends *A*, *B*, I laid ſucceſſively on the middle of it at *C*, 1, 2, 3, 4, 5, 6, 7, and 8 pounds; and I found the middle point *C* to deſcend through the ſpaces 1, 2, 3, 4, 5, 6, 7, and 8, reſpectively. And repeating the ſame experiment with the weights 3, 6, 9 lb. they all deſcended through ſpaces, either accurately or very nearly as the numbers 1, 2, 3. I tried the ſame thing with ſprings of metal, and found the ſpace through which they were bent, proportional to the weight ſuſpended. I alſo tried ſeveral experiments of this kind with wires, hairs, and other elaſtic flexible bodies, by hanging weights at them: and I found that the increaſe of their lengths, by ſtretching, was, in each of them, proportional to the weights hung at them: except when they were going to break, and then the increaſe was ſomething greater. It may be obſerved, that none of theſe bodies regained their firſt figure, when the weights were taken off, except well tempered ſprings; ſo that there are no natural bodies perfectly elaſtic. And even ſprings are obſerved by experience to grow weaker by often bending; and by remaining ſome time unbent, will recover part of their ſtrength: and are ſomething ſtronger in cold than in hot weather. But at any time a ſpring, and all ſuch bodies obſerve this law, that they have the leaſt reſiſtance when leaſt bent, and in all caſes are bent through ſpaces nearly proportional to the weights or forces applied. And therefore I think this law is ſufficiently eſtabliſhed, that the reſiſtance, any of theſe bodies makes, is proportional to the ſpace through which it is bent, or that it exerts a force proportional to the diſtance it is ſtretched to. FIG. 109. 110.

The knowledge of this property of ſpringy bodies is of great uſe in mechanics, for by this means a ſpring may be contrived to pull at all times with equal ſtrength, as in the fuſee of a watch; or it may be made to draw in any proportion of ſtrength required.

The action of a ſpring may be compared to the lifting up a chain of weights, lying upon a plane, or to the lifting a cylinder of timber out of the water endways.

FIG.

PROP. LXXVII.

111. *To find the lateral ſtrength of any beam of timber, whoſe tranſverſe ſection is any figure whatſoever.*

Let *ERG* be the ſection of the beam in the place where it breaks. Draw the ordinates *IN*, *in*, infinitely near each other, and parallel to the baſe *RG*.

Put $ER = d$ $EI = x$
$RG = b$ $IR = v$
$IN = y$

The abſolute ſtrength of one fibre of the wood $= 1$.

When the beam breaks, it is done by the ſeparation of the parts of the wood at *E*. Therefore *QRE* muſt be eſteemed a bended leaver, whoſe fulcrum is at *R*. When the beam breaks, the fibres at *E* are ſtretch to their full ſtrength, but thoſe nearer *R* are leſs ſtretched, and exert leſs force or reſiſtance in proportion to their diſtance from *R* (by the laſt Prop.); and therefore the reſiſtance of a fibre at $I = \frac{v}{d}$; and the reſiſtance of all the fibres in the parallelogram *In*, $= \frac{v}{d} \times In$; and the power of all the fibres in the parallelogram, in regard to the brachium *IR*, is $= \frac{vv}{d} \times In$. And the ſum of all the powers in the whole ſection = ſum of all the $\frac{vv}{d} \times In$.

Let *g*, *p*, be the diſtance of the center of gravity and percuſſion from *RG*, as the axis of motion.

Then (by Cor. 1. Prop. LVII.) the ſum of all the $vv \times In = gp \times$ ſum of all the $In = gp \times$ ſection *ERG*. And the ſum of all the $\frac{vv}{d} \times In = \frac{gp}{d} \times$ ſection *ERG*. Therefore the ſtrength of the beam at *E*, is $= \frac{gp}{d} \times$ ſection *ERG*.

112. *Cor.* 1. *If there be taken* $RO = \frac{gp}{d}$; *then all the fibres of the wood being ſuppoſed to be collected in O, and acting there with their full ſtrength; their total ſtrength at O, ſhall be equal to the ſtrength of the beam, at the ſection ERG.*

FIG. 112. For ſuppoſe O to be ſuch a point; then the ſtrength of the beam, or $\frac{gp}{d} \times$ ſection $ERG = RO \times$ ſection ERG; and $RO = \frac{gp}{d}$.

Cor. 2. *If the ſection be a parallelogram.* $g = \frac{1}{2}d$, *and* $p = \frac{2}{3}d$, *therefore* $RO = \frac{1}{3}d$.

In a circle whoſe diameter is ER, $g = \frac{1}{2}d$, $p = \frac{5}{8}d$, *and* $RO = \frac{5}{16}d$, $= \frac{1}{3}d$ *nearly, as in the parallelogram.*

In the periphery of a circle (the beam being a hollow cane) $g = \frac{1}{2}d$, *and* $p = \frac{3}{4}d$, *whence* $RO = \frac{3}{8}d = \frac{1}{3}d$ *nearly; as in the parallelogram.*

In a triangle whoſe baſe is at E parallel to the horizon, and vertex at R; $g = \frac{2}{3}d$, *and* $p = \frac{3}{4}d$, *and* $RO = \frac{1}{2}d$. *And its ſtrength* $= \frac{1}{2}d \times$ *area of the triangle.*

289. *Cor.* 2. *The ſtrength of a cylinder when twiſted, wreſted, or wrung round its axis, is equal to the lateral ſtrength of a triangular beam whoſe height = radius, baſe = circumference of the cylinder; and vertex of the triangle at D.*

For let $AD = r$, circ. $DEFG = c$, $Ap = v$, circ. $pqr = \frac{cv}{r}$. The reſiſtance of a fibre at D is 1, and at p is $= \frac{v}{r}$, and the reſiſtance of all the fibres in pqr is $= \frac{cvv}{rr}$; and the power of all the fibres in pqr, in regard to the brachium Ap is $= \frac{cvv}{rr} \times v$; and the total power of all the fibres in the whole ſection $DEFG$ is $= \frac{c}{rr} \times$ ſum of all the $v^3 = \frac{c}{rr} \times 1^3 + 2^3 + 3^3 + 4^3 \ldots$ to $r^3 = {}^{*}\frac{c}{rr} \times \frac{r^4}{4}$; therefore the ſtrength is $= \frac{crr}{4}$.

And in the triangle, the area being $\frac{cr}{2}$, and $d = r$, the ſtrength $= \frac{1}{2}d \times \frac{cr}{2} = \frac{crr}{4}$, the ſame as for the twiſted cylinder.

SCHOL.

This Cor. 2. does not agree ſo well with timber as metal, for the texture of wood is not the ſame in length as breadth. For all

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* *Ward's* Math. Guide, Part V. Lem. 4.

FIG. all wood is compoſed of long ſlender tubular fibres, joined together by a glutinous matter, which is eaſily ſeparated; and therefore wood is much more eaſily ſplit than broken.

PROP. LXXVIII.

113. *Given the weight that will break a beam laterally, to find how much will break it when drawn in direction of its length.*

Let DR be the beam.
Put l = its length DE,
W = weight applied at D, that can break it at E,
d = depth ER,
g = diſtance of the center of gravity of the ſection ERG from R,
p = diſtance of the center of percuſſion of ERG from RG,

Then $\frac{dl}{gp}W$ = weight that will break it when drawn in direction of its length.

But if the beam be ſupported at both ends, and the weight breaks it in the middle; g and p muſt be meaſured from the upper ſide, and take l for half its length, W for half the weight that breaks it.

For by Cor. 1. of the laſt Prop. if $RO = \frac{gp}{d}$, then all the fibres of the beam acting at O, will be equal to the ſtrength of the beam; and ſince W, applied at D, can break it in either caſe, therefore by the nature of the leaver, it will be, $l \times W = RO \times$ abſolute ſtrength $= \frac{gp}{d} \times$ abſolute ſtrength, therefore the abſolute ſtrength $= \frac{ldW}{gp}$, or the weight that can break it, when drawn in length.

Cor. Hence, if there be taken $RL = \frac{gp}{d}$, then the weight which being applied at L, will juſt break the beam horizontally, the ſame will juſt pull it aſunder, when applied lengthways.

For then $l = \frac{gp}{d}$, and weight W = reſiſtance at O = ſtrength of the whole beam.

Therefore

FIG. 113. Therefore if a piece of oak, an inch ſquare and a foot long, ſupported at both ends, bears 315 lb. before it breaks; it will bear, when drawn in length, 2835 lb. or 1 ton, 5 hundreds, 2 ſtone 7 pound; that is above a ton and a quarter.

SCHOL.

Here we all along ſuppoſe that the fulcrum at *R* remains fixt; but if it ſhould vary by the denting in of the parts at *R*, it will cauſe a little variation in the ſtrength, and make the beam ſomething weaker laterally. And that it will yield a little this way, is evident from experiments; for the hardeſt bodies, ſuch as glaſs in ſmall threads, may be extended in length, and conſequently may be contracted by a contrary force; and balls of glaſs or wood, let fall upon a hard body, will rebound; which they cannot do without the denting in of the parts.

PROP. LXXIX.

114. *If a weight be laid upon the ſtreight beam AB, ſupported at both ends, its bending or curvature will be nearly as the weight and length directly; and as the breadth and cube of the depth reciprocally.*

It is found (by Prop. LXXVI.) that if ſeveral weights be laid ſucceſſively upon a horizontal beam *AB*, the ſpace *CDB*, thro' which the point *D* deſcends, will be as the weight it bears. Now the parts at *e*, *f*, which are contiguous at the beginning of the deſcent, are gradually ſeparated; till at laſt the beam breaks. At which time, when it can bear no more, the infinitely ſmall diſtance *ef* is a given quantity. If *CD* be ſuppoſed very ſmall, then *CD* is as the curvature at *D*, and this curvature is as the infinitely ſmall angle *eDf*, that is, as $\frac{ef}{Df}$, and when it breaks, as $\frac{1}{Df}$.

Let *L* be the length of any beam, *b* its breadth, *Df* its depth, then (by Prop. LXXIII.) its ſtrength, or the weight it will bear, is as $\frac{b\times Df^2}{L}$. Therefore put $\frac{b\times Df^2}{L}$ for the greateſt weight, and

FIG. 114. $\frac{1}{Df}$ for the curvature when breaking; W for any other weight, and C the correſpondent curvature, And it will be as $\frac{b\times Df^2}{L} : \frac{1}{Df} :: W : C = \frac{LW}{b\times Df^3}$.

Cor. 1. *The quantity of deflexion CD of any beam is as the weight and cube of the length directly, and the breadth and cube of the depth reciprocally.*

For when CD is very ſmall, ADB is very near a circle, or nearer a parabola; ſuppoſe it a circle, and let its radius be R, then $2R\times CD = AD^2$ or $\frac{1}{4}LL$; therefore $\frac{CD}{LL} = \frac{1}{8R}$, whence $\frac{CD}{LL}$ is as the curvature C, that is as $\frac{LW}{b\times Df^3}$, or CD is as $\frac{WL^3}{b\times Df^3}$: And if ACB, the original poſition of the beam, is not a right line, yet CD will ſtill be of the ſame quantity.

Cor. 2. *In ſimilar homogenous ſtreight bodies the curvature is as the weight directly, and cube of the depth reciprocally; but the deflexion CD is as the weight directly, and depth reciprocally.*

Cor. 3. *In ſimilar bodies bending from a ſtreight line by their own weight, the curvature is given, and the deflexion is as the ſquare of the depth.*

Cor. 4. *In the utmoſt ſtrength of beams, or their breaking poſition, the curvature is reciprocally as the depth, and the deflexion as the ſquare of the length directly and the depth reciprocally.*

For then $b\times Df^2$ is as LW.

Cor. 5. *What is ſaid of ſtreight beams is equally true of any beams, in regard to the increaſe or variation of curvature, and to the deflection from their original poſition.*

SCHOL.

What is ſaid of beams of timber in this ſection, is equally applicable to any ſolid bodies, acted on in a like manner as by weights. There are ſome bodies in which a very little bending may have a great effect, as in the glaſſes of large teleſcopes. For (by Cor. 3. of the laſt Prop.) the deflexion from their true figure, ariſing from their own weight, is as the ſquare of the diameter, when the glaſſes are ſimilar. And though this be inſenſible in ſmall glaſſes, it may produce ſome ſenſible error in large ones; and the ſame may happen to them in grinding by too much preſſure.

From the foregoing propoſitions it follows, that if a certain beam of timber be able to ſupport a given weight, another beam of the ſame timber, ſimilar to the former, may be taken ſo great as to be able but juſt to bear its own weight; and any bigger beam cannot ſupport itſelf, but muſt break by its own weight; and any leſs beam will bear ſomething more. For the ſtrength being as the cube of the depth, and the ſtreſs being as the matter and length, is as the 4th power of the depth: it is plain the ſtreſs increaſes in a greater ratio than the ſtrength. Whence it follows, that a beam may be taken ſo large, that the ſtreſs may far exceed the ſtrength. And that of all ſimilar beams there is but one that will ſupport itſelf, and nothing more. Likewiſe if any beam can make 10 times its own weight, no other ſimilar beam will do the ſame. And the like holds in all machines, and in all animal bodies. And hence there is a certain limit, in regard to magnitude, not only in all machines and artificial ſtructures, but alſo in natural ones, which neither art nor nature can go beyond, ſuppoſing them made of the ſame matter, and in the ſame proportion of parts.

Hence it is impoſſible that mechanic engines can be increaſed to any bigneſs. For when they arrive at a particular ſize, their ſeveral parts will break and fall aſunder by their weight; neither can any buildings of vaſt bigneſs be made to ſtand, but muſt fall to pieces by their great weight, and go to ruin. Vaſt columns and pyramids will break by their weight and tumble down.

It is likewiſe impoſſible for nature to produce animals of any vaſt ſize at pleaſure, or any ſuch thing as giants, or men of prodigious ſtature; except ſome ſort of matter can be found to make the bones of, which is ſo much harder and ſtronger than any hitherto known; or elſe that the proportion of the parts be ſo much altered, and the bones and muſcles made thicker in proportion, which will make the animal diſtorted and of a monſtrous figure, and not capable of performing any proper action. And being made ſimilar, and of common matter, they will not be able to ſtand or move, but being burdened by their own weight muſt fall down.

Thus it is impoſſible that there can be any animal ſo big as to carry a caſtle upon his back, or any man ſo ſtrong as to remove a mountain, or pull up a large oak by the roots: nature will not admit

FIG. admit of theſe things, whence its impoſſible there can be animals of any ſort beyond a determinate bigneſs.

Fiſh may indeed be produced to a larger ſize than land animals, becauſe their weight is ſupported by the water; but yet they cannot be increaſed to immenſity, becauſe the internal parts will preſs upon one another by their weight and deſtroy their fabric.

On the contrary, when the ſize of animals is diminiſhed, their ſtrength is not dimiſhed in the ſame proportion as the weight. And therefore a ſmall animal will carry far more than its own weight, whilſt a great one cannot carry ſo much as its weight. And hence it is that ſmall animals are more active, will run faſter, jump farther, or perform any motion quicker, for their weight, than large animals; for the leſs the animal the greater the proportion of the ſtrength to the ſtreſs. And nature ſeems to know no bounds as to the ſmallneſs of animals, at leaſt in regard to their weight.

Neither can any two unequal and ſimilar machines reſiſt any violence alike, or in the ſame proportion; but the greater will be more hurt than the leſs. And the ſame is true of animals, for large animals by falling break their bones, whilſt leſſer ones falling higher receive no damage. Thus a cat may fall 2 or 3 yards high and be no worſe, and an ant from the top of a tower.

It is likewiſe impoſſible, in the nature of things, that there can be any trees of immenſe bigneſs; if there were any ſuch, their limbs, boughs, and branches, muſt break and fall down by their great weight. Thus it is impoſſible there can be an oak a quarter of a mile high; ſuch a tree cannot grow or ſtand, but its limbs will drop off by their weight. And hence likewiſe, leſſer plants can better ſuſtain themſelves than large ones can do.

Neither could a tree of an ordinary ſize be able to ſtand if it was compoſed of the ſame tender matter that ſome plants conſiſt of; nor ſuch a plant if it were much bigger than common. And that plants made of ſuch tender matter may better ſupport themſelves; nature has made the trunks and branches of many of them hollow, by which means they are both lighter and ſtronger.

The propoſitions before laid down concerning the ſtrength and ſtreſs of timber, *&c.* are alſo of excellent uſe in ſeveral concerns of life, and particularly in architecture; and upon theſe principles a great many problems may be reſolved relating to the due proportion of ſtrength in ſeveral bodies, according to their particular poſitions and weights they are to bear, ſome of which I ſhall briefly enumerate.

If a piece of timber is to be holed with a mortiſe hole, the beam will be ſtronger when it is taken out of the middle, than if it be taken out of either ſide. And in a beam ſupported at both ends, it is ſtronger when the hole is taken out of the upper ſide than the under one, provided a piece of wood is driven hard in to fill up the hole.

If a piece is to be ſpliced upon the end of a beam to be ſupported at both ends, it will be ſtronger when ſpliced on the under ſide of a beam, than on the upper ſide. But if the beam is ſupported only at one end, to bear a weight on the other, it is ſtronger when ſpliced on the upper ſide.

When a ſmall leaver, *&c.* is nailed to a body to remove it or ſuſpend it by, the ſtrain is greater upon the nail neareſt the hand or point where the power is applied.

If a beam is ſupported at both ends, and the two ends reach over the props, and be fixt down immoveable, it will bear twice as much weight as when the ends only lie looſe or free upon the ſupporters.

If a ſlender cylinder is to be ſupported by two pieces, the diſtance of the pins ought to be $\frac{5858}{10000}$ parts of the length of the cylinder, that is $\frac{3}{5}$ its length, the pins equi-diſtant from its ends, and then the cylinder will endure the leaſt bending or ſtrain by its weights.

115. By the foregoing principles it alſo follows, that a beam fixt at one end, and bearing a weight at the other, if it be cut in the form of a wedge and placed with its parallel ſides, parallel to the horizon, it will be equally ſtrong every where, and no ſooner break in one place than another.

116. If a beam has all its ſides cut into the form of a concave parabola, whoſe vertex is at the end, and baſe a ſquare, a circle, or any regular polygon, ſuch a beam fixt horizontal at one end, is equally ſtrong throughout for ſupporting its own weight.

By the ſame principles, if a wall faces the wind, and if the ſection of it be a right angled triangle, or the foreſide be perpendicular to the horizon, and the backſide terminated by a ſloping plane interſecting the other plane in the top of the wall, ſuch a wall will be equally ſtrong in all its parts to reſiſt the wind if the parts of the wall cohere ſtrongly together; but if it be built of looſe materials, it is better to be convex on the backſide in form of a parabola.

FIG. If a wall is to ſupport a bank of earth, or any fluid body, it ought to be built concave in form of a ſemi-cubical parabola, whoſe vertex is at top of the wall; this is when the parts of the wall ſtick well together; but if the parts be looſe, then a right line or ſloping plane ought to be its figure. Such walls will be equally ſtrong throughout.

All ſpires of churches in the form of cones or pyramids, are equally ſtrong in all parts to reſiſt the wind; but when the parts cohere not together, parabolic conoids are equally ſtrong throughout.

Likewiſe if there be a pillar erected in form of the logarithmic curve, the aſſymptote being the axis, it cannot be cruſhed to pieces in one part ſooner than in another by its own weight. And if ſuch a pillar be turned upſide down, and ſuſpended at the thick end in the air, it will be no ſooner pulled aſunder in one part than another by its own weight. And the caſe is the ſame if the ſmall end be cut off, and inſtead of it, a cylinder be added, whoſe height is half the ſubtangent.

117. Laſtly, let *AE* be a beam in form of a triangular priſm, and if $AD = \frac{1}{9}AB$, and $AI = \frac{1}{9}AC$, and the part *ADIF* be cut away parallel to the baſe, the remaining beam *DICEF* will bear a greater weight *P*, than the whole *ABCEG*, or the part will be ſtronger than the whole, which is a paradox in mechanics.

And upon the ſame principles an infinite number of queſtions of like kind may be reſolved, which are curious enough, and of great uſe in the common affairs of life.

All I ſhall here add, is the ſtrength of ſeveral ſorts of timber, and other bodies, as I have collected from experiments.

In the firſt edition of this book I had inſerted the ſtrength of ſome ſorts of wood, ſuch as I had made experiments upon; in all which, I gave the leaſt weight which the worſt of them was juſt able to bear; leſt any body computing the ſtrength of a beam, ſhould overcharge it with too much weight. And ſince that time, I have made a great many more experiments, not only upon many different ſorts of wood, but ſeveral other bodies, the reſult of which I ſhall here ſet down. A piece of good oak, an inch ſquare, and a yard long, ſupported at both ends, will bear in the middle, for a very little time, about 330 pounds *averd.* but will break with more than that weight. This is at a medium; for there are ſome pieces that will carry ſomething more, and others not ſo much: but ſuch a piece of wood ſhould not in practice be truſted for any length of time with above a third or a fourth part of that weight. For ſince this is the extreme weight which the beſt wood will bear, that of a worſe ſort muſt break with it. For I have found by experience, that there is a great deal of difference in ſtrength, in different pieces of the very ſame tree, ſome pieces I have found would not bear half the weight that others would do. The wood of the boughs and branches, is far weaker than that of the body; the wood of the great limbs, is ſtronger than that of the ſmall ones, and the wood in the heart of a ſound tree is ſtrongeſt of all. I have alſo found by experience, that a piece of timber, which has born a great weight for a ſmall time, has broke with a far leſs weight, when left upon it for a far longer time. Wood is likewiſe weaker when it is green, and ſtrongeſt when thoroughly dried, and ſhould be two or three years old at leaſt. If wood happens to be ſappy it will be weaker upon that account, and will likewiſe decay ſooner. Knots in wood weaken it very much, and this often cauſes it to break where a knot is. Alſo when wood is croſs-grained, as it often happens in ſawing; this will weaken it more or leſs, according as it runs more or leſs croſs the grain. And I have found by experience, that tough wood croſs the grain, ſuch as elm or aſh, is 7, 8, or 10 times weaker than ſtreight; and wood that eaſily ſplits, ſuch as fir, is 16, 18, or 20 times weaker. And for common uſe it is hardly poſſible to find wood but it muſt be ſubject to ſome of theſe things. Beſides, when timber lies long in a building, it is apt to decay or be worm eaten, which muſt needs very much impair its ſtrength. From all which it appears, that a large allowance ought to be made for the ſtrength of wood, when applied to any uſe, eſpecially where it is deſigned to continue for a long time.

The proportion of the ſtrength of ſeveral ſorts of wood, and other bodies that I have tried, will appear in the following table.

Box, yew, plumbtree, oak	11
Elm, aſh	$8\frac{1}{2}$
Walnut, thorn	$7\frac{1}{2}$
Red fir, hollin, elder, plane	7
Crabtree, appletree	
Beech, cherrytree, hazle	$6\frac{2}{3}$
Alder, aſp, birch, white fir	6
Willow or ſaugh	
Iron	107
Braſs	50
Bone	22
Lead	$6\frac{1}{5}$
Fine free-ſtone	1

In this table I have put ſeveral ſorts of wood into one claſs together, which I found to be pretty near of the ſame ſtrength, as I found ſometimes one ſort to exceed in ſtrength and ſometimes another, there being a great difference even in the ſame ſort of wood; and I do not doubt but other people that ſhall make experiments, will find them as different and various as I have done, and perhaps quite different from mine, juſt according to the goodneſs or badneſs of the wood they uſe. But I have contented myſelf to ſet down what I found from my own experience, as the reſult of a great many trials, without any regard to what other people have done or may do. What I ſhall further add, is this,

A cylindric rod of good clean fir, of an inch circumference, drawn in length, will bear at extremity, 400 lb. and a ſpear of fir 2 inches diameter, will bear about 7 ton, but not more.

A rod of good iron of an inch circumference, will bear near 3 ton weight.

A good hempen rope of an inch circumference, will bear 1000 lb. at extremity.

All this ſuppoſes theſe bodies to be ſound and good throughout, but none of theſe ſhould be put to bear more than a third or a fourth part of the weight, eſpecially for any length of time.

From what has been ſaid, if a ſpear of fir or a rope, or a ſpear of iron of d inches diameter, was to lift $\frac{1}{4}$ the extreme weight, then,

The fir would bear $8\frac{4}{5}\,dd$ hundred weight.
The rope $22\,dd$ hundred weight.
The iron $6\frac{3}{4}\,dd$ ton weight.

SECT. IX.

The properties of fluids, the principles of hydroſtatics, hydraulics, and pneumatics.

PROP. LXXX.

FIG.

Motion or preſſure in a fluid is not propagated in right lines, but equally 118.
all around in all manner of directions.

If a force act at *a* in direction *ab*, that motion can be directed no further than theſe particles lie in a right line as to *c*. But the particle *c* will urge the particles *d*, *f* obliquely, by which that motion is conveyed to *e*, *g*. And theſe particles *e*, *g*, will urge the particles *n*, *p*, and *r*, *s* obliquely, which lie neareſt them. Therefore the preſſure as ſoon as it is propagated to particles that lie out of right lines, begins to deflect towards one ſide and the other, and that preſſure being farther continued, will deflect into other oblique directions, and ſo on. Therefore the preſſure and motion is propagated obliquely ad infinitum, and will therefore be propagated in all directions.

Cor. If any part of a preſſure propagated through a fluid, be ſtopt 119.
by an obſtacle, the remaining part will deflect into the ſpaces behind the obſtacle. Thus if a wave proceeds from C, and a part goes through the hole A, it expands itſelf, and forms a new wave beyond the hole, which moves forward in a ſemi-circle whoſe center is the hole.

For any part of a fluid preſſing againſt the next is equally re-acted on by the next, and that by the next to it, and ſo on; from whence follows a lateral preſſure (equal to the direct preſſure) into the places behind the obſtacle.

FIG.

PROP. LXXXI.

A fluid can only be at reſt when its ſurface is placed in a horizontal ſituation.

120. For let *ABCD* be a veſſel of water or any fluid: and let *AB* be parallel to the horizon. Suppoſe the ſurface of the liquor to be in the poſition *FE*. Then becauſe the parts of the fluid are eaſily moveable among themſelves; therefore (by ax. 7.) the higher parts at *E* will, by their gravity, continually deſcend to the lower places at *F*. Alſo the greater preſſure under *E* and the leſſer under *F*, will cauſe the parts at *E* to deſcend, and thoſe at *F* to aſcend. And thus the higher parts of the fluid at *E* deſcending, and ſpreading themſelves over the lower parts at *F*, which are at the ſame time aſcending: the ſurface of the fluid will at laſt be reduced to a horizontal poſition *AB*. But being ſettled in this poſition, ſince there is no part higher than another, there is no tendency in any one part to deſcend, more than in another; and therefore the fluid will reſt in an horizontal poſition.

121. Cor. 1. *If the fluid does not gravitate in parallel lines, but towards a fixed point or center C; then the fluid can only be at reſt when its ſurface takes the form of a ſpherical ſurface AB, whoſe center is C.*

For if any parts of the ſurface of the fluid *A* or *B*, were further from *C* than the reſt, they would continually flow down to the places nearer *C*, towards which their weights are directed: till at laſt they would be all equi-diſtant from it.

Cor. 2. *Any fluid being diſturbed, will of itſelf return to the ſame level, or horizontal poſition.*

122. Cor. 3. *Hence alſo if a different fluid ABEF reſt upon the fluid ABCD; both the ſurface FE, and the ſurface AB that divides them, will lie in a level or horizontal ſituation, when at reſt.*

For if any part of the ſurface *AB* be higher than the reſt, it will deſcend to the ſame level; and ſince *FE* is alſo level, and therefore the heights *AF*, *BE* in every place equal; the preſſure of it on all the parts of the horizontal ſurface *AB*, will be equal. And therefore it cannot deſcend in one place more than another, but will continue level.

Cor.

FIG. 122. *Cor.* 4. *Hence water communicating with two places, or any way conveyed from one place to another; will riſe to the ſame level in both places. Except ſo far as it is hindered by the friction of the channel it moves thro', or perhaps ſome very ſmall degree of tenacity or coheſion.*

PROP. LXXXII.

In any fluid remaining at reſt; every part of it, at the ſame depth, is in an equal ſtate of compreſſion.

123. For let the plane *EF* be parallel to the ſurface *AB*. Then ſince the height of the fluid at all the points of *EF*, is equal; therefore the weights ſtanding upon any equal parts of *EF* are equal: and therefore the preſſure in all the points of *EF* is equal alſo.

Cor. 1. *A fluid being at reſt, the preſſure at any depth is as the depth.*

For this preſſure depends on the weight of the ſuperincumbent fluid, and therefore is as its height.

Cor. 2. *In any given place, a fluid preſſes equally in all directions.*

For (by Prop. LXXX.) as the preſſure in any place acts in all directions, it muſt be the ſame in all directions. For if it were leſs in one direction than another, the fluid would move that way, till the preſſure becomes equal. And then the fluid would be at reſt, and be equally compreſſed in all directions.

Cor. 3. *The preſſure is equal in every part of a plane drawn parallel to the horizon.*

Cor. 4. *When a fluid is at reſt, each drop or particle of it, is equally preſſed on all ſides, by the weight of the fluid above it.*

PROP. LXXXIII.

123. *If a fluid be at reſt in any veſſel, whoſe baſe is parallel to the horizon; the preſſure of the fluid upon the baſe, is as the baſe and perpendicular altitude of the fluid; whatever be the figure of the containing veſſel.*

CASE I.

Let *ABCD* be a cylinder or priſm, then (by Cor. 1. Prop. LXXXII.) the preſſure upon a given part of the baſe, (as a ſquare inch) is as the depth. And the preſſure upon the whole baſe is as the number of parts, or inches, contained in it; and therefore is as the baſe and altitude of the fluid.

CASE II.

124. Let the heights and baſes of the veſſels *ABC*, *DEF* be equal to thoſe of the cylinder *ABCD*; then ſince any part of the baſes *AB* or *DE* is equally preſt, as an equal part of the baſe *CD*; therefore the whole preſſure upon the baſes *AB* or *DE* is equal to the whole preſſure upon the baſe *CD*. And therefore is as the baſe and perpendicular height.

Cor. 1. *If two veſſels ABC, DEF of equal baſe and height, tho' never ſo different in their capacities; be filled with any the ſame fluid; their baſes will ſuſtain an equal quantity of preſſure; the ſame as a cylinder of the ſame baſe and height.*

Cor. 2. *The quantity of preſſure at any given depth, upon a given ſurface; is always the ſame, whether the ſurface preſſed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid continued upwards from the compreſſed ſurface, riſes perpendicularly into a rectilinear direction, or creeps obliquely thro' crooked cavities and canals; and whether theſe paſſages are regular or irregular, wide or narrow.* And hence

125. Cor. 3. *If ABDCF be any veſſel containing a fluid; and BL, ED, HFOK, and GC be perpendicular to the horizon, and GHAB the ſurface of the liquor; and FL, COD parallel to AB. Then the preſſure at L and F is as BL or HF; at D, O and C, as ED; at K as HK. And therefore the preſſure at L and F is the ſame. And the preſſures at D, O, C are equal.*

Cor. 4. *The preſſure is every where directed perpendicularly againſt* FIG.
the inner ſurface of the veſſel. Therefore at K it is directed down- 125.
ward, at L ſideways, and at F upwards. By Prop. IX.

Cor. 5. *If two veſſels AB, CD, communicate with one another by* 126.
the tube BC, and if any liquor be poured into one AB, it will riſe to the ſame height in the other CD, and will ſtand at equal heights in both; that is, AD will be a horizontal line.

For if the fluid ſtand at unequal heights, the preſſure in the higher will be greater than in the lower, and cauſe it to move towards the lower.

Cor. 6. *If two different fluids ſuſtain one another at reſt in two* 127;
veſſels AB, CD, that communicate; their height above their place of 128.
meeting, will be reciprocally as their denſities or ſpecific gravities.

Let the fluids join at *C*, and take the perpendicular height of *eC*, equal to that of *AB*. Then if the denſities of the fluids were equal they would ſuſtain one another at the equal heights *AB*, *Ce*. Therefore that the preſſure of the other fluid may be the ſame at *C*, its height muſt be ſo much greater as the denſity is leſs; that is, *CD* : *Ce* or *AB* :: denſity of *AB* : denſity of *CD*.

SCHOL.

The truth of the foregoing propoſitions may be eaſily proved experimentally. Take ſeveral tubes open at both ends, ſome ſtreight ſome crooked, with their low ends turned in all directions, and of ſeveral ſizes, regular and irregular. Put theſe into a veſſel of water to any depth, and the water will riſe up to the height of the external ſurface of the water in them all. But this is to be underſtood of ſuch tubes as are ſufficiently wide; for in capillary tubes immerſed in a veſſel of water, it riſes ſomething above the level, and that to heights reciprocally as the
diameters. Likewiſe, if water can riſe and be ſuſpended at the
height *B* in the capillary tube *AB*, it will be ſuſpended at the 129.
ſame height *B*, whilſt the part of the tube at *B* remains the ſame, whatever be the figure or wideneſs of the under part *CD*. And the aſcent and ſuſpenſion of water is the very ſame in vacuo. The ſame holds for any other fluids, but different fluids riſe to different heights. But quickſilver, inſtead of aſcending in a tube, ſinks in it, and has its ſurface depreſſed below the common ſurface, to depths which are reciprocally as the diameters of the

FIG. 129. tubes. But the forces by which fluids are ſuſpended in capillary tubes come under no hydroſtatic laws.

PROP. LXXXIV.

If a homogeneous body be immerſed in a fluid of the ſame denſity with itſelf, it will remain at reſt in any place and in any poſition; but a body of greater denſity than the fluid will ſink to the bottom, and a body of leſs denſity will riſe to the top.

CASE I.

130. Let the body *EGF* be immerſed in the fluid *AD*. Then ſince the body is of the ſame denſity as the fluid, therefore the body will preſs the fluid under it, juſt as much as the ſame quantity of the fluid put in its place. And therefore the preſſure of the body, together with that of the fluid above it, preſſes the fluid below as much as a column of the fluid of the ſame depth. Therefore the preſſure of the body at *F* againſt the fluid is equal to the preſſure of the fluid at *F* againſt the body. And therefore theſe two preſſures will remain in equilibrio, and the body will be at reſt.

CASE II.

If the body is more denſe, the preſſure againſt the fluid underneath is greater than that of an equal quantity of the fluid. Therefore the weight of the body will overcome the preſſure of the fluid under it, and it will ſink. But if the body be lighter, the preſſure of the fluid will overcome the weight of the body, and it will riſe to the top.

Cor. 1. *If ſeveral fluids of different denſities be mixt together in the ſame veſſel, the heavieſt will get to the loweſt place, and the lighteſt to the top; and thoſe of a mean denſity to the middle. And in any bodies whatever the heavieſt will be the loweſt.*

Cor. 2. *Hence bodies placed in fluids have a twofold gravity, the one true and abſolute, the other apparent or relative. Abſolute gravity is the force with which bodies tend downward; by this all ſorts of fluid bodies gravitate in their proper places, and their weight taken*

taken together compoſe the weight of the whole; for the whole is heavy as may be experienced in veſſels full of liquor. FIG. 130.

Relative gravity is the exceſs of the gravity of the body above that of the fluid. By this kind of gravity fluids do not gravitate in their proper places; that is, they do not preponderate, but hindering one another's deſcent, remain in their proper places as if they were not heavy.

Cor. 3. Hence an irregular body, or one that is heterogeneous, deſcending in a fluid, or if it move in any direction, and a line be drawn connecting the center of gravity and center of magnitude of the body, the body will ſo diſpoſe itſelf as to move in that line; and that the center of gravity will go foremoſt and the center of magnitude behind.

For there being more matter and leſs ſurface near the center of gravity, that part will be leſs reſiſted than near the center of magnitude; therefore the center of magnitude will be more retarded than the center of gravity, and will be left behind.

Cor. 4. Hence no body can be at reſt within a fluid, unleſs it be of the ſame ſpecific gravity as the fluid.

SCHOL.

What is here ſaid of bodies of greater denſity ſinking in a fluid, muſt be underſtood of ſuch as are ſolid. For if a body be hollow it may ſwim in a fluid of leſs denſity. But if the hollows or cavities be filled with the fluid, it will then ſink. Likewiſe bodies of greater ſpecific gravity being reduced to extremely ſmall particles, may then be ſuſpended in the fluid. But the forces by which this is done belong not to any laws of hydroſtatics.

PROP. LXXXV.

Bodies immerſed in a fluid and ſuſpended in it, loſe the weight of an equal bulk of the fluid.

For (by the laſt Prop.) if the body *EF* be of the ſame denſity as the fluid it loſes all its weight, and neither endeavours to aſcend or deſcend. Therefore, if it be lighter or heavier it

FIG. 130. only endeavours to aſcend or deſcend with the difference of the weights of the body and the fluid; and has therefore loſt the weight of as much of the fluid.

Cor. 1. *The fluid acquires the weight which the body loſes.*
For the ſum of the weights of the ſolid and fluid is the ſame, both before and after emerſion.

Cor. 2. *All bodies of equal magnitude immerſed in a fluid loſe equal weights, and unequal bodies loſe weights proportional to their bulks.*

Cor. 3. *The weights loſt by immerging one and the ſame body in different fluids are as the denſities of the fluids, or as their ſpecific gravities.*

Cor. 4. *Hence alſo, if two bodies of unequal bulks be in equilibrio in one fluid, they will loſe their equilibrium in another fluid of different denſity.*

SCHOL.

Since a body immerſed in a fluid loſes ſo much weight as that of an equal quantity of the fluid, therefore it tends downwards only with the difference of theſe weights; and this is the relative gravity of the body in the fluid. But if the body is ſpecifically lighter than the fluid, it ſeems to loſe more weight than it has, and hence the body will tend upwards with the difference of theſe weights. And this is the relative levity of the body in the fluid; ſuch as we ſee in feathers or ſmoke in the air, or cork in water.

PROP. LXXXVI.

The weight of a ſolid body floating upon a fluid is equal to the weight of a quantity of the fluid, as big as the immerſed part of the ſolid, cut off by the plane of the ſurface of the fluid.

For if the body be at reſt, the preſſure of the body upon the fluid underneath, is juſt the ſame as the preſſure of the fluid in the room of the immerſed part. And therefore the weight of one is equal to the weight of the other. FIG.

Cor. 1. *If the body be homogeneous, the weight or magnitude of the whole floating body is to the weight or magnitude of the part immerſed :: as the denſity or ſpecific gravity of the fluid is to the denſity or ſpecific gravity of the body.*

For the denſity of the fluid : denſity of the body :: weight (of the fluid equal to the immerſed part, or the weight) of the whole body : weight of the immerſed part.

Cor. 2. *If one and the ſame body float, or ſwim upon different liquids, the immerſed part in each liquid will be reciprocally as their denſities. And therefore a body will ſink deeper in a lighter fluid than in a heavier.*

PROP. LXXXVII.

If a floating body AFBE, or ſyſtem of bodies, be at reſt in a fluid, and D be the center of gravity of the whole body, and C the center of gravity of the fluid AFB equal to the immerſed part of the body; then, I ſay, the line CD will be perpendicular to the horizon. 131.

For as *C* is the center of gravity of the fluid *AFB*, it is the center of all the forces or weights of the parts of the water in *AFB*, tending downwards; but becauſe the body is at reſt, the ſame point *C* is alſo the center of all the preſſures of the fluid underneath tending upwards, by which the weight of the fluid *AFB* or of the body *AFBE* (equal to it by Prop. laſt) is ſuſtained. Therefore the ſum of all the forces tending upwards to *C*, is equal and contrary to the ſum of all the forces tending downwards from *D* (by Ax. 11.) becauſe that preſſure ſuſtains the body. But the weight of the body tending from *D* is perpendicular to the horizon. Therefore *CD* is perpendicular to the horizon.

Cor. If the whole body be as heavy or heavier than water, and be immerſed in it, the center of gravity will be the loweſt, and deſcend the foremoſt.

PROP.

PROP. LXXXVIII.

If a fluid, conſidered without weight, be encloſed in a veſſel, and ſtrongly compreſſed on all ſides, every part within it will be in the ſame compreſſed ſtate.

For if any particle was leſs preſſed than another, the greater preſſure would move the fluid towards the leſs compreſſed part, till their compreſſion became every where equal; and then the equal preſſures would ballance one another, and remain at reſt.

132. Cor. 1. *Hence any ſoft body as GHI, whoſe parts cannot be condenſed, being immerſed in a fluid encloſed in a veſſel, and ſtrongly compreſſed on every ſide, the body will retain its figure, and ſuffer no change from the compreſſion of the ambient fluid. And all its parts will remain at reſt among themſelves, and in the ſame compreſſed ſtate as the fluid.*

Cor. 2. *The motion of any included body as E, or of any number of bodies, will not be at all changed by the compreſſion of the fluid, but will remain the ſame as before.*

For the compreſſion acting every way alike, can make no alteration in the motion of bodies.

Cor. 3. *In an inflexible veſſel, a fluid will not ſuſtain a ſtronger preſſure on one ſide than another; but will give way to any exceſs of preſſure in a moment of time, and be reduced to an equality of preſſure.*

PROP. LXXXIX.

130. *If air, or any elaſtic fluid of ſmall denſity, be ſhut up in a cloſe veſſel, every part of it will be in the ſame compreſſed ſtate.*

For let *ABCD* be a veſſel full of encloſed air, then the air, at equal altitudes within the veſſel, will be in the ſame ſtate of compreſſion; and the compreſſion in the bottom of the veſſel can only exceed that at the top, by the weight of a column of air of the height

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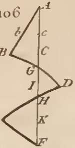

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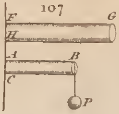

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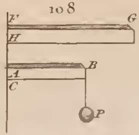

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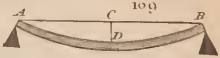

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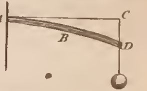

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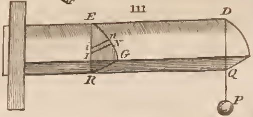

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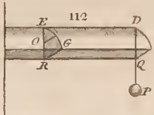

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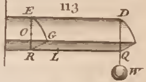

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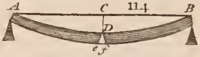

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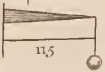

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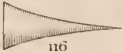

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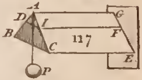

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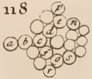

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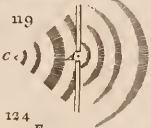

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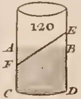

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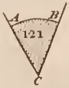

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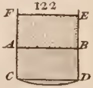

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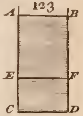

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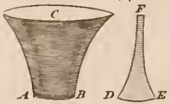

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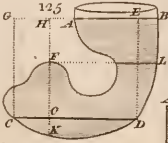

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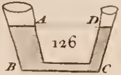

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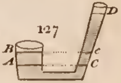

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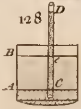

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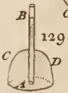

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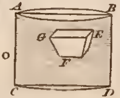

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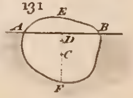

Pl.VI. pa. 120.

height of the veſſel *AC*, (by Cor. 1. Prop. LXXXII.) but the weight of ſuch a column of air is inſenſible in reſpect of the external preſſure, or the preſſure of the height of the atmoſphere. And therefore the compreſſion in every part of the veſſel may be looked upon to be the ſame.

Cor. 1. *In like manner the compreſſion of the air in any two places near the earth's ſurface is very nearly the ſame.*

For the difference is only the weight of a column of air, whoſe height is the difference of the heights of the two places, which is very inconſiderable.

Cor. 2. *If air be compreſſed in any veſſel by the preſſure of the external air, its elaſtic force is equal to the force and preſſure of the external air.*

Cor. 3. *The air does the ſame thing by its ſpring, as a non-elaſtic fluid does by its weight or preſſure.*

For the ſpring or elaſticity of the air is the force it exerts againſt the force of compreſſion, and therefore muſt be equal to it.

SCHOL.

That the air is a heavy, elaſtic, compreſſible body, is confirmed by many experiments made for that purpoſe. Its properties are theſe.

1. The air has ſome, though a very ſmall degree of weight, which is ſo ſmall, that it hardly becomes ſenſible, but in the weight of the whole atmoſphere, or body of air encloſing the earth.

2. The air is an elaſtic fluid, and capable of being condenſed and rarified. And when it is condenſed or forced into a leſs ſpace, its ſpring, or the force it exerts to unbend itſelf, is proportional to the force that compreſſes it. And the ſpace any given quantity takes up, is reciprocally as the compreſſing force; or its elaſticity is as its denſity.

3. All the air near the earth is in a compreſſed ſtate, by the weight of the atmoſphere or body of the air above, which compreſſes it. And from hence the denſity of the air grows continually leſs, the higher it is above the ſurface of the earth. The weight of the atmoſphere at the ſurface of the earth, is at a medium about 14½ lb. averd. upon every ſquare inch. But at different times it differs, by reaſon of winds, hot or cold weather, &c. But the height of the atmoſphere is uncertain, by reaſon it

grows continually more rare towards the top till it vaniſhes. The weight of the atmoſphere is equal to the weight of water 11 yards high.

4. The ſpring or elaſticity of the air is encreaſed by heat, and decreaſed by cold, ſo that if any quantity of air be encloſed in a veſſel, it will have a greater ſpring or preſſure when heated, and will loſe part of its ſpring by cold.

PROP. XC.

To find the ſpecific gravity of bodies.

CASE I.

If it be a ſolid body heavier than water, weigh it exactly, firſt in air, and then in water, or ſome fluid whoſe ſpecific gravity you know; and let

The abſolute weight of the body $= A$,
The weight in water, &c. $= B$,
The ſpecific gravity of water, &c. $= C$,
The ſpecific gravity of the body $= D$,

Then will $D = \frac{A}{A-B}C$, *the ſpecific gravity of the body.*

CASE II.

For a ſolid body lighter than water. Take any piece of metal and tie it to a piece of the light body, ſo that the compound may ſink in water; and putting A, C, D, as in Caſe I. and

$E =$ *weight of the metal in water.*
$F =$ *weight of the compound in water.*

Then $D = \frac{AC}{A+E-F}$, *the ſpecific gravity of the light body.*

CASE III.

For a fluid. Take a ſolid body of known ſpecific gravity, which will ſink in the fluid. And putting the ſame letters as in Caſe I *Then will* $C = \frac{A-B}{A}D$, *the ſpecific gravity of the fluid.*

Or thus:

Take a body that will ſink in the fluid and alſo in water, and let

A = *abſolute weight of the body,*
B = *its weight in water,*
G = *its weight in the fluid,*
C = *ſpecific gravity of water,*
Z = *ſpecific gravity of the fluid required,*

Then $Z = \frac{A-G}{A-B}C.$

But for mercury, or for powders, duſt, or ſmall fragments of bodies, you muſt uſe a glaſs or metal bucket, obſerving to balance its weight both in air and water. And for bodies that will diſſolve in water, weigh them in oil of turpentine inſtead of water. When the body is weighed in the fluid, it muſt be ſuſpended by a horſe hair, or a fine ſilk thread. Note, if the body and fluid be near the ſame ſpecific gravity, your work will be more exact.

To demonſtrate the rules, it is evident (by Prop. LXXXV.) that a body weighed in water, loſes the weight of as much water. Therefore in Caſe I. the weight of an equal quantity of water is $A-B$. But (by Def. 11.) the ſpecific gravities are as the weights of equal quantities of matter; therefore $A-B : A :: C : D$.

And in Caſe II. $F-E$ = weight of the light body in water, (which is negative when E is greater than F) and the weight of an equal quantity of water is $A-\overline{F-E}$ or $A+E-F$, therefore (as in Caſe I.) it is $A+E-F : A :: C : D$. And the rule is equally true, whether D be lighter or heavier than water.

In Caſe III. ſince $D = \frac{A}{A-B}C$, therefore $C = \frac{A-B}{A}D$. Or in the other rule; $A-B$ = weight of as much water, and $A-G$ = weight of as much of the fluid, and the ſpecific gravities being as the weights of equal quantities of the matter; therefore $A-B : A-G :: C : Z$.

Cor. 1. *Hence if a piece of metal, or any ſort of matter is offered, to know what ſort it is of. Find its ſpecific gravity by the rule above, which ſeek in the following table; and the neareſt to it gives the name of the body, or what kind it is of.*

Cor. 2. *And to find the ſolid content of a ſmall body heavier than water. Weigh it in air and water, and the difference of the weights reduced to grains, being divided by* 256; *the quotient is the cubic inches it contains.*

For a cubic inch of water weighs 256 grains; or a cubic foot weighs 76lb. troy, or 62 $\frac{1}{2}$ lb. averdupoiſe, which is but 254 grains to an inch.

Cor. 3. *Hence alſo the ſolidity of a body being known, the weight may be found, and the contrary. Thus, put* $n = 0.5275$ *ounces troy, or* 0.5787 *ounces averdupoiſe; and* $D =$ *ſpecific gravity of the body by the following table. Then as* $1 : nD :: $ *ſolid content in inches : weight in ounces; and one being given finds the other.*

For the weight of a cubic inch of water is .5275 oz. troy, or .5787 oz. averdupoiſe.

A TABLE of the ſpecific gravities of bodies.

SOLIDS.

Fine Gold	19.648
Standard Gold	18.888
Lead	11.340
Fine Silver	11.092
Standard Silver	10.536
Copper	9.000
Copper Halfpence	8.915
Fine Braſs	8.350
Caſt Braſs	8.100
Steel	7.850
Iron	7.644
Pewter	7.471
Tin	7.320
Caſt Iron	7.000
Lead Ore	6.200
Copper Ore	5.167
Lapis Calaminaris	5.000
Load-ſtone	4.930
Crude Antimony	4.000
Diamond	3.517
White Lead	3.160
Iſland Cryſtal	2.720

Marble

Marble	2.707
Pebble Stone	2.700
Coral	2.700
Jaſper	2.666
Rock Cryſtal	2.650
Pearl	2.630
Glaſs	2.600
Flint	2.570
Onyx-ſtone	2.510
Common Stone	2.500
Glauber Salt	2.250
Cryſtal	2.210
Oyſter Shells	2.092
Brick	2.000
Earth	1.984
Nitre	1.900
Vitriol	1.880
Alabaſter	1.874
Horn	1.840
Ivory	1.820
Brimſtone	1.800
Chalk	1.793
Borax	1.717
Allum	1.714
Clay	1.712
Dry Bone	1.660
Humane Calculus	1.542
Sand	1.520
Gum Arabic	1.400
Opium	1.350
Lignum Vitæ	1.327
Coal	1.250
Jet	1.238
Coral	1.210
Ebony	1.177
Pitch	1.150
Roſin	1.100
Mahogany	1.063
Amber	1.040
Brazil Wood	1.031
Box Wood	1.030
Common WATER	1.000
Bees Wax	.955
Butter	.940

FIG.

Logwood	·913
Ice	.908
Aſh (dry)	.838
Plumbtree (dry)	.826
Elm (dry)	.801
Oak (dry)	.800
Yew	·760
Crabtree	·700
Beech (dry)	·700
Walnut-tree (dry)	·650
Cedar	.613
Fir	·580
Cork	.238
New fallen Snow	.086

FLUIDS.

Quickſilver	14.000
Oil of Vitriol	1.700
Oil of Tartar	1.550
Honey	1.450
Spirit of Nitre	1.315
Aqua Fortis	1.300
Treacle	1.290
Aqua Regia	1.234
Spirit of Urine	1.120
Human Blood	1.154
Sack	1.033
Urine	1.032
Milk	1.031
Sea Water	1.030
Serum of human Blood	1.030
Ale	1.028
Vinegar	1.026
Tar	1.015
WATER	1.
Diſtilled Waters	·993
Red Wine	·990
Linſeed Oil	·932
Brandy	·927
Oil Olive	·913
Spirit of Turpentine	.874
Spirit of Wine	.866
Oil of Turpentine	.810
Common Air	.0012

In this table you have the mean ſpecific gravities. For there FIG. is ſome difference in different pieces of the ſame ſort of body, by reaſon of their different goodneſs, fineneſs, compactneſs, texture, dryneſs, being more or leſs free from mixture, &c. And ſometimes by a greater degree of heat or cold, which affect all bodies a little, from whence there will ariſe a ſenſible difference in different parcels of the ſame ſort of matter, in almoſt all bodies, whether ſolid or fluid. Particularly in wood there is great difference, for green wood is far heavier than dry wood, and ſome green wood will ſink in water, as elm.

PROP. XCI.

The center of preſſure of any plane ſuſtaining a fluid preſſing againſt it, is the ſame as the center of percuſſion, ſuppoſing the axis of motion to be at the interſection of this plane with the ſurface of the fluid.

The center of preſſure is that point againſt which a force being applied equal and contrary to the whole preſſure, it will juſt ſuſtain it, ſo as the body preſſed on, will incline to neither ſide. 133.

Let AF be the ſurface of the water, O the center of preſſure, draw AO, and parallel to AF, draw cbd. Then the preſſure againſt any ſmall part cd, is as cd and the depth of the fluid, that is as $cd \times Ab$. And the force to turn the plane about O, is $cd \times Ab \times bO$, or $cd \times Ab \times AO - cd \times Ab^2$. And the ſum of them all muſt be equal to 0. Therefore $AO = \frac{\text{ſum of all } cd \times Ab^2}{\text{ſum of all } cd \times Ab}$; and therefore (by Prop. LVII.) O is the ſame as the center of percuſſion.

Cor. 1. *The center of preſſure, upon a plane parallel to the horizon, or upon any plane where the preſſure is uniform, is the ſame as the center of gravity of that plane.*

For the preſſure acts upon every part, in the ſame manner as gravity does.

Cor. 2. *The quantity of preſſure upon any plane ſurface, is equal to that of the ſame plane, placed parallel to the horizon, at the depth*

FIG. 133. *depth where its center of gravity is. And the ſame is true of any number of ſurfaces taken together.*

For the whole preſſure is as the ſum of all the $Ab \times cd$; and upon the whole figure placed at the center of gravity, it is $ABC\times$diſtance of the center of gravity from A. But (by Cor. 3. Prop. XLIV.) theſe products are equal. And the ſame may be proved for ſeveral ſurfaces, or the ſurface of any ſolid, taking the center of gravity of all theſe ſurfaces.

PROP. XCII.

To find the center of equilibrium of a body, or a ſyſtem of bodies, immerſed in a fluid.

The center of equilibrium is the ſame with reſpect to bodies immerſed in a fluid, as the center of gravity is to bodies in free ſpace: It is a certain point, upon which if the body or bodies be ſuſpended, they will reſt in any poſition.

67. Let A, B, C be three bodies, or the quantities of matter in them; p, q, r their relative gravities in the fluid; $1=$abſolute gravity. Then pA, qB, rC are the weights of A, B, C in the fluid. Let G be the center of equilibrium. Then, by the ſame reaſoning as in Prop. XLVII. the ſum of the forces of A, B, C is $pA\times Aa + qB \times Bb + rC\times Cc = Gg \times \overline{pA + qB + rC}$, the ſum of the forces or weights when ſituated in G. *Whence* $Gg = \frac{Aa \times pA + Bb \times qB + Cc \times rC}{pA + qB + rC}$, *the diſtance of the center of equilibrium from ST, in the fluid.* And if any body, as A, is lighter than the fluid; then its relative gravity p will be negative. And if any body is ſituated on the other ſide of the plane, its diſtance from it muſt be taken negative.

Cor. If the body or bodies be homogeneous, the center of equilibrium is the ſame as the center of gravity.

SCHOL.

The relative gravity is found thus. Take the ſpecific gravity of the fluid from that of the body, and divide the remainder by the ſpecific gravity of the body. And theſe ſpecific gravities are had by Prop. XC.

PROP.

FIG. 81.

PROP. XCIII.

If a ſyſtem of bodies oſcillate in a fluid, without reſiſtance; to find the length of an iſocronal pendulum vibrating in vacuo.

Becauſe particles of different ſpecific gravities, placed in any given point, will require different times of vibrating in the fluid; therefore we muſt find the point where a particle of infinite denſity being placed, will vibrate in the ſame time as the ſyſtem: and this will be the center of oſcillation. For this particle will loſe nothing of its weight in the fluid; its relative gravity being the ſame as the abſolute. Whence the vibrations of this particle will be performed in the ſame time as in vacuo.

Let A, B, C be three bodies, or their quantities of matter; p, q, r their relative gravities in the fluid; $1 =$ abſolute gravity. Then pA, qB, rC are the weights of the bodies in the fluid. Let G be the center of equilibrium; and O the center of oſcillation ſought. Put $s = A \times SA^2 + B \times SB^2 + C \times SC^2$. Then (by the ſame reaſoning, and conſtruction, as in Prop. LVIII.) the angular velocities which the bodies A, B, C generate in the ſyſtem are, $\frac{-Se \times pA}{s}$, $\frac{Sn \times qB}{s}$, $\frac{Sd \times rC}{s}$; and the whole angular velocity generated by them all, is $\frac{-Se \times pA + Sn \times qB + Sd \times rC}{s}$. Likewiſe the angular velocity which the particle P, ſituated in O, generates in the ſyſtem, is $\frac{Sr \times P}{P \times SO^2}$ or $\frac{Sr}{SO^2}$. But their vibrations are performed alike: therefore their angular velocities muſt be equal. That is $\frac{-Se \times pA + Sn \times qB + Sd \times rC}{s} = \frac{Sr}{SO^2} = \frac{Sg}{SG \times SO}$. Whence $SO = \frac{Sg}{SG} \times \frac{s}{-Se \times pA + Sn \times qB + Sd \times rC}$. But (by Prop. XCII.) $-Se \times pA + Sn \times qB + Sd \times rC = Sg \times \overline{pA + qB + rC}$ therefore $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SG \times \overline{pA + qB + rC}}$, *the length of an iſocronal pendulum, out of the fluid.*

Cor.

FIG. 81. Cor. 1. *Hence if the bodies are homogeneous, then* $p = q = r$; *and* $SO = \dfrac{A \times SA^2 + B \times SB^2 + C \times SC^2}{p \times SG \times \overline{A+B+C}}$.

Cor. 2. *The ſyſtem makes an exceeding ſmall vibration in the fluid, in the ſame time that a ſimple pendulum, whoſe length is* $\dfrac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SG \times \overline{pA+qB+rC}}$, *makes a vibration in vacuo.*

For the velocity of the ſyſtem being very ſmall; the reſiſtance is inconſiderable.

Cor. 3. *Hence if* SA *be the length of a ſimple pendulum* (A), *vibrating in a fluid: then* $\dfrac{SA}{p}$ *is the length of an iſocronal pendulum in vacuo.*

For in a ſingle body, $SO = \dfrac{A \times SA^2}{SA \times pA}$ or $\dfrac{SA}{p}$.

Cor. 4. *And if* p *be negative, or the pendulum ſpecifically lighter than the fluid; the pendulum will turn upſide down, and vibrate upwards in the fluid. And the length of an iſocronal pendulum out of the fluid, will be* $\dfrac{SA}{p}$, *as before.*

SCHOL.

The center of percuſſion is the ſame in a fluid as out of it. For there is nothing concerned in that, but the quantities of matter and velocities; both of which are the ſame in the fluid, as out of the fluid.

The relative gravities p, q, r are found by the Schol. of the laſt Prop.

PROP. XCIV.

If a fluid runs thro' any tube, pipe, or canal, and always fills it; its velocity in any place will be reciprocally as the area of the ſection in that place.

Let

FIG. 134.

Let *AB*, *CD* be two ſections at *A* and *C*; and let the quantity of the fluid *ABDC*, in a very ſmall time, be tranſlated into the part *abdc* of the pipe. Draw *Pp*, *Qq* perpendicular to *AB*, *CD*, or parallel to the motion of the fluid; then *Pp*, *Qq* being indefinitely ſmall, will be the velocities of the fluid at *P* and *Q*, or the ſpaces deſcribed in that ſmall part of time. Then, becauſe the pipe is always full, the quantity $ABDC = abdc$. Take from both, the part *abDC*, which is common; and there remains $ABba = CDdc$, that is the ſection $APB \times Pp =$ ſection $CQD \times Qq$.

Cor. 1. *The quantity of motion of the fluid in the pipe* AD, *at any ſection* CD; *is equal to the motion of a cylinder of that fluid, whoſe baſe is* CD, *and length the ſame with the pipe from the beginning to that ſection, and velocity that of the fluid at* CD. *Or the quantity of motion at* CD, *is as the length of the pipe to that ſection* CD.

For ſince the velocity in *A* is as $\frac{1}{APB}$, the motion of *ABba* is $\frac{ABba}{APB}$, that is as *Pp*; and the motion of the whole, is as the ſum of all the *Pp*, or the length of the pipe, without any regard to the diameter *AB*. The cylinder, whoſe baſe is *CQD*, and height *PQ* is $= PQ \times CQD$; and its motion with velocity at *C*, is $= \frac{PQ \times CQD}{CQD} = PQ$ the length of the pipe, as before.

Cor. 2. *If water is driven through the pipe* PR *by ſome given force acting at* P, *and the length of the pipe* PR *be given; the quantity of water diſcharged at* R, *in a ſecond, or ſome given time, will be as the ſection at* R.

For if the force is given, the motion generated in a given time, will be given; and this motion, being as the quantity of water × velocity at *R*; therefore the quantity forced through *R*, will be reciprocally as the velocity, or directly as the ſection at *R*, by this Prop.

Cor. 3. *The velocity and quantity of motion, is the ſame very near in crooked tubes as in ſtreight ones; and in pipes divided into ſeveral branches; taking the ſection of all the branches as the ſection of one tube.*

FIG.

PROP. XCV.

135. *In any pipe whoſe ſection is ABCD, the ſtreſs or force to ſplit any part of the pipe at B, is equal to half the preſſure of the fluid upon the plane BD, drawn perpendicular to the curve in B, and of the ſame length as that part of the pipe.*

Let Ee be any ſmall part of the ſurface; draw EO perp. to the curve Ee. Draw En, en perp. to BO, and er perp. to EN. And let OE repreſent the preſſure of a particle of the fluid, then $EO\times Ee=$ preſſure upon Ee. The force OE may be divided into the two ON, NE; of which ON tends only to ſplit the tube at A, but that in direction NE is the force to ſeparate the parts at B. Therefore $EN\times Ee$ is the ſtreſs at B. But the triangles Eer, EON are ſimilar, and $Ee\times EN=EO\times er$, or $EO\times Nn$. Therefore the part of the preſſure on Ee, in direction NE, from whence the ſtreſs at B ariſes, is $=EO\times Nn$, that is $=$ to the preſſure upon the plane Nn. Conſequently the ſtreſs ariſing from the preſſure on BE is $=$ preſſure on BN, and from the preſſure on BA is $=$ preſſure on BO. And the ſtreſs at D by the preſſure on AD, is equal to the preſſure on OD.

Alſo we ſuppoſe the ſame forces acting in the ſemi-circle BCD: but theſe ſerve only to keep the forces, acting upon BAD, in equilibrio.

Cor. 1. *The ſtreſs on any part of a pipe full of water, is as the diameter of the pipe, and the perpendicular height of the water above that place. And conſequently the thickneſs of the metal ought to be in that ratio.*

Cor. 2. *In any concave ſurface, caſk, or veſſel, generated by revolving round an axis, and filled with a fluid; the ſtreſs as to ſplitting is equal to half the preſſure upon the plane paſſing through its axis. And the ſtreſs on both ſides at B and D, equal to the whole preſſure on that plane.*

Cor. 3. *Hence the internal preſſure on any length of the pipe, is to the ſtreſs it ſuffers as to ſplitting* :: *as* 2×3.1416, *to* 1.

Cor. 4. *Hence it follows, that the ſtreſs, ariſing from any preſſure, upon any part, to ſplit it longitudinally, tranſverſely, or in*

FIG. 135. *any direction, is equal to the preſſure upon a plane, drawn perpendicular to the line of direction. Thus the ſtreſs ariſing from the preſſure on BE is = preſſure on BN.*

Cor. 5. *And if the pipe be flexible; it will, by the preſſure, be put into a cylindrical form; or ſuch that the ſection is a circle.*

For if *BD* be greater than *AC*, there will be a greater preſſure in direction *OA* than in direction *OB*. And the greater preſſure will drive out the ſides *A* and *C*, till *AC* become equal to *BD*; and *ABCD* be a circle. Beſides, a circle is more capacious than any other figure. And if a pipe be not flexible, yet the preſſure of the fluid will always endeavour to put it into a circular figure.

Cor. 6. *And if an elaſtic compreſſed fluid be incloſed in a veſſel, flexible, and capable of being diſtended every way; it will form itſelf into a ſphere, for the ſame reaſon.*

PROP. XCVI.

If a cloſe flexible tube AB full of air, be immerſed wholly or in part in the water CDEF; the force to ſplit it, in any place O, is proportional to AO, its height from A. 136.

For the air compreſſed at *A* is in equilibrio with the external preſſure of the water. At *B* and *O* the air is in the ſame compreſſed ſtate as in *A*, but the external preſſure at *B*, is leſs by the weight of the column of water *AB*: and at any place *O*, the external preſſure is leſs than in *A*, by the weight of *AO*; therefore the internal preſſure at *O* exceeds the external, by the weight of the column of water *AO*. And the ſtreſs at *O* is as that exceſs.

Cor. 1. *The ſtreſs is greateſt at the top BG, and at A is nothing.*

Cor. 2. *If the tube be inflexible; the ſtreſs will be according to the ſtate of the compreſſed air within it. If the air within be the ſame as the external air at B, then the ſtreſs at O is as BO. If it be leſs then the external air, the increaſe of the ſtreſs will be alſo as BO; acting at the outſide. If it be of very great denſity, the increaſe of ſtreſs at O is as AO.*

For

FIG. 136. For the preſſure within is uniform; but without, it is as the depth of the water.

PROP. XCVII.

The quantity of a fluid flowing in any time through a hole in the bottom or ſide of a veſſel, always kept full; is equal to a cylinder whoſe baſe is the area of the hole, and its length the ſpace a body will deſcribe in that time, with the velocity acquired by falling through half the height of the liquor above the hole.

137. Let *ADB* be a veſſel of water, *B* the hole, and take $BC = BD$ the height of the water. And let the cylinder of water *BC* fall by its weight through half *DB*, and it will by that fall, acquire ſuch a motion, as to paſs through *DB* or *BC* uniformly in the ſame time, by Cor. 3. Prop. XIV. But (by Prop. LXXXIII. and Cor. 2.) the water in the orifice *B* is preſſed with the weight of a column of water, whoſe baſe is *B* and height *BD* or *BC*; therefore this preſſute is equal to the weight of the cylinder *BC*. But equal forces generate equal motions: therefore the preſſure at *B*, will generate the ſame motion in the ſpouting water, as was generated by the weight of the cylinder of water *BC*. Therefore in the time of falling through half *DB*, a cylinder of water will ſpout out, whoſe length (or the ſpace paſſed uniformly over) is *BC* or *BD*. And in the ſame time repeated, another equal cylinder *BC* will flow out, and in a third part of time, a third, *&c.* Therefore the length of the whole cylinder run out, will be proportional to the time, and conſequently the velocity of the water at *B* is uniform. Therefore in any time, the length of a cylinder of water ſpouting out, will be equal to the length deſcribed in that time, with the velocity acquired by falling through half *DB*.

Cor. 1. *Hence in the time of falling through half* DB, *a quantity of the fluid runs out, equal to a cylinder whoſe baſe is the hole; and length, the height of the fluid above the hole.*

Cor. 2. *The velocity in the hole* B *is uniform, and is equal to that a heavy body acquires by falling through half* DB.

Cor.

FIG. 138.

Cor. 3. *But at a ſmall diſtance without the hole, the ſtream is contracted into a leſs diameter, and its velocity increaſed; ſo that if a fluid ſpout through a hole made in a thin plate of metal, it acquires a velocity nearly equal to that, which a heavy body acquires by falling the whole height of the ſtagnant fluid above the hole.*

For ſince the fluid converges from all ſides towards the center of the hole *BF*; and all the particles endeavouring to go on in right lines, but meeting one another at the hole, they will compreſs one another. And this compreſſion being every where directed to the axis of the ſpouting cylinder; the parts of the fluid will endeavour to converge to a point, by which means the fluid will form itſelf into a ſort of a conical figure, at ſome diſtance from the hole as *BEGF*. By this lateral compreſſion, the particles near the ſides of the hole are made to deſcribe curve lines as *HE*, *KG*; and by the direct compreſſion, the fluid from the hole is accelerated outwards at *EG*; and thus the ſtream will be contracted at *E*, in the ratio of about $\sqrt{2}$ to 1, and the velocity increaſed in the ſame ratio.

It muſt be obſerved, however, that the particles of the fluid don't always move right forward; but near the edge of the hole, often in ſpiral lines. For no body can inſtantly change its courſe in an angle, but muſt do it gradually, in ſome curve line.

Cor. 4. *The fluid at the ſame depth, ſpouts out nearly with the ſame velocity, upwards, downwards, ſideways, or in any direction. And if it ſpout vertically, aſcends nearly to the upper ſurface of the fluid.*

Cor. 5. *The velocities of the fluid, ſpouting out at different depths, are as the ſquare roots of the depths.*

For the velocities of falling bodies are as the ſquare roots of the heights.

Cor. 6. *Hence if* $s = 16\frac{1}{12}$ *feet,* D = *depth of the veſſel to the center of the hole,* F = *area of the hole, all in feet,* t = *time in ſeconds. Then the quantity of water running out in the time* t, *by this Prop. will be* $tF\sqrt{2Ds}$ *feet, or* $6.128tF\sqrt{2Ds}$ *ale gallons.*

SCHOL.

There are ſeveral irregularities in ſpouting fluids, ariſing from the reſiſtance of the air, the friction of the tubes, the big-

neſs

FIG. 138. neſs and ſhape of the veſſel, or of the hole, *&c.* A fluid ſpouts fartheſt through a thin plate; if it ſpout through a tube inſtead of a plate, it will not ſpout ſo far; partly from the friction, and partly becauſe the ſtream does not converge ſo much, or grow ſmaller. A jet de'au ſpouts higher, if its direction be a little inclined from the perpendicular; becauſe the water in the uppermoſt part of the jet, falls down upon the lower part, and ſtops its motion. We find by experience, a fluid never ſpouts to the full height of the water above the hole; but in ſmall heights falls ſhort of it, by ſpaces, which are as the ſquares of the heights of the fluid. And all bodies projected upwards, fall ſhort of thoſe projected in vacuo, by ſpaces which are in the ſame ratio; from the reſiſtance of the air.

By experiments, if the height of a reſervoir be 5 feet, a jet will fall an inch ſhort; and the defect will be as the ſquare of the height of the reſervoir. But ſmall jets fail more than in that proportion, from the greater reſiſtance of the air.

PROP. XCVIII.

139. *If a notch or ſlit, fghi, in form of a parallelogram, be cut out of the ſide of a veſſel full of water, ADE; the quantity of water flowing out of it, will be* $\frac{2}{3}$ *the quantity flowing out of an equal orifice, placed at the whole depth gi, or at the baſe hi; in the ſame time: the veſſel being ſuppoſed to be always kept full.*

For draw the parabola *goh*, whoſe axis is *gi*, and baſe *hi*, and ordinate *ro*; then ſince the velocity of the fluid at any place *r*, is as $\sqrt{gr}$, (by Cor. 5. of the laſt Prop.) that is (by the nature of the parabola) as the ordinate *ro*; therefore *ro* will repreſent the quantity diſcharged at the depth or ſection *rn*. Alſo *hi* will repreſent the quantity diſcharged at the depth or baſe *hi*. Conſequently the ſum of all the ordinates *ro*, or the area of the parabola, will repreſent the quantity diſcharged at all the places *rn*. And the ſum of all the lines *hi* or *rn*, or the area of the parallelogram *fghi*, will repreſent the quantity diſcharged by all the ſections *rn*, placed as low as the baſe *hi*. But the parabola is to the parallelogram, as $\frac{2}{3}$ to 1.

FIG. 139.

Cor. 1. *Let* $s=16\frac{1}{12}$ *feet.* $D=gi$, *the depth of the ſlit.* $F=$ *area of the ſlit, ſhig. Then the quantity flowing out in any time or number of ſeconds* t, *is* $=\frac{2}{3}tF\sqrt{2Ds}$.

This follows from Cor. 6. of the laſt Prop.

Cor. 2. *The quantity of fluid diſcharged through the hole rnhi, is to the quantity which would be diſcharged through an equal hole placed as low as hi; as the parabolic ſegment robi, to the rectangle rnhi.*

This appears from the reaſoning in this propoſition.

SECT. X.

The reſiſtance of fluids, their forces and actions upon bodies; the motion of ſhips, and poſition of their ſails.

PROP. XCIX.

A body deſcending in a fluid, adds a quantity of weight to the fluid, equal to the reſiſtance it meets with in falling.

For the reſiſtance is equal to the gravity loſt by the body. And becauſe action and re-action are equal and contrary, the gravity loſt by the body, is equal to that gained by the fluid. Therefore the reſiſtance is equal to the gravity gained by the fluid.

Cor. 1. *If a body aſcends in a fluid; it diminiſhes the gravity of the fluid, by a quantity equal to the reſiſtance it meets with.*

Cor. 2. *This increaſe of weight ariſing from the reſiſtance, is over and above the additional weight mentioned in Cor.* 1. *Prop.* LXXXV.

Cor. 3. *If a heterogeneous body deſcend in a fluid; it will endeavour to move with its center of gravity foremoſt, leaving the center of gravity of as much of the fluid, behind.*

For the ſide towards the center of gravity contains more matter, and will more eaſily make its way through the fluid; and be leſs retarded in it.

PROP.

PROP. C.

If any body moves through a fluid; the reſiſtance it meets with, is as the ſquare of its velocity.

For the reſiſtance is as the number of particles ſtruck, and the velocity with which one particle is ſtruck. But the number of particles of the fluid which are ſtruck in any time, is as the velocity of the body. Therefore the whole reſiſtance is as the ſquare of the velocity.

Cor. 1. *The reſiſtances of ſimilar bodies moving in any fluids, are as the ſquares of their diameters, the ſquares of their velocities, and the denſities of the fluids.*

For the number of particles ſtruck with the ſame velocity, are as the ſquares of the diameters, and the denſities of the fluids.

Cor. 2. *If two bodies A, B, with the ſame velocity, meet with the reſiſtances p and q; their velocities will be as* $\frac{1}{\sqrt{p}}$ *and* $\frac{1}{\sqrt{q}}$, *when they meet with equal reſiſtances.*

For let b be the common velocity, then $p : bb :: q : \frac{bbq}{p}$, and $b\sqrt{\frac{q}{p}}$ = velocity of A to have the reſiſtance q; and ſince b = velocity of B to have the ſame reſiſtance q; therefore vel. A : vel. $B :: b\sqrt{\frac{q}{p}} : b :: \frac{1}{\sqrt{p}} : \frac{1}{\sqrt{q}}$.

PROP. CI.

The center of reſiſtance of any plane moving directly forward in a fluid, is the ſame as the center of gravity.

The center of reſiſtance is that point, to which if a contrary force be applied, it ſhall juſt ſuſtain the reſiſtance.

Now the reſiſtance is equal upon all equal parts of the plane, and therefore the reſiſtance acts upon the plane, after the ſame manner, and with the ſame force as gravity does; therefore the center of both the reſiſtance and gravity muſt be the ſame.

Cor. 1. *In any body moving through a fluid; the line of direction of its motion will paſs through the center of reſiſtance, and center of gravity of the body.*

For if it do not, the forces ariſing from the weight and reſiſtance, will not balance one another, which will cauſe the body to librate or oſcillate in the fluid; till by degrees the ſituation of theſe two centers will fall into the line of their motion.

Cor. 2. *And for the ſame reaſon; if a globe, moving in a fluid, oſcillates or turns round its axis; that ſide, which in oſcillating moves againſt the fluid, ſuffers a greater force or reſiſtance; and therefore the body is driven from that part, and made to recede from that ſide, and deflect to the other ſide; and perhaps deſcribe a curve line in the fluid.*

PROP. CII.

140. *If a non-tenacious fluid, ſuch as the wind, &c. move againſt the ſail SA or any plane ſurface, in direction WS; it ſhall urge it in a direction WA perpendicular to that ſurface, with a force, which is as the ſquare of the velocity, the ſquare of the ſine of the angle of incidence, the magnitude of the ſail, and the denſity of the fluid.*

Draw *WA*. *AC* perpendicular to *SA*, *SW*; and the force of the fluid upon *SA*, is as the force of one particle, and the number of them falling on *SA*.

But (by Cor. 1. Prop. IX.) the force of one particle is as its velocity × *S*. incidence *WSA*.

And the number of them (ſuppoſing the denſity to be given) is as their velocity × *CA*, or (ſuppoſing the ſail *SA* given) as the velocity × *S*. *WSA*.

Therefore the force of the fluid upon the ſail *SA*, is as the ſquare of the velocity, and the ſquare of the ſine of *WSA*.

Increaſe the denſity of the fluid, and the magnitude of the ſail, in any ratio; and its evident the force of the fluid againſt the ſale, will be increaſed in the ſame ratio.

Cor. 1. *And if a thin body move in a fluid at reſt; the ſame law holds in reſpect of the reſiſtance it meets with, in the direction perpendicular to its ſurface.* FIG. 140.

Cor. 2. *If a fluid with a given velocity move in direction WS, againſt the ſail SA; its force to move the ſail in any given direction SB, is as the ſquare of the S.WSA × by the S.ASB. And if WSB be a right angle, as S.WSA × $\frac{1}{2}$ ſine of twice WSA.* 141.

For let *SD*, *DB* be perpendicular to *SA*, *SB*. Then *SD* is the whole force acting at *S*, and *SB* the force in direction *SB*, and *SB* is as *S.SDB* or *ASB*. And if *ASB* be the comp. of *WSA*, then *S.WSA* × coſ. *WSA* is as *S*. twice *WSA*, by trigonometry.

Cor. 3. *The force of a fluid in direction WS, to move the ſail or body SA in the ſame direction WS; is (ceteris paribus) as the cube of the ſine of incidence WSA.*

For then *WSB* will be one continued ſtreight line.

Cor. 4. *But the force of a given ſtream of a fluid, againſt any ſail SA, to move it perpendicular to its ſurface, is ſimply as the S. angle of incidence: but to move it in the ſame direction with itſelf, as the ſquare of the S. incidence: all things elſe remaining the ſame.*

This follows from Cor. 1. Prop. IX. and Cor. 2. of this.

SCHOL. I.

If the angle *WSB* be given; the fluid will have the greateſt force poſſible againſt the ſail, to move it in direction *SB*; when its poſition is ſuch, that the ſine of the diff. of the angles $\overline{WSA-ASB}$, may be $\frac{1}{3}$ the ſine of the ſum *WSB*.

SCHOL. II.

If the fluid be tenacious it will urge the body in the ſame direction with itſelf, and with a force which is as the ſine of incidence: or univerſally, as the ſine of incidence, the ſquare of the velocity, the magnitude of the ſail, and denſity of the fluid. 140.

For by reaſon of the tenacity of the fluid, the ſail is acted on by both the forces *WA*, *AS*, which are equivalent to *WS*.

FIG.

PROP. CIII.

142. *If a very thin and light body SA, plain on both ſides, be placed in a very denſe fluid, which moves in direction WS; and the body can make little or no way thro' the fluid, but only in the direction of its length SA. And if the body be obliged to move parallel to itſelf in a given direction SD. I ſay the body will be ſo moved in the fluid, that its abſolute velocity will be* $= \frac{S.WSA}{S.ASD} \times$ *velocity of the fluid.*

Draw *DT* parallel to *AS*, and produce *WS* to *T*. Then whilſt a part of the fluid moves from *S* to *T*, the body will be moved into the line *TD*; and ſince *SD* is the direction of its motion, the point *S* will be found in *D*. And therefore the velocities of the fluid, and of the body, will be as *ST* to *SD*; that is, as *S.TDS* or *DSA*, to *S.STD* or *WSA*.

Cor. 1. *If WS the direction of the fluid, is perpendicular to SD the direction of the body; then the velocity of the body SA will be = tan. WSA × velocity of the fluid.*

For $\frac{S.WSA}{S.ASD} = \frac{S.WSA}{cof.\ WSA} =$ tan. *WSA*, radius being = 1.

Cor. 2. *And hence if the body SA continually turn round an axis parallel to WS; then the velocity of SA in direction perpendicular to WS, will be as the tan. WSA × velocity of the fluid.*

For *SA* in this caſe will always have the ſame poſition to the direction of the fluid, as before.

143. *Cor.* 3. *If a very thin body SA be obliged to move parallel to itſelf, through a very denſe fluid at reſt; and if it be drawn with a given velocity in direction always parallel to SW. Its abſolute velocity in the fluid, will be reciprocally as the coſ. WSA, and in direction SA.*

Draw *AC* perpendicular to *SW*. Then by reaſon of the denſity and reſiſtance of the fluid, the body will not be able to move laterally, but only in direction *SA*. But the velocities of the point *S* in directions *SW*, *SA* are as *SC* to *SA*, or as coſ. *CSA* to rad. Therefore vel. in direction $SA = \frac{rad.}{coſ.\ CSA} \times$ vel. in direction *SW*.

PROP.

PROP. CIV.

If a plane ſurface SA*, moving parallel to itſelf, with velocity and direction* SD*, be acted upon by a fluid moving with velocity and direction* WS*. And if* WF *be drawn parallel and equal to* SD*; and* FS *drawn. I ſay the fluid acts upon the plane in the angle* FSA*, with the relative velocity* FS*.* 144.

For complete the parallelogram $WSDF$; and let the body be at reſt, and the fluid move with the contrary motion DS or FW; and then their relative motions will be the ſame as before: and the fluid will have the two motions FW, FD in reſpect of the body SA at reſt. Therefore (by Cor. 2. Prop. VII.) the motion compounded out of theſe is FS; which is the abſolute motion of the fluid, ſuppoſing the body at reſt; or the relative motion of it, in reſpect of the moving body; and therefore acts on it in the angle FSA.

Cor. If F *falls in the line* SA*, then the fluid acts not at all upon the body. And if it fall on the contrary ſide of it; then the fluid acts on the contrary ſide of the body* SA*.*

SCHOL.

The fluid will move with the ſurface in direction SD, with the greateſt force; when it has ſuch a poſition, that the ſine of the diff. of the angles, $\overline{FSA-ASD}$, may be $\frac{1}{3}$ the ſine of the angle FSD.

And when the angle WSA is given, the fluid will have the greateſt force upon the ſail SA, to move it in direction SD, when the $S.$ angle ASD is equal to $\frac{WS}{WF} \times \frac{1}{3}$ the $S.$ of WSA.

PROP. CV.

Let SA *be the ſail of a ſhip,* SD *the poſition of her keel;* SK*,* DK *perpendicular to* SA*,* SD*. And if* DE*,* DS *be as the reſiſtances the ſhip has ahead and aſide, with equal velocities; and if* DC *is a mean* 145.

FIG. 145. *a mean proportional between DE and DK; then SC will be the way of the ſhip nearly.*

For let SK perpendicular to SA repreſent the force of the wind upon the ſail. The force SK is reſolved into the forces SD, DK; SD is the direct force, and DK the force producing her lee way. By Prop. C. her reſiſtance ahead with velocity SD : reſiſt. ahead with vel. $DE :: SD^2\ DE^2$, and reſiſt. ahead with vel. DE : reſ. aſide with vel. $DE :: DE : SD$, and reſ. aſide with vel. DE : reſ. aſide with vel. $DC :: DE^2 : DC^2$.

Therefore ex equo.

Reſ. ahead with vel. SD : reſ. aſide with vel. $DC :: SD^2 \times DE^3 : DE^2 \times SD \times DC^2 :: SD \times DE : DC^2$.

But the reſiſtances are as the forces producing them, therefore $SD : DK :: SD \times DE : DC^2 = DE \times DK$.

Cor. 1. *Let r = ſhip's reſiſtance ahead, R = ſhip's reſiſtance aſide, with the ſame velocity. Then $R : r ::$ radius $\times$ cotan. ASD : tan. ſquare of DSC, the leeway.*

For let rad. $= 1$. tan. $DSK = t$. Then $1 : t :: SD : DK = t \times SD$; and $SD : DC$ or $\sqrt{DE \times DK}$ or $\sqrt{t \times SD \times DE} :: 1 :$ tan. $DSC = \sqrt{\frac{t \times DE}{SD}}$.

Cor. 2. *Hence the tangent of the lee-way, in the ſame ſhip, is as the ſquare root of the cotangent of the angle ASD, which the ſail makes with the keel. Therefore if the lee-way be known for any poſition of the ſail, it will be known for all.*

SCHOL.

The lee-way of a ſhip is generally ſomething more than is here aſſigned; becauſe her hull and rigging will make her drive a little to the leeward, directly from the wind.

PROP. CVI.

146. *If the wind with a given velocity, in direction WS, fall on the ſail SA of a ſhip, making little or no lee-way; it will urge the ſhip in direction of the keel SD, with a force, which is as $\overline{S.WSA}^2 \times S.ASD$.*

Draw

FIG. 144.

Draw SC perpendicular to SA, and CD to SD. And (by Prop. CII.) the force acting upon the ſail in direction SC, is as the ſquare of the ſine of WSA. But the forces in directions SC and SD are as SC to SD, or as radius 1 to the ſine of SCD or ASD. Therefore the force in direction $SD = S.ASD \times$ force in direction $SC = S.ASD \times S.^2WSA$.

Cor. 1. *The force acting in direction DC perpendicular to the keel, is as* $\overline{S.WSA}^2 \times cos.\ ASD$.

Cor. 2. *The force in direction SD will be univerſally as* $\overline{S.WSA}^2 \times S.ASD$, *and the ſquare of the velocity of the wind, and magnitude of the ſail.* 146.

Cor. 3. *The velocity of the ſhip in direction SD, is as* $S.WSA \times \sqrt{S.ASD} \times$ *velocity of the wind.*

For the ſquare of the velocity of the ſhip in any direction, is as the reſiſtance in the water, or (its equal) the force of the wind upon the ſail in that direction; that is (by Cor. 2.) as $\overline{S.WSA}^2 \times S.\ ASD$, and the ſquare of the velocity of the wind. The denſity, and ſail being given.

Cor. 4. *Let the angle WSA be given. And if SDC be a ſemicircle deſcribed on any given line SC; then the force in any direction SD of the keel, is as the cord SD; and the velocity as* $\sqrt{SD}$.

Cor. 5. *The velocity of the ſhip to windward, is as* $S.WSA \times \sqrt{S.ASD} \times cos.\ WSD$.

For draw SP perpendicular to WS, and DG to SP; and the velocities in directions SD, GD are as SD to GD, or as radius 1 to $S.DSG$: therefore the velocity in $GD = S.DSG \times S.WSA \times \sqrt{S.ASD}$.

Cor. 6. *The force of the ſail SA to turn the ſhip about, is as* $\overline{S.WSA}^2 \times cos.\ ASD$.

This appears by Cor. 1. ſuppoſing the ſail placed in the head of the ſhip.

PROP.

PROP. CVII.

If a ſtream of any fluid as water, flows directly againſt any plane ſurface; its force againſt that plane is equal to the weight of a column of the fluid, whoſe baſe is the ſection of the ſtream; and its length twice the height deſcended by a falling body, to acquire the velocity of the fluid.

Let $s = 16\frac{1}{12}$ feet, the height deſcended by a falling body in 1 ſecond.
v=velocity of the fluid, or the ſpace it deſcribes in one ſecond.
B=baſe of the cylinder or column of water.

Then $2s$=velocity generated by gravity in falling through s. Therefore (by Cor. 1. Prop. XIV.)

$4ss : s :: vv : \frac{vv}{4s} =$ height fallen to gain the velocity v.

And $\frac{vv}{2s} =$ twice that height. Alſo $\frac{vv}{2s}B =$ a cylinder of twice that height.

Now the motion which the cylinder's weight will generate in 1 ſecond, is $2s \times \frac{vv}{2s}B$; or vvB; the motion being as the body × by the velocity. And the force of the fluid againſt the plane, is equal to the reſiſtance of the plane. And the motion deſtroyed in 1 ſecond by the reſiſtance of the plane, is $v \times Bv$ or vvB; which was alſo the motion generated by the weight of the cylinder $\frac{vv}{2s}B$, in the ſame time. But equal forces in the ſame time generate or deſtroy equal motions. Therefore the weight of the cylinder $\frac{vv}{2s}B =$ force of the fluid againſt the plane.

Cor. 1. *The force of a ſtream of water againſt any plane, is equal to the weight of a column of water, whoſe baſe is the ſection of the ſtream, and height* $\frac{vv}{2s}$; *or the height of the water, if it flow through a hole at the bottom of a reſervoir.*

It follows from this Prop. and Prop. XCVII. Cor. 2.

Cor. 2. *Moreover if any part of the water lie upon the plane; the force will be augmented by the weight of ſo much water.*

Cor. 3. *The forces of different ſtreams of water againſt any plane, are as their ſections and the ſquares of the velocities.*

Cor. 4. *If the plane be alſo in motion; the relative velocity of the water againſt the plane, muſt be taken inſtead of the abſolute velocity.*

SCHOL.

A cubic foot of water contains 6.128 ale gallons, and weighs $92\frac{1}{2}$ lb. aver.

The denſity of water to that of air at a mean is as 850 to 1. Sir Iſaac Newton found the reſiſtance of water to that of air (by the oſcillations of a pendulum) to be as 446 to 1. or between 446 and 478 to 1; or at a mean as 460 to 1. See Schol. Prop. XXXI. Book II. Principia.

LEMMA.

If the quadrant EDA revolve about the radius CA, and deſcribe an hemiſphere; and from all the points of its ſurface, as D, d, perpendiculars DB, db, be let fall upon the baſe EC. I ſay the ſum of all the perpendiculars BD, in the ſurface EDA, is to the ſum of as many radii CD; as 1 to 2. 147.

For take Dd infinitely ſmall, and complete the ſquare $CAHN$, and draw CH, alſo draw $DFGR$, $dfgr$ parallel to EC. By the ſimilar triangles CDF, Dnd; $DF\times Dd = CD\times nd$; alſo $BD = CF = FG$.

The ſurface of the ſpherical annulus $DdfF$ is $3.1416\times 2DF\times Dd$ or $3.1416\times 2CD\times nd$, that is (becauſe $3.1416\times 2CD$ is given) as nd or Ff. And the ſum of all the BD's in the annulus is as $BD\times$ by its ſurface, that is as $BD\times Ff$, or $FC\times Ff$. Therefore the ſum of all the BD's in the annulus is expreſſed by the area $FfgG$. And for the ſame reaſon, the ſum of as many radii, by $FfrR$. Therefore the ſum of all the BD's in the hemiſphere: is to the ſum of as many radii :: as the ſum of all the $FfgG$: to the ſum of all the $FfrR$ :: that is as the triangle CAH : to the ſquare $CAHN$, or as 1 to 2.

PROP. CVIII.

If a cylinder moves uniformly forward, in direction of its axis, in a fluid of the ſame denſity; it meets with a reſiſtance equal to 148.

FIG. *the force which can generate its motion, in the time it deſcribes twice its length.*

148. Let *AB* be the cylinder moving from *A* towards *G*, and take *FBCG* equal to *ASBF*. And let us firſt ſuppoſe that the cylinder *AB*, whilſt it moves forward, puſhes againſt the ſeveral parts of the fluid, and drives them ſucceſſively before it, in direction of its axis, from the ſeveral places through which it paſſes. So that in equal times it moves equal quantities of the fluid, and communicates to them the ſame velocity that it moves with. It is evident that the cylinder, after it has moved uniformly forward, the length of its axis has removed the cylinder of the fluid *FBCG* equal to itſelf *ASBF*, and has communicated a motion to it equal to its own. And ſince action and re-action are equal, the force that uniformly generated this motion, is equal to the uniform reſiſtance the cylinder ſuffered in the mean time. And therefore the reſiſtance is equal to the force by which its own motion can be generated, in the time it deſcribes its length.

All this is true, upon ſuppoſition that every particle of the fluid is driven directly forward, with the ſame velocity the cylinder has. But ſince, in reality, the motion generated in the
147. fluid is not directly forward, but (by Prop. LXXX.) diverges
148. on all ſides, and in all manner of directions *CD*, *Cd*, *&c.* Therefore if the quadrant *AE* be divided into an infinite number of equal parts, *Dd*, and to all the points *D*, *d*, the radii *CD*, *Cd*, *&c.* be drawn, repreſenting the motions of the particles in all directions; and from any one *D*, the perp. *DB* be drawn on *EC*. Then the motion *CD* ($=CA$) is reſolved into the two motions *CB*, *BD*; of which *CB* does not affect the cylinder; and the direct motion of the particle *D* is only *BD*, which is leſs than *CD*. Therefore the force to generate this motion, and conſequently the reſiſtance of a particle at *D* (equal to this force) muſt be leſs than before in proportion of *CD* to *BD*. Therefore the former reſiſtance, when all the particles are driven directly forward; to the reſiſtance when they diverge on all ſides; is as the ſum of all the radii *CD*, drawn to every point of the ſurface of a ſphere, to the ſum of all the correſponding ſines *BD*; that is (by the Lem.) as 2 to 1. Therefore the reſiſtance the cylinder meets with now, is but half the former reſiſtance. Conſequently, ſince the force to generate any motion is reciprocally as the time; the reſiſtance will be equal to the force that can generate its motion, in the time that it deſcribes twice its length.

FIG. 148.

Cor. 1. *If a cylinder moves in direction of its axis, in a fluid of the ſame denſity, and with the velocity acquired by falling in vacuo, from a height equal to its length: it meets with a reſiſtance equal to its weight.*

For the force that generates its motion, in the time of its moving twice its length (or of falling through once its length), is its gravity.

Cor. 2. *If a cylinder moves uniformly forward in any fluid; its reſiſtance is to the force by which its whole motion may be generated, in the time of moving twice its length; as the denſity of the fluid, to the denſity of the cylinder.*

For if the denſity of the fluid be increaſed in any ratio; the reſiſtance will be increaſed in the ſame ratio.

Cor. 3. *The reſiſtance of a cylinder moving in any fluid, is equal to the weight of a cylinder of that fluid, of the ſame baſe, and its length equal to the height a body falls in vacuo, to acquire its velocity.* By Cor. 1.

Cor. 4. *Let* $s = 16\frac{1}{12}$ *feet,* $B =$ *baſe of the cylinder,* $v =$ *its velocity, or the ſpace deſcribed in* 1 *ſecond. Then its reſiſtance is* $=$ *weight of the cylinder* $\frac{vv}{4s}B$, *of the fluid.*

SCHOL.

If the cylinder move in a fluid incloſed in a veſſel; inſtead of the abſolute velocity, the relative velocity in the fluid muſt be taken, in order to find the reſiſtance. And beſides, if the veſſel be narrow, the reſiſtance will be increaſed more or leſs, becauſe the fluid, being confined by the veſſel, cannot then diverge in all directions. And if it be ſo confined, that it cannot diverge at all, but is obliged to move directly forward; the reſiſtance then will be double; which is the greateſt it can poſſibly have, or the utmoſt limit of its reſiſtance. Alſo by comparing the laſt Cor. with Cor. 1. Prop. CVII. it appears that the force of a cylinder of water againſt a plane, is double the reſiſtance an equal cylinder would meet with, moving in water with the ſame velocity. And this will not appear ſtrange, when we conſider, that in the firſt caſe the whole motion of the water is deſtroyed by the reſiſtance of the plane; but in the latter caſe, the water diverges every way from the moving cy-

FIG. 148. linder, and does not partake of its direct motion. But if the water was not ſuffered to diverge, but was driven directly forward with the motion of the cylinder; the reſiſtance would then be doubled; and theſe two caſes would become the ſame.

LEMMA.

147. *If the quadrant ADE revolve about the radius CA, and generate an hemiſphere; and on every point B of the baſe, perpendiculars BD be drawn. I ſay the ſum of all the* BD^2 *on the baſe, is to the ſum of as many* CD^2, *as* 1 *to* 2.

Let $CD=r$, $CB=x$, $BD=y$, $c=3.1416$; then $2cx=$ circumference of BC. Then,

The ſum of all the rr: to the ſum of all the yy, in the annulus Bb, is as $2cx\times rr$: to $2cx\times yy :: rrx : yyx$.

And ſum of all the rr : ſum of all the yy, in the hemiſphere, is as ſum $rrx\times Bb$: ſum $yyx\times Bb$, on the baſe;

Or as ſum of $rrx\times Bb$: ſum of $\overline{rr-xx}\times x\times Bb$;

Or as the ſum of $rrx\times Bb$: ſum $rrx\times Bb$ − ſum $x^3\times Bb$, in the baſe.

† But the ſum of all the $x\times Bb=\overline{1+2+3+4,\ \&c.\ \text{to}\ r\times 1}=\frac{1}{2}rr$.

† And ſum of all the $rrx\times Bb=\frac{1}{2}r^4$. putting $Bb=1$.

† Alſo the ſum of all the $x^3\times Bb=\overline{1^3+2^3+3^3+4^3,\ \&c.\ \text{to}\ r^3\times 1}=\frac{1}{4}r^4$.

Therefore the ſum of all the rr : ſum of all the yy, in the hemiſphere; is as $\frac{1}{2}r^4 : \frac{1}{2}r^4-\frac{1}{4}r^4$, or as $\frac{1}{2}$ to $\frac{1}{4}$, that is as 2 to 1.

PROP. CIX.

If a globe move uniformly forward in a compreſſed infinite fluid; its reſiſtance is to the force by which its whole motion may be deſtroyed or generated, in the time of deſcribing $\frac{8}{3}$ *parts of its diameter; as the denſity of the fluid, to the denſity of the globe, very nearly.*

149. Let the globe move in the direction CA. Draw the tangent DH, and BDG parallel to CA, and GH perp. to DH; and let GD

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† See *Ward*'s Math. Guide, Part V.

FIG. 149.

GD be the force of a particle of the fluid againſt the baſe *B*, in direction *GD*: then *GH* will be the force acting againſt *D*, in direction *DC*. And this force is to the force in direction *GD* as *DC* to *DB*. Whence the force againſt *B*, is to the force againſt *D*, in direction *GD*; in a ratio compounded of *GD* to *GH*, and *DC* to *DB*; that is as DC^2 to DB^2. Therefore the force of all the particles of the fluid againſt the baſe, is to their force againſt the convex ſurface; as the ſum of all the DC^2, to the ſum of all the DB^2 on the baſe; that is (by the Lem.) as 2 to 1. Therefore the reſiſtance of the ſurface of the ſphere, is but half the reſiſtance of the baſe, or of a cylinder of the ſame diameter.

Now the globe is to the circumſcribing cylinder as 2 to 3; and half of that force (which can deſtroy all the motion of this cylinder, whilſt it deſcribes 2 diameters) will deſtroy all its motion, whilſt it deſcribes 4 diameters. And therefore the ſame force that deſtroys the cylinder's motion, in the time of moving 4 diameters, will deſtroy the globe's motion whilſt it moves $\frac{2}{3}$ of this length, or $\frac{8}{3}$ of its own diameter. But (by Cor. 2. Prop. CVIII.) half the reſiſtance of the cylinder, that is the reſiſtance of the globe, is to this force; as the denſity of the fluid, to the denſity of the cylinder or globe.

Cor. 1. *The reſiſtance of a ſphere is but half the reſiſtance of a cylinder, of the ſame diameter.*

Cor. 2. *The reſiſtance of a globe moving in any fluid, is equal to the weight of a cylinder of that fluid, of the ſame diameter; and its length equal to half the height, through which a body falls in vacuo, to acquire the velocity of the globe.* By Cor. 3. Prop. CVIII.

Therefore if $s=16\frac{1}{12}$ *feet,* $v=$*velocity of a globe, or the ſpace it moves in* 1 *ſecond,* $D=$*its diameter: then its reſiſtance is equal to the weight of a cylinder of the fluid, of the ſame diameter* D, *and its length* $\frac{vv}{8s}$. *And if* $v=4\sqrt{\frac{Ds}{3}}$, *its reſiſtance is equal to the weight of an equal globe of the fluid.*

Cor. 3. *The greateſt velocity a globe can obtain, by deſcending in a fluid; is that which it would acquire by falling in vacuo, through a ſpace that is to* $\frac{4}{3}$ *the diameter; as the difference between the denſity of the globe and the denſity of the fluid, is to the denſity of the fluid.*

For let *G*, *F* be the denſities of the globe and the fluid; *D* the diameter of the globe. Then ſince a globe is equal to a

FIG. 149. cylinder whoſe height is $\frac{2}{3}D$. Therefore the weight of the globe $=$ weight of a cylinder of the fluid, whoſe length is $\frac{2}{3}D \times \frac{G}{F}$. And (by Prop. LXXXV.) the weight of the globe in the fluid is $=$ weight of a cylinder of the fluid, whoſe length is $\frac{2}{3}D \times \frac{G-F}{F}$. But (by Cor. 2.) the reſiſtance of the globe moving with the velocity acquired by falling in vacuo, through the height $\frac{4}{3}D \times \frac{G-F}{F}$ is $=$ weight of a cylinder of the fluid whoſe length is $\frac{2}{3}D \times \frac{G-F}{F}$. Therefore the weight of the globe in the fluid is equal to the reſiſtance: and conſequently it cannot accelerate the globe.

And hence if $v = 4\sqrt{\frac{G-F}{3F}Ds}$, *the reſiſtance is equal to the weight of the globe in the fluid.*

Cor. 4. *Two equal and homogeneous globes moving in a reſiſting medium; will, in times that are reciprocally as the firſt velocities, deſcribe equal ſpaces; and loſe a given part of their motions.*

For the motion loſt, in deſcribing two very ſmall equal ſpaces, is as the reſiſtance and time; that is (becauſe the ſpace is given) as the ſquare of the velocity directly and the velocity inverſely; that is directly as the velocity. And ſo in deſcribing any ſpaces, the motion loſt will always be as the firſt motion; and the time reciprocally as the firſt velocity.

Cor. 5. *Two homogeneous globes, moving with equal velocities in a fluid; loſe equal velocities in deſcribing ſpaces proportional to their diameters.*

For the velocity loſt in each, by deſcribing two ſmall ſpaces proportional to the diameters; will be as the reſiſtance and time directly, and the body inverſely; that is (becauſe the reſiſtance is as the ſquare of the diameter, and the time as the diameter), as the cube of the diameter directly, and the cube of the diameter inverſely: therefore the velocity loſt is equal in both. And the like for any ſucceeding correſpondent parts.

SCHOL.

The reſiſtance of fluids is of three kinds. 1. *Tenacity* or *coheſion* or the parts of the liquor, which is the force by which the particles of the fluid ſtick together, and cauſes them not to ſeparate eaſily; and this is the ſame for all velocities. 2. *Friction*

tion or *attrition*, where the parts of the fluid do not ſlide freely by one another; and this is as the velocity. 3. The *denſity* or *quantity of matter* to be removed; and this is as the ſquare of the velocity. The two former kinds are very ſmall in all fluids, except viſcid and glutinous ones; and upon this account the foregoing theory regards only the laſt kind. And therefore the reſiſtance there deſcribed is the very leaſt the body can poſſibly meet with. But ſince all fluids have ſome ſmall degree of friction and tenacity, they will increaſe the reſiſtance a little. Alſo when the velocity is very great; the compreſſion of the fluid ought to be ſo too, to cauſe the fluid to return with equal eaſe behind the moving body; and when this does not happen, the reſiſtance is increaſed upon that account. For a fluid yielding to a projectile, does not recede ad infinitum; but with a circular motion comes round to the places which the body leaves. Likewiſe when bodies move in a ſtagnant fluid near the ſurface; the fluid cannot dilate itſelf upwards, to give way to the moving body; and this will conſiderably increaſe the reſiſtance. Alſo if a body moves in a fluid incloſed in a veſſel; the relative velocity of the body in the fluid muſt be eſteemed its true velocity. But the reſiſtance it meets with will be increaſed, becauſe the fluid has not liberty to diverge every way. And the ſtraiter the veſſel, the more is the reſiſtance increaſed; and it may by this means be increaſed till it be near double; beyond which it cannot go. For all that a body can do is to drive the fluid wholly before it, without any diverging. So that the leaſt reſiſtance a globe can have is the ſame as is laid down in Cor. 2. of the laſt Prop. and the greateſt can never exceed the double of it; ſo that it will always be between theſe limits. If the fluid in which the body moves be elaſtic and ſpring from the body; the reſiſtance will be greater than if it was non-elaſtic. But theſe irregularities are not conſidered in the foregoing theory.

There are ſome bodies that may be reckoned in a middle ſtate between ſolids and fluids. And in ſome of theſe the tenacity and friction is ſo great, as in many caſes far to exceed the reſiſtance ariſing from their denſity only. For example, it appears by experiments, that if a hard body be ſuſpended at ſeveral heights, and be let fall upon any ſoft ſubſtance, ſuch as tallow, ſoft clay, wax, ſnow, &c. it will make pits or impreſſions, which are as the heights fallen, that is as the ſquares of the velocities. Likewiſe nails give way to a hammer in a ratio which is as the ſquare of the velocity. Comparing this with Schol. Prop. XIV. It appears that in theſe caſes, the reſiſtance

FIG. 149. reſiſtance is the ſame for all velocities: which argues a very great degree of tenacity. Again, bodies projected into earth mixt with ſtones; the impreſſions are found to be between the ſimple and duplicate ratio of the velocities. Therefore in this caſe, the reſiſtance is in a leſs ratio than the ſimple ratio of the velocity: and therefore theſe ſort of bodies have both friction and tenacity. And in different ſorts of bodies, there is great difference and variety in their nature and conſtitution.

Tenacity may be compared to the force of gravity, which is always the ſame; with this difference, that tenacity acts always contrary to the motion of the body, and when the body is at reſt, it is nothing. *Attrition* may be compared to the motion of a body ſtriking always a given number of particles of matter in a given time, with any velocity: and therefore the reſiſtance of ſuch a body will be as the velocity.

SECT. XI.

Methods of communicating, directing, and regulating any motion in the practice of mechanics.

PROP. CX.

To communicate motion from one body to another, or from one place to another.

1. The eaſieſt and ſimpleſt method of communicating motion from one thing *A* to another *B*, is by a rope or a leaver *AB*, reaching between the two places, or things. 150.

2. Motion is communicated from one wheel or roller *DC* to another *AB*, by a perpetual or endleſs rope *ABCD*, going once or oftner about them: or if you will, by a chain. That the rope ſlip not, make knots on it, and channels in the wheels, if neceſſary. 151.

3. Motion is communicated from one wheel *ABC*, to another *DEF*; by the teeth in the two wheels working together. 152. Or thus, where the axis of *A* having but one tooth; one revolution of it anſwers to the motion of only one tooth in *B*. 153.

4. Motion is communicated from one place to another, by one or more beams or leavers, *MB*, *BC*, *CE*, *EF*, *FH*, *&c.* moveable about the centers *A*, *B*, *C*, *D*, *E*, *F*, *G*; of which *A*, *D*, *G*, *&c.* are fixed. Here if the point *M* be moved, the point *H* will be moved; for *MB*, *BC*, *CE*, *&c.* all move one another to the laſt, *FH*. 154.

5. Motion may alſo be communicated from *A* to *B*, by a pinion at *A*, and a ſtreight ruler with teeth, which bite one another. 155.

PROP.

PROP. CXI.

By help of one uniform motion given; to produce another, either uniform or accelerated.

152. 1. A uniform motion is produced in the wheel *DEF*; by moving the wheel *ABC* uniformly, which carries it. Alſo a uni-
151. form motion is produced in wheels moving by cords, as *AB*, *CD*: for one being moved uniformly, moves the other alſo uniformly.

156. 2. The wheel *BF* may be made to move uniformly about the center *C*, by the motion of the wheel *BD*. On the baſe *BF* with the generating circle *BD*, deſcribe the epicycloidal tooth *BE*. Then the point *B* of the wheel *AB*, moving uniformly about the center *A*, and paſſing over the tooth *BE*; will move the wheel *BF* uniformly about *C*. Here the acting tooth *AB* ought to be made crooked as *Ab*, that it touch not the end *E*, of the tooth *BE*, if it act on the concave ſide. Or elſe the plane of the wheel *BD* muſt be raiſed above the plane of *BF*, and a tooth made at *B* to bend down perpendicular to the plane of the wheel, as *AG*, to catch the tooth *BE*.

157. 3 The leaver *AB* may be made to move up and down with either a uniform or accelerated motion, after this manner. Let *AE* be a wheel whoſe axis is parallel to the leaver, and directly above it. Take any arch *N*4, and divide it into any number of equal parts at 1, 2, 3, *&c.* through which from the center *O*, draw *Oa*, *Ob*, *Oc*, *Od*; and make 1*a*, 2*b*, 3*c*, 4*d*, *&c.* reſpectively equal to 1, 2, 3, 4 equal parts. And through the points *N*, *a*, *b*, *&c.* draw the curve *Nabcd*. Then the part *NdF* being made of ſolid wood, and fixed to the wheel; and the wheel being turned uniformly about, in the order *ENA*; the part *NF* will give a uniform motion to the leaver *AB*, about the center of motion *C*. And you may fix as many of theſe teeth to the wheel as you will.

Again, in the tooth *AD*, if *A*1, 12, 23, *&c.* be taken equal, and 1*a*, 2*b*, 3*c*, 4*d*, *&c.* be taken equal to 1, 4, 9, 16, *&c.* equal parts; and the curve *Aabcd* be drawn, and the tooth formed. Then the leaver will be moved with a uniformly accelerated motion.

The accelerated motion is proper for lifting a given weight at the end *B*, as a hammer; or for working a pump, by a chain going over the end *B*.

4. The

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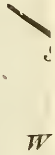

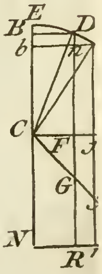

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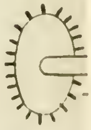

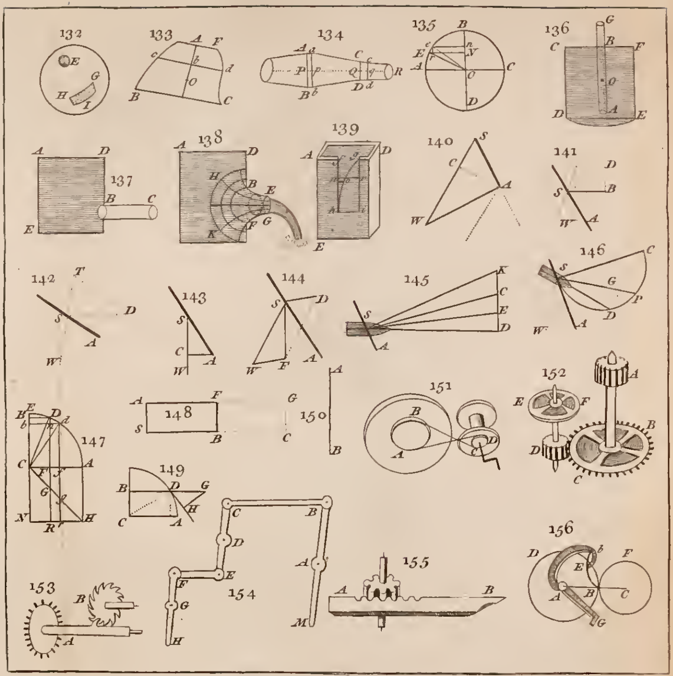
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Pl. VII *pa.* 162.

FIG. 158. 4. The leaver *AB*, may alſo be moved thus, by help of a machine *GFD*, moving uniformly along *GD*. Make *HI*, *IF* right lines; and make as many ſuch teeth as you will; and theſe will give a uniform motion to the leaver.

Make the curves *EFE* all parabolas, equal and equi-diſtant; whoſe vertices are at *F*; and their baſes meet at *E*, and theſe will make the leaver riſe and fall with an accelerated motion. Such parabolic teeth as theſe may be placed on a wheel, whoſe axis is perpendicular to the horizon.

159. 5. One wheel may move another with an accelerative motion thus. On the circle or wheel *EF*, take *Ea*, *ab*, *bc*, *&c.* equal to each other. And on the edge of the wheel *BD* take *B*1, a very ſmall part; and 13, 35, 57, *&c.* 3, 5, 7, *&c.* times *B*1, ſuppoſe the plane of the wheel *EF* to be extended as far as the marks 1, 3, 5, 7, *&c.* then turn the wheel *EF*, till *E* fall on *a*; then mark the point 1 on the plane of the wheel *EF*. Then turn *EF* till *E* comes to *b*; and mark the point 3 on the plane of the wheel *EF*. Likewiſe let *E* come to *c*, *d*, *&c.* and mark the points 5, 7, *&c.* on the plane of the wheel *EF*; then *E* 1 3 5 7 *r* is the figure of the tooth of the wheel *EF*, which being uniformly moved, will move *DB* with an accelerative motion.

PROP. CXII.

To change the direction of any motion.

160. 1. The direction of any motion may be changed, by the leaver of the firſt kind, for the two ends have oppoſite motions. Likewiſe a bended leaver will change the direction to any other direction.

161. 2. The direction of motion may be changed by the help of pulleys, with a rope going over them. Thus the direction *AB* is changed ſucceſſively into the directions *BC*, *CD*, *DE*, *EF*, *FG*.

162. 3. The direction may be changed by wheels, whoſe axles are perpendicular to one another. Thus the direction *AB* is changed into the direction *EF*; by the wheel *C*, working in the crown wheel *D*.

163. 4. The direction may be changed, by making the lanthorn *B*, inclined in any given angle, to be moved by the coggs of the wheel

FIG. wheel *A*. Here the rungs at *F*, where they work, muſt be paral-
163. lel to the plane of the wheel *A*, or perpendicular to the coggs.
164. The ſame thing may be done by wheels with teeth, as *C*, *D*.

In both caſes the axles of the two wheels muſt be in one plane.

PROP. CXIII.

To regulate any motion, or to make it uniform.

165. 1. Any motion is made uniform, by the help of a pendulum *AB*, ſuſpended at *A* and vibrating. As the pendulum vibrates, it cauſes *CDE* to vibrate alſo, about the axis *DE*. The weight *I* carries the wheel *R*, and *R* moves *LF*. Now whilſt the pendulum vibrates towards *M*, a tooth of the wheel *GF* goes off the pallat *I*, and another catches the pallat *H*; and when the pendulum returns towards *N*, it draws the pallat *H* off the tooth, and another catches the pallat *I*; and ſo on alternately. So that at every vibration of the pendulum, a tooth goes off one or other of the palats.

166. 2. A uniform motion is effected by the pendulum *CP*, vibrating in the arch *NM* about the center of motion *C*. As the pendulum vibrates, it cauſes the piece *ADE* to vibrate along with it about the axis of motion *DE*. By this motion the leaf *a* catches hold of a tooth of the horizontal wheel *GF*, in its going; and the leaf *b* of another tooth, in returning. A wheel with a weight is applied to the pinion *L*, to keep the pendulum going.

167. 3. A pendulum may alſo be applied thus for the ſame purpoſe. *FG* is a thick wheel, or rather a double wheel, whoſe axis is parallel to the horizon. *nP* a pendulum vibrating upon the axis *DE*, which is parallel to the planes of the wheel *FG*; *anb* two wings perpendicular to *DE*, and to *nP*; 1, 1, 1 pins in the rim *G*; and 2, 2, 2 pins in the rim *F*. Theſe pins are in the planes of the wheel; but not perpendicular to the circumference, but inclined in an angle of about 45 degrees, and the pins in one end are againſt the ſpaces in the other; *ab* is parallel to the axis of the wheel *FG*, but neither in the ſame horizontal or perpendicular plane; but almoſt the radius of the wheel below, and ſomething more forward. Whilſt the pendulum *P* vibrates in the arch *MN*, about the axis *DE*, the wing *a* catches hold of a tooth in the end *F*; and when it returns, the wing *b* catches hold of a tooth in the end *G*. Thus the pins acting

ing alternately againſt the wings *a*, *b*, keep the pendulum going, by help of the weight *W*.

FIG. 167. 168. 169. 4. A ſteady motion is continued by applying the heavy wheel *ABC*, to the machine : or the croſs bar *DE* loaded with two equal weights at *D* and *E*. Or a cylinder of ſome heavy matter may be applied ; being made to revolve about its axis. By theſe the force of the power, which would be loſt, is kept in the wheel, and is equally diſtributed in all parts of the revolution. Such a wheel is of great uſe in ſuch machines as act with unequal force at different times, or in different parts of a revolution. For by its weight it conſtantly goes on at the ſame rate, and makes the motion uniform, and every where equal. By reaſon of its weight a little variation of force will not ſenſibly alter its motion : and its friction, and the reſiſtance of the air will hinder it from accelerating. If the machine ſlackens its motion, it will help it forward ; if it tends to move too faſt, it will keep it back.

Every ſuch regulating wheel ought to be fixed upon that axis, where the motion is ſwifteſt. And ought to be the heavier, the ſlower it is deſigned to move ; and the lighter, the ſwifter the motion is. And in all caſes the center of motion muſt be in the center of gravity of the wheel. And the axis may be placed parallel to the horizon, as well as perpendicular to it.

If the machine be large, and the axis of the heavy wheel be perpendicular to the horizon ; the heavy wheel may be made to roll on the ground, round that axis ; by putting the wheel upon another axis fixed in the former at right angles to it ; and thus the weight is taken off the firſt axis. And two ſuch wheels may be applied on oppoſite ſides.

170. 5. Any ſwift motion may be moderated by a fly *AB*, moveable about the axis *CD*. This is made of thin metal ; at *s* is a ſpring to keep the axis and fly pretty ſtiff together. This bridles the rapidity of the motion of the machine, to which it is applied, by reaſon of its great reſiſtance in the air ; and therefore it hinders the motion from accelerating beyond a certain degree. This ſort of fly is uſed in clocks, and is ſo uſeful in any motion that requires to ſtop, or move a contrary way.

None of theſe regulating wheels or flies add any new power to the machine ; but rather retard the motion by their friction and reſiſtance.

PROP.

FIG.

PROP. CXIV.

To deſcribe ſeveral ſorts of knots.

As ropes are made uſe of in ſeveral ſorts of machines, and eſpecially aboard of ſhips; it is proper for a mechanic to know how to tye them together. Therefore I ſhall here deſcribe ſeveral ſorts of knots, not ſo much to teach how to tye them, as to ſhew the form they appear in, when they are tyed. For the method of tying them is beſt learned from thoſe that can tye them already.

171. 1. *A thumb knot.* This is the ſimpleſt of all; and is uſed to tye at the end of a rope, to hinder its opening out. Alſo it is uſed by taylors at the end of their thread.

172. 2. *A loop knot.* This is uſed to join pieces of ropes together.

173. 3. *A draw knot*, is the ſame as the laſt; only one (or both) of the ends returns the ſame way back, as *a b c d e*. By pulling at *a* the part *bcd* comes through, and the knot is looſed.

174. 4. *A ring knot.* This ſerves alſo to join pieces of rope together.

175. 5. *Another knot* for tying ropes together. This is made uſe of when any rope is often to be looſed.

176. 6. *A running knot*, to draw any thing cloſe. By pulling at the end *a*, the rope is drawn through the loop *b*, and the part *cd* is drawn cloſe about a beam, *&c.*

177. 7. *Another knot*, to tye any thing to a poſt; here the end may be put through as oft as you will.

178. 8. *A very ſmall knot.* There is a thumb knot made at the end of each piece; and the end of the other is to go through it. Thus the rope *ac* runs through the loop *d*, and *bd* through *c*. And then drawn cloſe by pulling at *a* and *b*: if the ends *e*, *f* be drawn, the knot will be looſed again.

179. 9. *A fiſher's knot*, or *water knot.* This is the ſame as the 4th, only the ends are to be put twice through the ring, which in that was but once; and then drawn cloſe.

180. 10. *A maſhing knot* for nets; and is to be drawn cloſe.

181. 11. *A barber's knot*, or a knot for cawls of wigs. This muſt be drawn cloſe.

194. 12. *A bowline knot.* When this is drawn cloſe, it makes a loop that will not ſlip, as fig. *R*. This ſerves to hitch over any thing.

193. 23. *A wale knot* is made with the three ſtrands of a rope, ſo that it cannot ſlip. When the rope is put through a

hole, this knot keeps it from ſlipping through, 'tis repreſented at *S*, fig. 193. If the three ſtrands are wrought round once or twice more, after the ſame manner, it is called *crowning*. By this means the knot is made bigger and ſtronger. A thumb knot art. 1, may be applied to the ſame uſe as this. FIG. 193.

Concerning the ſtrength of ropes, ſee the latter end of Sect. VIII.

SECT. XII.

The powers and properties of compound engines; of forces acting within the machine; of friction.

PROP. CXV.

In any compounded machine, if the power and weight keep the machine, and all its parts, in equilibrio. Then the power it to the weight, in the compound ratio of the power to the weight in every ſimple machine, of which the whole is compoſed.

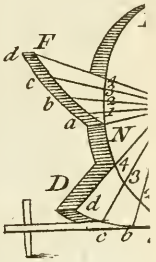
For let the compound machine be divided into all its ſimple mechanic powers; and in the firſt let the power be to the weight as *A* to *B*. Then conſidering the weight *B* in the firſt, as the power in the ſecond, to which it is equal (by Ax. 3.) let that power in the ſecond machine be to the weight as *B* to *C*. Then ex equo, the firſt power *A* is to the ſecond weight *C*; in the compound ratio of *A* to *B* and *B* to *C*. In like manner, if the weight in the ſecond be taken for the power in the third, and this power be to the weight as *C* to *D*; then the firſt power *A* is to the laſt weight *D*, in the compound ratio of *A* to *B*, *B* to *C*, and *C* to *D*; and ſo on thro' the whole.

Cor. In any machine compoſed of wheels; the power is to the weight; in the compound ratio of the diameter of the axel where the weight is applied, to the diameter of that where power is applied; and the number of teeth in the pinion of each axis, beginning at the power, to the number of teeth in each wheel they work in, till you come at the weight.

Or, inſtead of the teeth, you may take their diameters.

PROP.

G.



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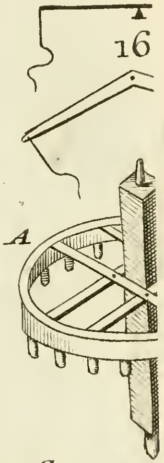

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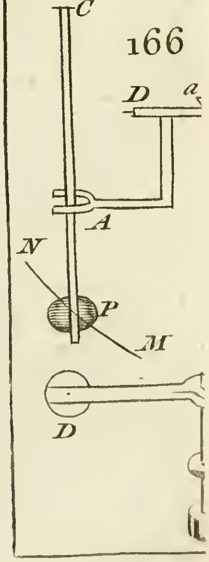

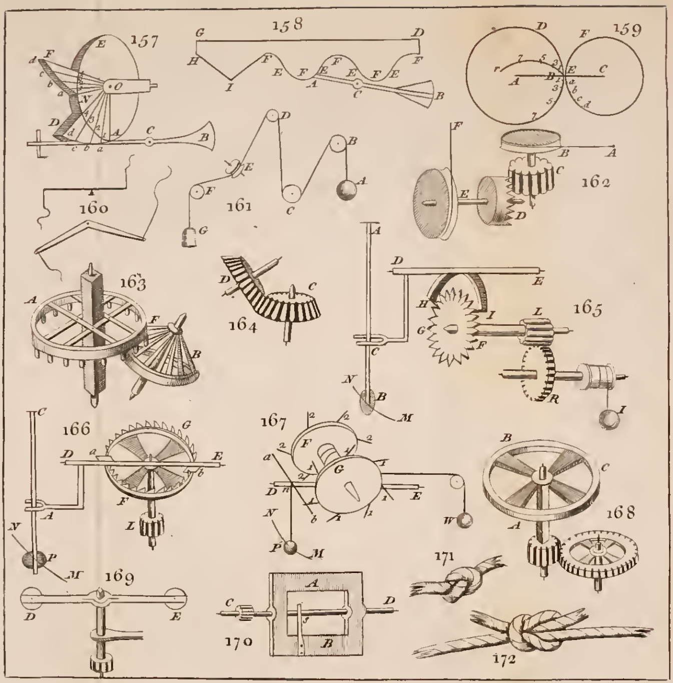

Pl.VIII. pa 168.

PROP. CXVI.

If the power and weight be in equilibrio on any machine; if they be put in motion, the velocity of the weight will be to that of the power; as the power is to the weight.

For ſince they are in equilibrio; one of them cannot of itſelf move the other; therefore if they be put into motion, the momentum or quantity of motion of the weight, will be equal to that of the power: and therefore their velocities will be reciprocally as their quantities.

Cor. 1. *Hence it follows, that if any weight is moved by help of a machine; what is gained in power is loſt in time.*

For in whatever proportion the power is leſs than the weight; in the ſame proportion will the weight be ſlower than the power.

Cor. 2. *Hence the motion of the weight is not at all increaſed by any engine, or mechanical inſtrument; only the velocity of the weight is ſo much diminiſhed thereby; that the quantity of motion of the weight, may not exceed the quantity of motion of the power. And therefore it is a vain fancy for any one to think that he can move a great weight with a little power, and with the ſame velocity as with a greater power.*

For the advantage gained by the power is loſt by the velocity. If any power is able to raiſe a pound with a given velocity; it is impoſſible by the help of any machine whatever, that the ſame power can raiſe two pounds with the ſame velocity. Yet it may, by help of a machine, be made to raiſe 2 pounds with half that velocity; or even 1000 pounds with the thouſand part of the velocity. But ſtill there is no greater quantity of motion produced, when 1000 pounds weight is moved, than when 1 pound: the 1000 pounds being proportionally ſlower. The power and uſe of machines conſiſts only in this, that by their means the velocity of the weight may be diminiſhed at pleaſure, ſo that a given weight may be moved with a given power; or that with a given force any given reſiſtance may be overcome. Mechanic inſtruments being only the means whereby one body communicates motion to another: and not deſigned to produce a motion that had no exiſtence before.

Cor. 3. Hence alſo it is plain, that a given power or quantity of force, applied to move a heavy body by help of a machine, can produce no greater quantity of motion in that body, than if that force was immediately applied to the body itſelf. Nor not quite ſo much, by reaſon of the friction and reſiſtance of the engine. And if the power be given, you may chuſe whether you will move a greater weight with a leſs velocity, or a leſs weight with a greater velocity. But to do both, is utterly repugnant to the eternal laws of nature.

PROP. CXVII.

182. *If any machine CD, is to be moved by the help of leavers, wheels, &c. And if the power that moves it acts intirely within the machine, and exerts its force againſt ſome external object B. Then the force applied within to move the machine, will be juſt the ſame as if the machine was at reſt, and the object B was to be moved: ſuppoſing B to be as eaſily moved as the machine.*

For ſuppoſe firſt, the leaver *AFB* to be fixt, and to make a part of the machine; and let the external force acting at *B*, which is capable to move the machine, be 1. Now ſuppoſe the leaver *AFB*, moveable about *F*; and a force applied at *A*, ſo great, as to act at *B*, with the force 1. Then the action and re-action at *B* being the ſame as before, it is plain the machine will be moved as before. But the force now acting at *A*, is $\frac{BF}{AF} \times 1$; juſt the ſame as if the point *F* were fixt, and *B* was to be moved. And if more leavers, or any number of wheels be added, the thing will ſtill be the ſame.

Otherwiſe.

Let the abſolute force to move the machine be 1, and the force acting at *A* be *f*; and let us firſt conſider it as acting out of the machine. Then *B* being fixt, is the fulcrum; therefore the force acting at *F*, is $\frac{AB}{FB} \times f$. Now if the acting force be conſidered in the machine, it will not be urged forward with all this force, for the re-action will be equal to *f*, the power at *A*.

A. Therefore the abſolute force the machine is moved with, is $\frac{AB}{FB}\times f - f$ or $\frac{AB-FB}{FB}\times f$, that is $\frac{AF}{FB}\times f$, but this is $=$ 1, therefore $f=\frac{FB}{AF}\times 1$. FIG. 182.

Cor. 1. *Hence if the abſolute external force, to move any body or machine, be given; and the machine is to be moved by an internal power: that power may be found, by ſuppoſing the machine at reſt, and the external object B was to be removed, and to require the ſame abſolute force to move it.*

For it is the ſame thing, as to the power, whether the machine, or the external object be moved, whilſt the other is at reſt.

Cor. 2. *If the power acting within the machine be not communicated to ſome external object, it will have no force at all to move the machine. And any force that both begins and ends within it, does nothing at all to move it.*

For the power acting only againſt ſome part of the machine, will be deſtroyed by the contrary and equal re-action. And the body being acted on by theſe equal and contrary forces, will not be moved at all. Thus if a man, ſitting in the head of a boat, pull the ſtern towards him by a rope; the boat will not be moved at all out of its place, by that force.

PROP. CXVIII.

To determine the friction, and other irregularities in mechanical engines.

The propoſitions hitherto laid down, ſuppoſe all bodies perfectly ſmooth, that they ſlide over one another without any friction, and move freely without any reſiſtance. But ſince there is no ſuch thing as perfect ſmoothneſs in bodies; therefore in rubbing againſt one another they meet with more or leſs friction, according to their roughneſs; and in moving in any medium, will be reſiſted according to the denſity of the medium. Even ropes going over pullies cannot be bended without ſome force.

Among machines, ſome have a great deal more friction than others, and ſome very little. Thus a pendulum has little or no friction, but what ariſes from the reſiſtance of the air. But a carriage has a great deal of friction. For upon plain ground a loaded cart requires the ſtrength of ſeveral horſes to draw it along; and all or moſt of this force is owing to its friction. All compounded machines have a great deal of friction, and ſo much the more, as they conſiſt of more parts that rub againſt one another. And there is great variety in ſeveral ſorts of bodies, as to the quantity of friction they have, and even in the ſame bodies under different circumſtances: upon which account it will be impoſſible to give any ſtanding rules, by which its quantity can be exactly determined. All we can do is to lay down ſuch particular rules, as have been deduced from experiments made upon particular bodies; which rules will require ſome variation under different circumſtances; according to the judgment and experience of the artiſt.

1. Wood and all metals when oiled or greaſed have nearly the ſame friction. And the ſmoother they are, the leſs friction they have. Yet metals may be ſo far poliſhed as to increaſe friction, by the coheſion of their parts.

Wood ſlides eaſier upon the ground in wet weather than in dry; and eaſier than iron in dry weather. But iron ſlides eaſier than wood in wet weather. Lead makes a great deal of reſiſtance. Iron or ſteel running in braſs, makes the leaſt friction of any. In wood acting againſt wood, greaſe makes the motion twice as eaſy, or rathar $\frac{2}{3}$ eaſier. Wheel naves greaſed or tarr'd, go 4 times eaſier than when wet.

Metals oiled make the friction leſs than when poliſhed, and twice as little as when unpoliſhed.

In general, the ſofter or rougher the bodies, the greater is their friction.

2. As to particular caſes: a cubic piece of ſoft wood of eight pound weight, moving upon a ſmooth plane of ſoft wood, at the rate of three feet per ſecond; its friction is about $\frac{1}{3}$ the weight of it. But if it be rough, the friction is little leſs than half the weight.

Upon the ſame ſuppoſition, other ſoft wood upon ſoft wood very ſmooth; the friction is about $\frac{1}{4}$ the weight.

Soft wood upon hard, or hard upon ſoft, $\frac{1}{5}$ or $\frac{1}{6}$ the weight.

Hard wood upon hard wood, $\frac{1}{7}$ or $\frac{1}{8}$ the weight.

Poliſhed ſteel moving on ſteel or pewter, $\frac{1}{4}$ the weight: moving on copper or lead, $\frac{1}{5}$ the weight: on braſs $\frac{1}{6}$ the weight. Metals of the ſame ſort have more friction than different ſorts.

The friction, *ceteris paribus*, increaſes with the weight, almoſt in the ſame proportion. The friction is alſo greater with a greater velocity, but not in proportion to it, except in very few caſes. A greater ſurface alſo cauſes ſomething more friction, with the ſame weight and velocity. Yet friction may ſometimes be increaſed, by having too little ſurface to move on: as upon clay, *&c.* where the body ſinks.

3. The friction ariſing from the bending of ropes about machines, differs according to their ſtiffneſs, the temper of the weather, degree of flexibility, *&c.* but, *ceteris paribus*, the force or difficulty of bending a rope, is as the ſquare of the diameter of the rope, and its tenſion, directly; and the diameter of the cylinder, or pulley, it goes about, reciprocally.

A rope of 1 inch diameter, whoſe tenſion, or weight drawing it, is 5lb. going over a pulley 3 inches diameter; requires a force of 1lb. to bend it.

4. The reſiſtance of a plane moving through a fluid, is as the ſquare of the velocity: and (putting $v =$ velocity in feet, in a ſecond) it is equal to the weight of a column of the fluid, whoſe baſe is the plane, and height $\frac{vv}{64}$. And in a globe it is but half ſo much.

5. The friction of a fluid running through a tube is as the velocity and diameter of the tube.

But the friction is greater in reſpect to the quantity of the fluid, in ſmall tubes, than in large ones; and that reciprocally as their diameters. But the abſolute quantity of the friction in tubes, is but very ſmall, except the velocity be very great, and the tube very long.

But if a pipe be divided into ſeveral leſſer ones, whoſe number is n; the reſiſtance ariſing from the friction will be increaſed as $\sqrt{n}$. For the area of the ſection of any one pipe, will be $\frac{1}{n}$; and the friction, being as the circumference, will be as $\frac{1}{\sqrt{n}}$; and therefore the friction in all of them, will be $\frac{n}{\sqrt{n}}$, or as $\sqrt{n}$.

6. As to the mechanic powers. The ſingle leaver makes no reſiſtance by friction. But if by the motion of the leaver in lifting, the fulcrum or place of ſupport be changed further from the weight; the power will be decreaſed thereby.

7. In any wheel of a machine, running upon an axis; the friction on the axis is as the weight upon it, the diameter of the axis, and the angular velocity. This ſort of friction is but ſmall.

8. In the pulley, if p, q be two weights, and q the greater; and if $W = \frac{4pq}{p+q}$, then W is the weight upon the axis of the ſingle pulley. And it is not increaſed by the acceleration of the weight q, but remains always the ſame.

The friction of the pulley is very conſiderable, when the ſheaves rub againſt the blocks; and by the wearing of the holes and axles.

The friction on the axis of the pulley is as the weight W, its angular velocity, the diameter of the axis directly, and the diameter of the pulley inverſely. A power of 100 lb. with the addition of 50 lb. will but draw up 500 lb. with a tackle of 5.

And 15 lb. over a ſingle pulley will draw up only 14 lb.

9. In the ſcrew there is a great deal of friction. Thoſe with ſharp threads have more friction than thoſe with ſquare threads. And endleſs ſcrews have more than either. Screws with a ſquare thread raiſe a weight with more eaſe than thoſe with a ſharp thread.

In the common ſcrew, the friction is ſo great, that it will ſuſtain the weight in any poſition given, when the power is taken off. And therefore the friction is at leaſt equal to the power. From whence it will follow, that in the ſcrew.

The power muſt be to the weight or reſiſtance; at leaſt as twice the perpendicular height of a thread, to the circumference deſcribed by one revolution of the power; if it be able to raiſe the weight, or only ſuſtain it. This friction of the ſcrew is of great uſe, as it ſerves to keep the weight in any given poſition.

10. In the wedge, the friction is at leaſt equal to the power, as it retains any poſition it is driven into. Therefore in the wedge,

The power muſt be to the weight; at leaſt as the baſe to the height; to overcome any reſiſtance.

11. To find the friction of any engine, begin at the power, and conſider the velocity and the weight at the firſt rubbing part; and eſtimate its quantity of friction, by ſome of the foregoing articles. Then proceed to the next rubbing part, and do the ſame for it. And ſo on through the whole.

And note, ſomething more is to be allowed for increaſe of friction, by every new addition to the power.

Cor. Hence will appear the difficulty, or rather impoſſibility of a perpetual motion; or ſuch a motion as is to continue the ſame for ever,

ever, or at leaſt as long as the materials will laſt, that compoſe the moving machine.

For ſuch a motion as this ought continually to return undiminiſhed, notwithſtanding any reſiſtance it meets with, which is impoſſible. For although any body once put into motion, and moving freely without any reſiſtance, or any external retarding force acting upon it, would for ever retain that motion. Yet in fact we are certain, that no body or machine can move at all, without ſome degree of friction and reſiſtance. And therefore it muſt follow, that from the reſiſtance of the medium, and the friction of the parts of the machine upon one another, its motion will gradually decay, till at laſt all the motion is deſtroyed, and the machine is at reſt. Nor can this be otherwiſe, except ſome new active force, equal to all its reſiſtance, adds a new motion to it. But that cannot be from the body or machine itſelf; for then the body would move itſelf, or be the cauſe of its own motion, which is abſurd.

PROP. CXIX.

To contrive a proper machine that ſhall move a given weight with a given power; or with a given quantity of force, ſhall overcome any other given reſiſtance.

If the given power is not able to overcome the given reſiſtance, when directly applied, that is, when the power applied is leſs than the weight or reſiſtance given; then the thing is to be performed by the help of a machine made with *leavers*, *wheels*, *pullies*, *ſcrews*, &c. So adjuſted, that when the weight and power are put in motion on the machine; the velocity of the power may be at leaſt ſo much greater than that of the weight; as the weight and friction of the machine taken together, is greater than the power. For on this principle depends the mechaniſm or contrivance of mechanical engines, uſed to draw or raiſe heavy bodies, or overcome any other force. The whole deſign of theſe being to give ſuch a velocity to the power in reſpect of the weight; as that the momentum of the power may exceed the momentum of the weight. For if machines are ſo contrived, that the velocities of the agent and reſiſtant are reciprocally as their forces, the agent will juſt ſuſtain the reſiſtant: but with a greater degree of velocity will overcome it. So that if the

FIG. the exceſs of velocity in the power is ſo great, as to overcome all that reſiſtance which commonly ariſes from the friction or attrition of contiguous bodies, as they ſlide by one another, or from the coheſion of bodies that are to be ſeparated, or from the weights of bodies to be raiſed. The exceſs of the force remaining, after all theſe reſiſtances are overcome, will produce an acceleration of motion proportional thereto, as well in the parts of the machine, as in the reſiſting body. Now how a machine may be contrived to perform this to the beſt advantage, will appear from the following rules.

1. Having aſſigned the proportion of your power and the weight to be raiſed: the next thing is to conſider how to combine leavers, wheels, pulleys, *&c.* ſo that working together they may be able to give a velocity to the power, which ſhall be to that of the weight, ſomething greater than in the proportion of the weight to the power. This done, you muſt eſtimate your quantity of friction, by the laſt Prop. and if the velocity of the power be to that of the weight, ſtill in a greater proportion, than the weight and friction taken together, is to the power; then your machine will be able to raiſe the weight. And note, this proportion muſt be ſo much greater, as you would have your engine work faſter.

2. But the proportion of the velocity of the power and weight, muſt not be made too great neither. For it is a fault to give a machine too much power, as well as too little. For if the power can raiſe the weight, and overcome the reſiſtance, and the engine perform its proper effect in a convenient time, and works well, it is ſufficient for the end propoſed. And it is in vain to make more additions to the engine, to increaſe the power any further: for that would not only be a needleſs expence, but the engine would loſe time in working.

3. As to the power applied to work the engine, it may be either a living power, as men, horſes, *&c.* or an artificial power, as a ſpring, *&c.* or a natural power, as wind, water, fire, weights, *&c.*

When the quantity of the power is known; it matters not as to the effect, what kind of power it is. For the ſame quantity of any ſort will produce the ſame effect. And different ſorts of powers, may be applied in an equal quantity, a great variety of ways.

The moſt eaſy power applied to a machine is weight, if it be capable of effecting the thing deſigned. If not, then wind, water, *&c.* if that can conveniently be had, and without much expence.

A ſpring is alſo a convenient moving power for ſeveral machines; but it never acts equally as a weight does; but is ſtronger when much bent, than when but a little bent, and that in proportion to the degree of bending, or the diſtance it is forced to. But ſprings grow weaker by often bending, or remaining long bent; yet they recover part of their ſtrength by lying unbent.

The natural powers, wind and water, may be applied with vaſt advantage to the working of great engines, when managed with ſkill and judgment. The due application of theſe has much abridged the labours of men; for there is ſcarce any labour to be performed, but an ingenious artificer can tell how to apply theſe powers to execute his deſign, and anſwer his purpoſe. For any conſtant motion being given, it may by a due application be made to produce any other motions we deſire. Therefore theſe powers are the moſt eaſy and uſeful, and of the greateſt benefit to mankind. Beſides, they coſt nothing, nor require any repetition or renewing, like a weight or a ſpring, which require to be wound up. When theſe cannot be had, or cannot ſerve our end, we have recourſe to ſome living power, as men, horſes, *&c.*

4. Men may apply their ſtrength ſeveral ways, in working a machine. A man of ordinary ſtrength turning a roller by the handle, can act for a whole day againſt a reſiſtance equal to 30lb. weight; and if he works 10 hours in a day, he will raiſe a weight of 30 lb. $3\frac{1}{2}$ feet in a ſecond; or if the weight be greater, he will raiſe it ſo much leſs in proportion. But a man may act, for a ſmall time, againſt a reſiſtance of 50 lb. or more.

If two men work at a windleſs, or roller, they can more eaſily draw up 70lb. than one man can 30lb. provided the elbow of one of the handles be at right angles to that of the other. And with a fly, or heavy wheel applied to it a man may do $\frac{1}{3}$ part more work; and for a little while act with a force, or overcome a continual reſiſtance of 80lb. and work a whole day when the reſiſtance is but 40lb.

Men uſed to carrying, ſuch as porters, will carry, ſome 150lb. others 200 or 250lb. according to their ſtrength.

A man can draw about 70 or 80lb. horizontally; for he can but apply about half his weight.

If the weight of a man be 140lb. he can act with no greater a force in thruſting horizontally, at the height of his ſhoulders, than 27 lb.

As to horſes. A horſe is, generally ſpeaking, as ſtrong as 5 men. A horſe will carry 240 or 270lb.

A horſe draws to greateſt advantage, when the line of direction is a little elevated above the horizon, and the power acts againſt his breaſt. And can draw 200 lb. for eight hours in a day at 2½ miles an hour. If he draw 240 lb. he can work but ſix hours, and not go quite ſo faſt. And in both caſes, if he carries ſome weight, he will draw better than if he carried none. And this is the weight a horſe is ſuppoſed to be able to draw over a pulley out of a well. In a cart, a horſe may draw 1000 lb.

The moſt force a horſe can exert is, when he draws ſomething above a horizontal poſition.

The worſt way of applying the ſtrength of a horſe, is to make him carry or draw up hill. And three men in a ſteep hill, carrying each 100 lb. will climb up faſter than a horſe with 300 lb.

Though a horſe may draw in a round walk of 18 feet diameter; yet ſuch a walk ſhould not be leſs than 25 or 30 feet diameter.

5. Every machine ought to be made of as few parts, and thoſe as ſimple as poſſible, to anſwer its purpoſe; not only becauſe the expence of making and repairing will be leſs, but it will alſo be leſs liable to any diſorder. And it is needleſs to do a thing with many, which may be done with fewer parts.

6. If a weight is to be raiſed but a very little way, the leaver is the moſt ſimple, eaſy, and ready machine. Or if the weight be very great, the common ſcrew is moſt proper. But if the weight is to be raiſed a great way, the wheel and axle is a proper power, and blocks and pullies are eaſier ſtill; and the ſame may be done by help of the perpetual ſcrew.

Great wheels to be wrought by men or cattle, are of moſt uſe and convenience, when their axles are perpendicular to the horizon; but if by water, *&c.* then it is beſt to have their axles horizontal.

7. As to the combination of ſimple machines together, to make a compound one. Though the leaver when ſimple, cannot raiſe a weight to any great height; and in this caſe is of little ſervice; yet it is of great uſe when compounded with others. Thus the ſpokes of a great wheel are all leavers perpetually acting; and a beam fixed to the axis to draw the wheel about by men or horſes, is a leaver. The leaver alſo may be combined with the ſcrew, but not conveniently with pullies or with the wedge. The wheel and axle is combined with great advantage with pullies. The ſcrew is not well combined with pullies; but the perpetual ſcrew combined with the wheel, is very ſerviceable. The wedge cannot be combined with any other me-

chanical power; and it only performs its effect by percuſſion; but this force of percuſſion may be increaſed by engines.

Pullies may be combined with pullies, and wheels with wheels. Therefore if any ſingle wheel would be too large, and take up too much room, it may be divided into two or three more wheels and trundles, or wheels and pinions, as in clockwork; ſo as to have the ſame power, and perform the ſame effect.

In wheels with teeth, the number of teeth that play together in two wheels, ought to be prime to each other, that the ſame teeth may not meet at every revolution. For when different teeth meet, they by degrees wear themſelves into a proper figure. Therefore they ſhould be contrived, that the ſame teeth meet as ſeldom as poſſible.

8. The ſtrength of every part of the machine ought to be made proportional to the ſtreſs it is to bear. And therefore let every leaver be made ſo much ſtronger, as its length and the weight it is to ſupport is greater. And let its ſtrength diminiſh proportionally from the fulcrum, or point where the greateſt ſtreſs is, to each end. The axles of wheels and pullies muſt be ſo much ſtronger, as they are to bear greater weight. The teeth of wheels, and the wheels themſelves, which act with greater force, muſt be proportionally ſtronger. And in any combination of wheels and axles, make their ſtrength diminiſh gradually from the weight to the power, ſo that the ſtrength of every part be reciprocally as the velocity it has. The ſtrength of ropes muſt be according to their tenſion, and that is as the ſquares of their diameters, (ſee the end of Sect. VIII.) And in general whatever parts a machine is compoſed of, the ſtrength of every particular part of it muſt be adjuſted to the ſtreſs upon it, according to Sect. VIII. Therefore in ſquare beams the cubes of the diameters muſt be made proportional to the ſtreſs they bear. And let no part be ſtronger or bigger than is neceſſary for the ſtreſs upon it: not only for the eaſe and well-going of the machine, but for the diminiſhing the friction. For all ſuperfluous matter in any part of it, is nothing but a dead weight upon the machine, and ſerves for nothing, but to clog its motion. And he is by no means a perfect mechanic, that does not only adjuſt the ſtrength to the ſtreſs, but alſo contrive all the parts to laſt equally well, that the whole machine may fail together.

9. To avoid friction as much as poſſible, the machine ought not to have any unneceſſary motions, or uſeleſs parts; for a multiplicity of parts, by their weight and motion, increaſe the friction. The diameters of the wheels and pullies ought to be

large,

FIG. large, and the diameters of the arbors or ſpindles they run on, as ſmall as can be conſiſtent with their ſtrength. All ropes and cords muſt be as pliable as poſſible, and for that end are rubbed with tar or greaſe; the teeth of wheels muſt be made to fit and fill up the opens; and cut in the form of epicycloids. All the axles, where the motion is, and all teeth where they work, and all parts that in working rub upon one another, muſt be made ſmooth: and when the machine goes, muſt be oiled or greaſed. If a joint is to go pretty ſtiff and ſteady, rub a little greaſe upon it.

183. The axis *a* of a wheel, may have its friction diminiſhed, by cauſing it to run on two rollers, *B*, *C*, turning round with it, upon two centers.

184. Likewiſe inſtead of the teeth of wheels, one may place little wheels as *A*, *B*, running upon an axis in its center. And this will take away almoſt all the friction of the teeth. And in lanterns or trundles, the rounds may be made to turn about, inſtead of being fixed.

In all machines with wheels, the axes or ſpindles ought not to ſhake, which they will do, if they be too ſhort. And their ends ought juſt to fill their holes.

When the teeth of a wheel are much worn away, it makes that wheel move irregularly about, increaſes the friction, and requires more force; and may cauſe the teeth of two wheels to run foul upon one another, and to ſtop their motion, and endanger breaking the teeth. To prevent this, proper care ſhould be taken to dreſs the teeth, and keep them to their proper figure.

10 When any motion is to be long continued; contrive the power to move or act always one way, if it can be done. For this is better and eaſier performed than when the motion is interrupted, and the power is forced to move firſt one way, and then another. Becauſe every new change of motion requires a new additional force to effect it. Beſides, a body in motion cannot ſuddenly receive a contrary motion, without great violence. And the moving any part of the machine contrary ways by turns, with ſudden jerks, tends only to ſhake the machine to pieces.

11. In a machine that moves always one way, endeavour to have the motion uniform. Some methods of doing this may be ſeen in Prop. CXIII. and if one uniform motion be required to produce a motion either uniform or accelerated, ſome light may be had from Prop. CXI. Likewiſe how to communicate motion, conſult Prop. CX. And to change the direction, ſee Prop. CXII.

12. But when the nature of the thing requires that a motion is to be ſuddenly communicated to a body, or ſuddenly ſtopt: to prevent any damage or violence to the engine, by a ſudden jolt; let the force act againſt ſome ſpring, or beam of wood, which may ſupply the place of a ſpring.

13. In regard to the ſize of the machine; let it be made as large as it can conveniently. The greater the machine, the exacter it will work, and perform all its motions the better. For there will always be ſome errors in the making, as well as in the materials; and conſequently in the working of the machine. The reſiſtance of the medium in ſome machines has a ſenſible effect. But all theſe mechanical errors bear a leſs proportion to the motion of the machine, in great machines than in little ones, being nearly reciprocally as their diameters; ſuppoſing they are made of the ſame matter, and with the ſame accuracy, and are equally well finiſhed. Therefore in a ſmall machine they are more ſenſible; but in a great one almoſt vaniſh. Therefore great machines will anſwer better than ſmaller, in all reſpects, except in ſtrength; for the greater the machine the weaker it is, and leſs able to reſiſt any violence.

14. For engines that go by water, it is neceſſary to meaſure the velocity and force of the water. To get the velocity, drop in pieces of ſticks, *&c.* and obſerve how far they are carried in a ſecond, or any given time.

But if it flow through a hole in a reſervoir or ſtanding receptacle of water. The velocity will be found from the depth of the hole below the ſurface; by Cor. 2. Prop. XCVII. And its force by Cor. 1. Prop. CVII.

Thus let $s = 16\frac{1}{12}$ *feet*, $v =$ velocity of the fluid per ſecond. $B =$ the area of the hole. $H =$ height of the water; all in feet. Then the velocity $v = \sqrt{2sH}$; and its force $=$ the weight of the quantity $\frac{vv}{2s}B$ or HB of water, or $= \frac{62\frac{1}{2}}{112}HB$ hundred weight; becauſe a cubic foot is $= 62\frac{1}{2}$ lb. averd. Alſo a hogſhead is about $8\frac{1}{2}$ feet, or 531 lb. and a ton is 4 hogſheads.

When you have but a ſmall quantity of water, you muſt contrive it to fall as high as you can, to have the greater velocity, and conſequently more force upon the engine.

15. If water is to be conveyed through pipes to a great diſtance, and the deſcent be but ſmall; ſo much larger pipes muſt be uſed, becauſe the water will come ſlow. And theſe pipes ought not to be made ſtraiter in ſome places than others; for

the quantity of water conveyed through them, depends upon the bigneſs of the bore at the ſtraiteſt place.

Pipes of conduct coming directly from an engine, ſhould be made of iron, with flanches at the ends to ſcrew them together, with lead between, or elſe of wood; for lead pipes will bulge out at every ſtroke of the engine and burſt. But pipes next a jet muſt be lead. Pipes ſhould not turn off at an angle, but gradually in a curve: pipes of elm will laſt 20 or 30 years in the ground. But they muſt be laid ſo deep, that the froſt may not reach them, or elſe the water muſt be let out; otherwiſe the froſt will ſplit them.

The thickneſs of any pipe muſt be as the diameter of the bore, and alſo as the depth from the ſpring. For a lead pipe of 6 inches bore, and 60 or 70 feet high, the thickneſs muſt be half an inch. And in wooden pipes 2 inches.

Water ſhould not be driven through pipes faſter then 4 feet per ſecond, by reaſon of the friction of the tubes. Nor ſhould it be much wire-drawn, that is, ſqueezed through ſmaller pipes; for that creates a reſiſtance, as the water-way is leſs in narrow pipes.

And in pump work, where water is conveyed through pipes to higher places, the bores of the pipes ſhould not be made too ſtrait upwards; for the ſtraiter they are near the top, the leſs water will be diſcharged. Nor ſhould the pipe that brings the water into the pump be too ſtrait, for the ſame reaſon. The wider theſe are, the eaſier the pump works.

When pipes are wind-bound, that is, when air is lodged in them that the water can hardly paſs, it muſt be diſcharged thus. Going from the ſpring till you come to the firſt riſing of the ground, dig it open till the pipe be laid bare; then with a nail driven into it, at the higheſt part, or rather a little beyond, make a hole in the top; and all the air will blow out at the hole, and when the water comes, batter up the hole again. Do the ſame at every eminence, and all the air will be diſcharged. If the water runs faſt through the pipes, the air will be beyond the eminence; but ſtopping the water, the air will aſcend to the higheſt part. If air be driven in at firſt along with the water, the nail-hole muſt be left open, or a cock placed there to open occaſionally. Sometimes a ſmall leaden pipe is placed over the other, communicating with it in ſeveral places, in which is a cock at top to open upon occaſion.

192. 16. When any work is to be performed by a water wheel moved by the water running under it, and ſtriking the paddles

FIG. 192. or laddle boards. The channel it moves in ought to be ſomething wider than the hole of the adjutage, and ſo cloſe to the floats on every ſide, as to let little or no water paſs; and when paſt the wheel, to open a little that the water may ſpread. It is of no advantage to have a great number of floats or paddles, for theſe paſt the perpendicular are reſiſted by the back water, and thoſe before it are ſtruck obliquely. The greateſt effect that ſuch a wheel can perform, in communicating any motion, is when the paddles of the wheel move with $\frac{1}{3}$ the velocity of the water; in which caſe, the force upon the paddles is $\frac{4}{9}$ only; ſuppoſing the abſolute force of the water againſt the paddles, when the wheel ſtands ſtill, to be 1. So that the utmoſt motion which the wheel can generate, is but $\frac{4}{27}$ of that which the force of the water againſt the paddles at reſt, would produce. This is when the wheel is at the beſt; but oftentimes far leſs is done.

Machines to raiſe water, though well made, ſeldom loſe leſs than $\frac{1}{5}$ the computed quantity of water to be raiſed. The beſt contrived engine is ſcarce $\frac{1}{5}$ part better than the worſt contrived engine, when they are equally well executed.

A man with the beſt water engine cannot raiſe above one hogſhead of water in a minute, 10 feet high, to work all day.

30. 17. When a weight is to be raiſed with a given corporeal power, by means of the wheel and axle; ſo that the weight may receive the greateſt motion poſſible in a given time. The radius of the wheel and axle, and the weight to be raiſed, ought to be ſo adjuſted, that the radius of the axle (EF) : may be to the radius of the wheel (AB) :: as $\frac{2}{5}$ the power (P) : to the weight to be raiſed (W) : or, which comes to the ſame thing, the velocity gained by the power in deſcending muſt be $\frac{2}{5}$ the velocity which would be gained by gravity in the ſame time.

This only holds good, when the power is a heavy body, as well as the weight: but does not take place, when the power is ſome immaterial active force, ſuch as that of an elaſtic medium, the ſtrength of a ſpring, &c. whoſe weight is inconſiderable.

18. *Theſe principles alſo are very uſeful, and neceſſary to be known, where water works are concerned.*

The preſſure of the atmoſphere upon a ſquare inch, is 14.7 lb. *averd.* at a medium.

The weight of a column of water, equal to the weight of the atmoſphere, is 11 $\frac{1}{4}$ *yards.*

A cubic foot of water weighs $62\frac{1}{2}$ lb. *averd.* and contains 6.128 *ale gallons.*

An ale gallon of water contains 282 *inches*, and weighs 10.2 lb. *averd.*

A tun of water ale meaſure, weighs 1.1 *tun averd.* at 63 gallons the hogſhead.

A cylinder of water a yard high, and *d* inches diameter, contains $\frac{1}{10}$ *dd ale gallons*; and weighs $1\frac{1}{45}$ *dd pounds averd.*

SECT. XIII.

The deſcription of compound machines or engines, and the method of computing their powers or forces: with ſome account of the advantages and diſadvantages of their conſtruction.

PROP. CXX.

To deſcribe ſeveral ſorts of engines, and to compute their forces or effects.

There are two things required to make a good mechanic or engineer. The firſt is a good invention for contriving all the parts of a machine, to perform its motions and effects in the moſt ſimple and eaſy manner. The next is, to be able to compute the power or force of it; to know whether it can really perform the effect expected from it or not. The foundation of both theſe has been already laid down in this book. What ſeems to be neceſſary farther, is to give ſome examples in practice, by ſhewing the conſtruction of ſeveral mechanic engines, and computing their powers. As there is great ſkill and ſagacity in contriving fit and proper ways to perform any motion, ſo this is principally to be attained by practice, and a thorough acquaintance with machines of ſeveral kinds. I ſhall therefore give the mechanical conſtruction of ſeveral ſorts of machines, made for ſeveral different purpoſes, which will aſſiſt the reader's invention, and give him ſome idea how he may proceed in contriving a machine for any end propoſed. Of which I ſhall only give a ſhort explanation of the principal parts, not troubling the reader with any deſcription of their minuter parts, nor how they are joined together, or ſtrengthened, *&c.* It is ſufficient here to ſhew the diſpoſition and nature of the principal members: the reſt belongs to carpenters, joiners, ſmiths, *&c.* and is eaſily underſtood by any one.

To compute their powers.

1. As to ſimple machines, they are eaſily accounted for, and their forces computed, by the properties of the mechanic powers.

2. For compound machines; ſuppoſe any machine divided into all the ſimple ones that compoſe it. Then begin at the power and call it 1; and by the properties of the mechanic powers, find the force with which the firſt ſimple machine acts upon the ſecond, in numbers. Then call this force 1, and find the force it acts upon the third, in numbers. And putting this force 1, find the force acting on the fourth, in numbers; and ſo on to the laſt. Then multiply all theſe numbers together, the product will give the force of the machine, ſuppoſing the firſt power 1.

3. When pullies are concerned in the machine, all the parts of the ſame running rope, that go and return about ſeveral pullies, freely and without interruption, muſt be all numbered alike for the force. And if any rope act againſt ſeveral others, it muſt be numbered with the ſum of all theſe it acts againſt.

4. In a combination of wheels; take the product of the number of teeth in all the wheels that act upon and drive others, for the power; and the product of the teeth in all the wheels moved by them, for the weight. Or inſtead of the teeth take the diameters.

Or thus,

When a machine is in motion, if you meaſure the velocity of the weight, and that of the power, in numbers; then the firſt number to the ſecond, gives the proportion of the power to the weight.

Otherwiſe thus,

In wheel work, there are always two wheels fixt upon one axis, or elſe one wheel, and a pinion, trundle, or barrel, which ſupplies the place of a wheel. Of theſe two, call that wheel the *leader*, which is acted on by the power, or by ſome other wheel; and the other, on the ſame axis, call the *follower*, which drives ſome other forward. Then having either the number of teeth, or the diameter, of each, *take the product of all the leaders, for the weight; and the product of all the followers, for the* [illegible] Here the leader receives the motion, and the follower gives it.

5. And if the velocity of the power or weight be required. FIG. *Take the product of all the leaders, for the velocity of the power; and the product of all the followers, for the velocity of the weight.*

Other things that are more complex and difficult, muſt be referred to the general laws of motion.

EXAMPLE I.

Sciſſars, *pinchers*, &c. may be referred to the leaver of the firſt kind. A *handſpike* and *crow* are leavers of the firſt kind. *Knives* fixt at one end, to cut wood, bread, *&c.* are leavers of the ſecond kind. The *bones* in animals, alſo *tongs*, are leavers of the third kind. A *hammer* to draw a nail is a bended leaver.

Ex. II.

A *windleſs*, and a *capſtain* in a ſhip, and a *crane* to draw up goods out of a ſhip or boat, may be referred to the wheel and axel.

Ex. III.

All *edge tools* and inſtruments with a *ſharp point*, to cut, cleave, ſlit, chop, pierce, bore, *&c.* as *knives*, *hatchets*, *ſciſſars*, *ſwords*, *bodkins*, &c. may be reduced to the wedge.

Ex. IV.

The *bar AB* bearing a weight *C*, may be referred to the leaver; where the weight upon *A* : to the weight upon *B* :: is as *BC* : to *AC*. 186.

Ex. V.

Likewiſe if two horſes draw the weight *W*, in the directions *A* 1, *B* 2, by help of the *ſwingtree AB*; this may be referred to the leaver. And the ſtrength or force at *A*, to that of *B* :: is as *BC* : to *AC*. 187.

Ex. VI.

ACB is a *balance*, where the brachia *AC*, *CB* being equal, the weights in the two ſcales *D*, *E* will be equal. The properties of a good balance are, 1. That the points of ſuſpenſion of the ſcales, and the center of motion of the beam, be in one right line. 2. That the brachia or arms be exactly of equal length from the center of motion. 3. That they be as 188.

FIG. 188. long as poſſible with conveniency. 4. That there be as little friction as poſſible in the motion. 5. That the center of gravity of the beam be in, or but very little below, the center of motion. 6. That they be in equilibrio when empty.

If one brachium AC be longer than the other CB, then the weight in the ſcale E muſt be greater than that in D, to make an equilibrium. And then you will have a deceitful balance, which being empty, or loaded with unequal weights, ſhall remain in equilibrio. For $AC : CB ::$ weight in E : weight in D; by the property of the leaver. But changing the weights from one ſcale to the other will diſcover the deceit; for the balance will be no longer in equilibrio.

Ex. VII.

190. The *ſteelyard AB* is nothing but a leaver, whoſe fulcrum is C, the center of motion. If the weight P placed at D reduces the beam AB to an equilibrium; and there be taken the equal diviſions $D\,1$, $1\,2$, $2\,3$, $3\,4$, *&c.* then the weight P placed ſucceſſively at 1, 2, 3, 4, *&c.* will equi-ponderate with weights as W, ſuſpended at B, which are alſo as the numbers 1, 2, 3, 4, *&c.* reſpectively. Moreover if the diviſions $D1$, $1\,2$, $2\,3$, *&c.* be each $= CB$. Then if P be ſucceſſively placed at 1, 2, 3, *&c.* the weight W to balance it, will be reſpectively equal to P, $2P$, $3P$, *&c.* that is to 1, 2, 3 pounds, *&c.* if P is a pound.

For by the property of the leaver $CP \times P + CD \times P = CB \times W$, that is $PD \times P = CB \times W$. And $CB : PD :: P : W$, univerſally. Whence, if DP or $D1 = CB$, then $W = P$. If DP or $D2 = 2CB$, then $W = 2P$, *&c.* But if CB be greater than $D1$, $1\,2$, *&c.* then will the conſtant weight P be greater than W, $2W$, &c.

The properties neceſſary for a ſteelyard to have, are theſe.

1. That the fixt weight P being placed at D, where the diviſions begin, ſhall make the beam in equilibrio.

2. That the diviſions $D\,1$, $1\,2$, $2\,3$, *&c.* be equal to one another.

3. That CB may be of any length, provided the weight P be rightly adjuſted to it: *viz.* ſo that $CB : D1 :: P : 1$ pound, if W be pounds. Or $CB : D1 :: P : 1$ ſtone, if W be ſtones.

4. That the beam be ſtrait, and the upper edge in a line with the centers C, B.

5. That it move eaſily and freely on its center C.

Many ſteelyards are likewiſe graduated on the under ſide, FIG. which may be uſed by turning them upſide down. Generally 190. one ſide is for ſmall weights, and the other for great ones. And each ſide is adjuſted by the foregoing rules; and all the crooks hanging at it (except the moveable one for the weight) muſt go to the weight of the beam.

Ex. VIII.

Let AB be a *cheeſe preſs*; CE, FG are leavers moveable about 189. the points D, E, F, G, by applying the hand at C. S the ſtone or weight. H the cheeſe.

If $CD = 5$, $DE = 2$, $FG = 6$, $GH = 2$, $FR = 1$, $FH = 4$. Then in the leaver CE, D is the fulcrum. Call the power at C, 1; then the force at E or F is $\frac{5}{2}$. And in the leaver FG, whoſe fulcrum is G; if the power at F be 1, the force at R is $\frac{6}{5}$, therefore the power at C, to the weight S, is as 1 to $\frac{5}{2} \times \frac{6}{5}$ or 3. Alſo the weight of the ſtone at R, to the preſſure at H, as 2 to 5, or 1 to $\frac{5}{2}$. And the power at C, is to the preſſure at H, as 1 to $3 \times \frac{5}{2}$ or $7\frac{1}{2}$.

Ex. IX.

Let EG be a *ſpinning wheel*. Diameter of the rim $EF = 18$. 191. Diameter of the twill $ab = 2$. Diameter of the whorle $cd = 3$. $EabF$ the band going about the twill. $EcdF$ the band going about the whorle. Therefore whilſt the rim makes 1 revolution, the twill makes 9, and the whorle and feathers 6. Therefore there are 3 revolutions of the twill, for 2 of the feathers n. And conſequently the difference of the revolutions which is 1, is the quantity taken up by the twill, whilſt the thread trn is twined by theſe 2 revolutions of the feathers. The greater the difference of the revolutions of the twill and feathers, the more the wheel takes up. And the nearer an equality, the more ſhe twines. If they make equal revolutions in the ſame time, ſhe will not take up at all. And if the feathers make no revolutions, ſhe will twine none. The greater the proportion of the rim to the whorle and twill, the faſter ſhe will do both.

Ex. X.

A machine to raiſe a weight by the force of the running water HI, carrying the wheel LK, by means of the floats F, F. Let the diameter of the wheel LK be 10; of GB, 2; of DC, FIG. 192. 11; of *AE*, 3. Let the power of the water againſt the floats *F*, be 1. Then the force at *B* to move the wheel *CD*, will be 5; again if the power at *B* be 1, the force at *A* will be $3\frac{2}{3}$. Therefore the force of the water, to the weight *W*, is as 1 to $5\times3\frac{2}{3}$, or as 1 to $18\frac{1}{3}$.

When the wheels and axles and weight are ſo adjuſted, that the velocity of the floats at *F*, is $\frac{1}{3}$ the velocity of the water there; then the weight *W* will have the greateſt motion of aſcent poſſible. For if any one thing be changed, whether the weight, or the diameter of any wheel or axle, whilſt the reſt remain the ſame, the motion will be leſſened.

Ex. XI.

193. In the machine *FB*, which raiſes the weight *W*, by means of the wheel *EG*, and the perpetual ſcrew *BE*. Let the circumference deſcribed by the power *C* be 30 inches, the diſtance of two threads of the perpetual ſcrew *E*, be 1 inch. Diameter of the wheel $EG=5$ feet, of $DA=2$. Therefore if the power at *C* be 1, the force acting at *E* to turn the wheel *EG* will be 30. And if the power at *E* be 1, the force at *D* will be $2\frac{1}{2}$. Therefore the power at *C*, to the weight *W*, is as 1 to $30\times2\frac{1}{2}$ or 1 to 75.

Note, it is the ſame thing whether *CB* be ſtrait or crooked, whilſt the diſtance *BC*, in a ſtrait line is the ſame; and in meaſuring, you muſt always take the ſtrait line *BC*.

Ex. XII.

194. In a machine compounded of wheels to raiſe a weight; let $AB=5$, diameter of the barrel $MN=2$, the number of teeth in the wheels and nuts, as follows; $CD=10$, $CE=40$, $FG=12$, $FH=50$, $KI=12$, $IL=64$. Then the power applied to *B*, is to the weight *W*, as $1\times10\times12\times12$ to $5\times40\times50\times64$; that is as 1440 to 640000, or as 1 to $444\frac{4}{9}$.

But if the power was at *W*, to move the weight *B*; then the ratio will be inverted. For then the power will be to the force at *B*, as 444 to 1. Or if the velocity of *B* was required, you will have the velocity of *W* to that of *B*, as 1 to 444.

Ex. XIII.

195. A machine to raiſe a weight by help of the *triangle ABEF*, the windleſs *CC*, and two pullies *P*, *Q*. Let the diameter

FIG. 195. *HG* where the rope goes, be$=2$, radius $CD=5$. Then if the power at *D* be 1, the force at *H* is 5. And if the force at *H*, drawn by one rope, be 1, the force at *W* drawn by two ropes, will be 2. Therefore the power at *D*, to the weight *W*, is as 1 to 2×5 or 10. If the leg *AB* be wanting, the other two may be ſet againſt a wall, or upheld by ropes, and then it is called a pair of *ſheers*.

Ex. XIV.

196. If the weight *A* is to be lifted by the 3 pullies *C*, *D*, *E*, of which *C* is fixt. Call the power at *B*, 1. Then the force ſtretching *AE* is 1; and both together is equal to the force of $DE=2$; and force $DA=2$; whence, force $DC=4$; likewiſe force $CA=4$. Therefore the whole force acting at *A* is $1+2+4=7$, and the power at *B* to the weight *A*; as 1 to 7.

Ex. XV.

197. In this machine, *hACD* is a running rope fixt at *D*, *B* a fixt pulley. Let the power at *h* pulling the rope *hA* be 1. That on *AC* 1, and *CD* 1. Then will *AB* be 2, and *BC* 2, *BE* 4. And the weight *W* oppoſing *AC*, *BC* and *DC*, will be $1+2+1=4$. Whence the power at *h*, to the weight *W*, is as 1 to 4.

Ex. XVI.

198. Another machine with pullies. *A* a fixt pulley; the ends of the ſeveral ropes are fixt at *B*, *C*, *D* *E*. Suppoſe the power at $M=1$, then the force on *AF*, *FB* is 1; on *FG*, *GC*, 2; on *GH*, *HD*, 4; on *HI*, *IE*, 8. But the weight *P* acts againſt *HI*, *IE*, and is therefore$=16$: and the power is to the weight, as 1 to 16.

SCHOL.

39. In a ſingle pulley, as fig. 39. if a given power at *P* was to be a weight or heavy body, which was to raiſe ſome other weight *W*, there will be the greateſt motion generated in *W*, in any given time, when $W=\frac{2}{5}P$.

42. And in a combination of pullies, as fig. 42. if a weight *P* was to raiſe another weight *W*; and if velocity of *W* : velocity of $P :: \frac{2}{5}P : W$; then *W* will be the weight which will acquire the greateſt motion in a given time, by that given power *P*.

FIG. 199.

Ex. XVII.

Let *DE* be a *boat* rowed by oars; and let *ABC* be one oar. Here the power acts at *A*, and the pin *B* will be the fulcrum; and the force at *C*, acting againſt the water, is that which gives her motion. Let the power at *A* be 1; then the force at *C*, by which the boat is moved, is $\frac{AB}{BC}$. Whence the longer *AB*, or the ſhorter *BC* is, ſo much more power there is at *A* to move her forward.

Therefore long oars have the diſadvantage of loſing power. Yet the oars may be too ſhort, as well as too long. For if they be very ſhort, the motion of the boat will allow little time to ſtrike, and they will have but ſmall force to act againſt the water with, in ſo ſmall a time, as well as from the ſlow motion of the end *C*; which is a diſadvantage on the other ſide.

Ex. XVIII.

200. Let *FR* be *a boat or a ſhip*, *AS* a ſail. Suppoſe a plumb-line drawn through the center of gravity of the ſection of the ſhip and water; and another line *BO*, parallel to the horizon and to the axis or keel of the ſhip, and to paſs through the center of preſſure or reſiſtance of the ſhip, which ſhe has by the water in her motion. Let this interſect the former plumb-line in *O*. Through *C* the center of gravity of the ſail, draw *CD* perpendicular to the ſail; and *CB* perpendicular to *BO*, and *AS* in the plane of the triangle *CBD*.

Then if *DC* be the force of the wind againſt the ſail *AS*, then *DB* is the force generating her progreſſive motion, and *BC* is the force lifting the ſhip upwards. Now the force *DB*, acting at *C*, in direction *DB*, endeavours to turn the ſhip round an axis paſſing through *O*, with a force which is equal to the abſolute force *BD* × by the diſtance *CB*, or $CB \times BD$; and this is the force by which her head is depreſſed. Likewiſe the force *BC*, in direction *BC*, endeavours to turn the ſhip round an axis at *O*, the contrary way; and that with the force $BC \times$ diſtance *BO*, or $BC \times BO$; and this is the force that raiſes her head. Therefore the force to raiſe her head is to the force to depreſs it, as $CB \times BO$ to $CB \times BD$, or as *BO* to *BD*.

Hence, if the point *D* fall before *O*, then the ſail endeavours to raiſe the ſhip's head; if it be behind *O*, it endeavours to ſink it. If it be in *O*, it will keep her ſteady. And the height

of the ſail *AS* contributes nothing to her progreſſive motion; FIG. 200.
the ſame ratio of the abſolute to the progreſſive force, remains ſtill as *CD* to *DB*.

Ex. XIX.

EF is a *cart* or *carriage*, *BD* a rub for the wheel *CAD* to paſs over, *AB* the horizontal plane; *DB*, *AC* perpendicular, and *OD* parallel to *AB*. *C* the center of the wheel. Then the horizontal force required to pull the wheel over the rub *BD*, is as $\frac{DO}{CO}$. And the difficulty of going over rubs increaſes in a greater ratio than that of their heights. Alſo the higher the wheels, the more eaſily they paſs over them; but then they are more apt to overturn. To draw the cart with the leaſt power over the rub *BD*, it ſhould not be drawn in the horizontal direction *AB* or *OD*; but in the direction *AD*. The advantage of high wheels is, that they paſs the rubs moſt eaſily, and they have alſo leſs friction, and ſink leſs in the dirt, and more eaſily preſs down an obſtacle. But their diſadvantage is, that they eaſily overturn; they alſo make cattle draw too high; for they can apply their ſtrength beſt when they draw low and upward, as in the direction *AD*; which is the advantage of low wheels. Yet if the wheels are high, they may be made to draw low, by fixing the limmers or traces as far below the axle as you will. which will then be an equal advantage with low wheels. For the power not pulling at the wheel, but at the carriage, may draw from any part of it. There is another advantage in ſmall wheels, that they are better to turn with. 201.

A *waggon* with 4 wheels is more advantageous than a cart with only 2 wheels, eſpecially on ſand, clay, &c. Narrow wheels and narrow plates are a diſadvantage; the broader the wheels, the leſs they ſink, and therefore require leſs draught, and alſo cut the roads leſs; yet they take up a great deal of dirt, which clogs the carriage. There is a great deal of friction in all carriages, as is evident by the force required to draw them upon plain ground. And for that reaſon, experience can only inform us, how much force is able to draw any carriage. To make the reſiſtance as ſmall as can be, axles of iron, running in braſs boxes in the wheel naves, go the eaſieſt.

The ſpokes in the wheel ought to be a little inclined outwards; that when a wheel ſinks into a rut, the ſpokes (bearing then the greateſt weight) may be nearly perpendicular to the horizon.

FIG. 201. The underſide of the axle-tree, where the wheels run, ought to be nearly in a right line; if they ſlant much upward towards the ends, the wheel will work againſt the lin pin. Yet this cauſes the wheels to be further aſunder at top than at bottom in the rut, becauſe the ends of the axle-tree are conical; which is an inconvenience.

Ex. XX.

202. Suppoſe the *waggon FG* is moved forward, by a power acting within it. Which power turns the wheel *DE* by the ſpokes *AD*, *AD*, &c. and *DE* turns the wheel *IC*, which carries the waggon. Let the power at *A* be 1, then the force acting at *E* will be $\frac{DA}{DE}$; alſo if the power at *E* be 1, the force at *C*, by which the waggon is moved, will be $\frac{BE}{BC}$. Therefore the power at *A*, to the force by which the waggon can be moved, is as 1 to $\frac{DA \times BE}{DE \times BC}$. Or the power is to that force, as $DE \times BC$ to $DA \times EB$. It will be the ſame thing, if inſtead of teeth, the wheel *DE* carries *EB* by a chain going round them. You muſt ſuppoſe the like wheels on the oppoſite ſide.

Hence if the abſolute force to move the waggon without, be 1; the force within, applied at *A*, to move it, will be $\frac{DE \times BC}{BE \times DA}$.

Ex. XXI.

203. *ABCD* are the *ſails of a windmill*, all alike inclined to their common axis, and facing the wind, and turning about in the order *ABCD*. *WC* the direction of the wind parallel to the axis *EH*. Since *WC* is perpendicular to *EC*, draw *CF* in the ſail perpendicular to *EC*; then the angle *WCF* will be the angle of incidence of the wind upon the ſail. Therefore the force of the wind to turn the ſails about the axis *EH*, is as the ſquare of the ſine of the angle *WCF* × by its coſine. And the force acting againſt the mill, in direction of the axis *EH*, is as the cube of the ſine of *WCF*. Now ſince the force of the wind to turn the ſails round, is as $\overline{S.WCF}^2 \times$ coſ. *WCF*; therefore when that force is the greateſt, the angle *WCF* will be 54° : 44′.

And this is the moſt advantageous poſition of the ſails to move them from reſt; and would always be ſo, if the wind ſtruck them in the ſame angle when moving as when at reſt. But by reaſon of the ſwift motion of the ſails, eſpecially near the end *G*, the wind ſtrikes them under a far leſs angle; and not only ſo, but as the motion at the end *G* is ſo ſwift, it may ſtrike them on the backſide. Therefore it will be more advantageous to make the angle of incidence *WCF* greater, and ſo much more as it is further from *E*. Therefore at the places *n*, *o*, *G*, the tangents of the angles ought to be nearly as the diſtances, *En*, *Eo*, *EG*. And therefore the ſails ought to be twiſted, ſo as at *r* to lie more ſharp to the wind, and at *G* almoſt to face it. And by that means they will avoid the back wind. FIG. 203.

Ex. XXII.

GB is a common *ſucking pump*; *GKL* the handle; *CD* the bucket; *E*, *F* two clacks opening upwards. When the end *L* is put down, the end *G* raiſes the ſucker or bucket *CD*, and the valve or clack *F* ſhuts; and the water above the bucket being raiſed, the weight of the atmoſphere is taken off the water underneath in the pump. Then the preſſure of the external air in the pit or well *MN*, raiſes the water up the pump, opens the valve *E*, and aſcends thro' the hole *B* into the body of the pump *DB*. Again, when the handle *L* is raiſed, the bucket *CD* deſcends, the valve *F* opens, and lets the water aſcend through it, and the preſſure of the water ſhuts the valve *E*, ſo that the water cannot return through *B*. Then whilſt the end *L* is put down again, the ſucker *CD* is raiſed again, together with the water above it, whilſt more aſcends through *B*. So that at every ſtroke of the handle, water is raiſed into the pump, till at laſt it flows through the pipe *H*. 204.

If the bucket *CD* be more than 30 or 32 foot from the ſurface of the water *MN* in the pit, no water will aſcend above it; for the preſſure of the atmoſphere reaches no farther. Therefore it muſt be always within that diſtance, or this pump is uſeleſs for raiſing water.

The weight of water which the bucket lifts at each ſtroke is that of a column of water, whoſe height is *MH*, and its diameter that of the bore of the pump at *CD*, where the bucket goes. Therefore as *GK* to *KL* :: ſo the power applied at *L*, to that weight. Therefore it ſignifies nothing where the bucket is placed, as to the weight of water. If a leak happens in the barrel of the pump below the bucket *CD*, the air will get in and hinder the

work-

FIG. 204. working of the pump: If above *CD*, only ſome will be loſt; therefore *CD* ſhould be placed low; but then it will be bad to come at to repair it.

The bucket, ſucker, or piſton, is to be ſurrounded with leather to fit exactly, and muſt move freely up and down in the barrel, and muſt alſo exactly fill it. Of valves or clacks, ſome are flat, made of leather; others are conical: and they muſt all fit very cloſe, and move freely. To balance the weight of water, the handle *KL* is commonly made heavy, as of iron, with a knob at the end *L*.

The bore of the pipe at *B* ſhould not be too ſtrait; the wider it is, the more freely the water aſcends, and the eaſier the pump works. Likewiſe the longer ſtroke the pump makes, the more water is raiſed by the ſame power; there being leſs water loſt, by the valves ſhutting.

Calculation of a common pump.

Suppoſe *LK*, 3 feet; *KG*, 8 inches.
$h = HM$ the height from the water in yards:

Then the diameter of the bore at *D* will be $= \sqrt{\frac{100}{h}}$ inches.

And a ſingle perſon will raiſe $\frac{80}{h}$ hogſheads of water in an hour.

In many pumps for common uſe, it is not neceſſary to draw a great quantity of water; and then a ſmaller bore will ſerve, as 3 or 4 inches; which will make the pump go ſo much the lighter.

Ex. XXIII.

206. If a man ſitting in the ſcale *E*, be in equilibrio with a weight in the ſcale *A*; and if he thruſt againſt the beam *CB*, with a ſtick or otherwiſe, in direction *ED*, and by that means thruſts out the ſcale *E* to the poſition *BE*. Then the man in the ſcale *E* will over-balance the other ſcale *A*, and raiſe the weight. For let *EL* be perpendicular to *FB*; then the force at *E* to turn the ſcales is to the contrary force at *F*, as *CL* to *CF* or *CB*. For it is the ſame thing, as if *E* was ſuſpended at *L*.

And when the perpendicular obſtacle *GH* hinders the ſcale from going out; and the center *C* is always kept ſteady. Yet the ſcale *E* will ſtill preponderate. For let *ED* be the force acting againſt *D*; this is equivalent to the two forces *EB*, *BD*, acting

at E and D. The force BD tending to or from the center, does FIG.
nothing. But the force EB at E, acting at the diſtance CB, its 205.
power to bring down the ſcale E is $CB \times BE$. And the ſame force acting at D, its power to puſh up the ſcale is $CD \times BE$. And their difference $DB \times BE$ is the abſolute force to thruſt down the ſcale. And this force is to the whole thruſting force DE, as DB to DE. And if D were on the other ſide of C, the force would ſtill be $DB \times BE$, or $\overline{CB+CD} \times BE$.

But if the ſcale E was not moveable about B, as if it were tied by the cord DE; then no force acting againſt any part of the beam FB, could have any effect to deſtroy the equilibrium.

Ex. XXIV.

Suppoſe a man A ſtanding upon the plank CB, ſupported 207.
only at the end C, and pulling the end B towards him by the rope EB, in order to keep himſelf and the plank from falling.

Imagine the man and the plank to be one body; then the action and re-action in direction EB, deſtroy one another, and his pulling does nothing. It would therefore be in vain for him to endeavour to ſupport himſelf by that force; for both he and the plank muſt fall down together towards B, by their own weight.

Ex. XXV.

CD is a machine with two wheels fixed to an axis DF, round 208.
which goes a cord $GDFE$. There is a power at E endeavouring to draw the machine towards E, in a direction parallel to the horizon HO, by the cord EF, going under the axis DF. In the radius AH of the wheel, take AB equal to the radius of the axle DF, towards H, becauſe the ſtring goes below it. Then the force to move the machine, is the ſame as if the ſtring was fixed at B; where H is the fulcrum, A the weight. Then the force to move the machine towards E, with the given power E, will be as BH. Therefore it would be in vain, by pulling at the ſtring, to endeavour to make the body roll towards D, the contrary way. But if DF was greater than the diameter of the wheel, that is, if B falls beyond H, then the force drawing towards E, would move the body towards D the contrary way.

If the direction of the power DE be elevated above the horizon, as fe; then the machine could approach or recede, till the direction of the ſtring ef fell upon the point of contact H, and there it would reſt.

Ex.

Ex. XXVI.

209. AB is an *artificial kite*, kept up by the wind blowing in direction IVC, by drawing the ſtring $AIBIH$, fixed at A and B. The kite will gain ſuch a poſition, that HI produced will paſs through the center of gravity of its ſurface at C. Draw CO perpendicular to BA, and DO perpendicular to the horizon HO. Then OC is the direction of the force of the wind acting againſt the kite; and the force of the wind to keep her up, is as the ſquare of the ſine of the angle $ACIV$ or COD. Now if DO repreſent the given weight of the kite, CO will be the force of the wind acting againſt her, and CD the force pulling at the ſtring. The tail EF (with a bullet F at the end) being always blown from the wind, keeps her head always towards the wind.

As the direction of the thread always paſſes through C, therefore the angle ACH, and conſequently HCO, will always be the ſame at all altitudes. And ſhe can never aſcend ſo high, till the angle of altitude CHO be equal to ACH. And hence it follows, that the leſs the angle HCO is made, the higher ſhe will riſe. And likewiſe the greater the wind is, or the lighter the kite, *ceteris paribus*, the higher ſhe will riſe.

210. After a like manner a machine as ab may be contrived, to keep at the top of a running water, being held by the ſtring de tyed to a ſtone and ſunk to the bottom: ab is a thin board, b a piece of lead to ſink the end b, but the whole muſt be lighter than water, cd an iron pin fixed at C. Or the machine may have a looſe tail at b, heavier than water, as in the kite.

Ex. XXVII.

211. If AB is a machine to be moved by a power acting at C out of the machine, in direction DC. DF, GI two leavers within the machine, moveable about the two fixed fulcrums E, H.

Call the power at C, 1; then the force at F to move the leaver GI, is $\frac{DE}{EF}$. Then if the force at F be 1, that at the obſtacle I out of the machine is $\frac{GH}{HI}$. Therefore if the power at C be 1, the force acting againſt the obſtacle at I, or which is the ſame thing, the force urging forward the machine towards B, is $\frac{DE \times GH}{EF \times HI}$. But the power at C draws back the machine with the force 1. Therefore the abſolute force urging forward the machine is $\frac{DE\times GH}{EF\times HI} - 1$ or $\frac{DE\times GH - EF\times HI}{EF\times HI}$. FIG. 211.

Note, if the force at F be 1, the force againſt H is $\frac{FI}{HI}$. But this is not the force urging forward the machine, but to tear her in pieces, or to ſeparate the fulcrums E, H, from one another.

If there had been three leavers, and the power at D; the third had been directed towards K, the way the machine goes; then the power 1 muſt be added to the force at I, and the whole is the force urging forward the machine.

Hence, if the abſolute direct force to move the machine be 1, the power applied at D, which is able to move it, will be $\frac{EF\times HI}{DE\times GH - EF\times HI}$. But if the power at D act within the machine, this power could only be $\frac{EF\times HI}{DE\times GH}$; ſince there is then no force to be deducted for drawing back the machine.

Ex. XXVIII.

DABH is a *wooden bridge.* *AC*, *AD*, *AB*, *BH*, *BO*, beams of timber. *DE*, *EL*, *SR*, *RH*, braces to ſtrengthen the angles *A*, *B*. The ſtreſs upon any of the angles, is, *ceteris paribus*, ſo much greater, as the angle is greater. But the ſtrength on any angle *A*, is as the perpendicular *AP*. 212.

Ex. XXIX.

AB a *ſailing chariot.* *CDEF* horizontal ſails, ſo contrived that the ſails *D* facing the wind may expand, and thoſe going from the wind may contract. The ſails are turned about by the wind coming from any coaſt. Theſe ſails turn the axis and trundle *GH*. And the trundle turns the wheel *IL* by the cogs in it. Therefore the chariot may move in any direction. *R* is a rudder to ſteer with. 213.

Suppoſe the chariot to go againſt the wind. Let *D* be the center of preſſure of the two ſails *C*, *D*, the wind blows on. And let the power, (that is the force of the wind acting againſt the ſails) be 1, then the force acting againſt the teeth in *IL*, is $\frac{GD}{OH}$. And this force being 1, the force at *L* is alſo 1. There-

FIG. 213. fore the power at *D* to the force at *L*, is as 1 to $\frac{GD}{OH}$; or as *OH* to *GD*. Now ſince the maſt is ſtrained by the power falling on the ſails, therefore by this power *OH*, the chariot is urged backward. And by the force at *L* which is *GD*, it is urged forward. Let *R* be the force of the wind upon the body of the chariot, together with the friction in moving. Therefore if *GD* is greater than the radius $OH+R$, the chariot will move forward againſt the wind; if leſs, backward. But if they be equal, it will ſtand ſtill.

Ex. XXX.

214. *FG* a *chariot* or waggon to ſail againſt the wind. *S* the ſails of a windmill, turning in the order 1, 2, 3. As the ſails go round, the pinion *A* moves *B*, and the trundle *C* moves *D*, which has both teeth and cogs. *D* by its teeth moves *E*; and the trundle *E* fixt to the axle-tree carries round the wheels *H*, *I*, which move the waggon, in direction *HG*.

The ſails are ſet at an angle of 45°, ſo the force to turn them, and the force in direction of the axis, will be equal. This waggon will always go againſt the wind, provided you give the ſails power enough, by the combination of the wheels. But then her motion will be ſo much ſlower.

Ex. XXXI.

215. Let *AB* be part of a *rope*, *cd*, *cd*, &c. the particular ſtrands running about in a ſpiral manner. Let *FH* be the axis of the rope, the angle *GFH* or *HFK* the obliquity of the ſtrands. Draw *KH*, *GH* parallel to *FG*, *FK*, and draw *GEK*. Then the tenſion of the rope in direction *HF*, is to the ſtreſs on all the ſtrands in direction *FG*; as *FH* to $FG+FK$ or $FG+GH$, that is, as *EF* to *FG*. Therefore the abſolute force by which the rope is ſtretched, is to the ſtrain or ſtreſs upon all the ſtrands, or upon the twiſted rope; as *FE* is to *FG*; and ſo is the length of any part of the rope, to the correſpondent length of a ſtrand.

Hence ropes the leaſt twiſted are ſtrongeſt and bear the moſt weight; and the harder they are twiſted the ſooner they will break. And for the ſame reaſon if they be double twiſted, they will be weaker ſtill. But as it is very difficult to make all the fibres pull equally without twiſting, and impoſſible to make a rope hold together without it. Therefore it is neceſſary, it have as much, as to prevent the fibres from drawing out; and a ſmall degree will not much impair its ſtrength. A rope conſiſting of ſeveral ſtrands, is thicker when twiſted, than when untwiſted. FIG. 215.

Ex. XXXII.

ABC a *ſyphon* or crane. If the ſhorter end *AB* be immerſed in a veſſel of water *AD*, then by applying the mouth to the end *C*, and ſucking till the water comes, it will continue to flow out at the end *C*, as long as that end is lower than the ſurface of the water at *D*. If there be a mouth-piece at *E*, then ſucking at *E* (whilſt the end *C* is ſtopt with the finger) will make the water flow, when the finger is taken off. And when the water is begun to flow, the hole at *E* ſhould be ſtopt up, or elſe the water will flow no longer, than till the ſurface at *D* be as low as *E*. 216.

The reaſon of its flowing is this: the perpendicular height of the column of water *BC* being greater than that of *BD*; the preſſure at *C* is greater than at *D*; and the preſſure of the atmoſphere being the ſame at *D* and *C*; therefore the greater weight at *C* will make it flow out there; whilſt the preſſure of the atmoſphere at *D* forces more water up the tube *DB*; and ſo keeps it continually running as long as there is any water, and the end *C* continues lower than the ſurface at *D*. But if *C* is higher than *D*, the water will return back into *BD*. But if the height *DB* exceed the preſſure of the atmoſphere, which is 30 or 32 feet; then it cannot be made to flow out at the end *C*; or if there be a hole in the ſyphon higher than the ſurface at *D*, the air will get in, and the water will return through *BD*. Or if the ſyphon be very wide, the air will inſinuate itſelf into the end *C*, between the water and the tube, which will hinder it from running. To prevent which the end *C* may be immerſed into another veſſel of water, lower than the ſurface at *D*. If the ends of the ſyphon be turned up, as *F*, *G*, then the water will remain in the ſyphon, after it has done working, which in the other will all run out.

Ex. XXXIII.

CDLF a veſſel of water; *AB* a tube open at both ends, and about $\frac{1}{8}$ inch diameter. *AE* a quantity of mercury put into the tube. Then ſtopping the end *B*, let the other end *A* be immerſed deep enough in the water. Then opening the end *B*, the mercury will ſink ſo deep in the tube, till the height of the 217.

FIG. 217. the water *AB* be 14 times the height of the mercury *AE*: and then the mercury will be at reſt.

For the ſpecific gravities of water and mercury being 1 and 14, the column of water *AB* will be equal in weight with the column of mercury *AE*. Therefore the preſſures at *A* being equal, they will ſuſtain one another.

Ex. XXXIV.

218. *A*, *B*, are two *barometers*; *ed* is a tube, its bore $\frac{1}{4}$ or $\frac{1}{5}$ inch diameter, at leaſt, cloſe at top, and communicating with the veſſel *C*, with mercury in it. *C* is open to the external air. The uſe of this inſtrument is to ſhew the weight of the atmoſphere, and its variations. This tube and veſſel with mercury, is put into a frame, and hung perpendicular. Near the top of the tube is placed a ſcale of inches, by which the height of the mercury in the tube is known, and likewiſe a ſcale for the weather. At the top of the tube above the mercury is a vacuum. Now the atmoſphere preſſing upon the ſurface of the mercury at *C*, keeps the mercury ſuſpended at the height *d* in the tube, which therefore will be higher or lower according to the weight of the atmoſphere. The height of the mercury in the tube is generally 28, 29, or 30 inches; ſeldom more. If any air get into the tube it ſpoils the machine. Leſt the quickſilver ſtick to the glaſs, it is good to drum a little with the fingers upon it, in making any obſervation.

Rules for obſervation of the weather.

1. The riſing of the mercury preſages fair weather. It riſes and ſtands higheſt in ſerene, ſunſhiny, droughty weather: and in calm froſty weather it generally ſtands high. In thick foggy weather it often riſes.

2. The falling of the mercury denotes foul weather. It generally falls or ſtands low, in rainy, windy, or ſnowy weather.

3. In windy weather the mercury ſinks loweſt of all; and riſes faſt after ſtorms of wind.

4. In very hot weather the falling of the mercury foreſhews thunder.

5. In winter, the riſing foretels froſt; and falling in froſty weather foretels thaw.

6. In continual froſt, the riſing preſages ſnow. At other times, it generally falls in ſnowy weather.

7. When the mercury riſes after rain, expect ſettled ſerenity; if it deſcends after rain, expect broken ſhowery weather. FIG. 218.

8. When foul weather happens ſoon after the falling of the mercury, or fair weather after its riſing, expect but little of it.

9. In foul weather, riſing faſt and high, and continuing ſo 2 or 3 days, before the foul weather be quite over, expect a continuance of fair weather to follow.

10. In fair weather falling faſt and low, and continuing 2 or 3 days before the rain comes, expect a great deal of wet, and probably high winds.

11. Unſettled motion of the mercury denotes unſettled weather.

12. The greateſt height of the mercury is upon eaſterly and north-eaſterly winds.

13. The alterations are greater in northerly parts, than in the more ſoutherly; and there is little or no variation within the tropics.

Ex. XXXV.

ABE is an *artificial fountain.* *A* is an open veſſel, *B* a cloſe one; *E* may be made cloſe by ſtopping the hole *C*; theſe veſſels all communicate by the tubes *F*, *G*. The tube *F* reaches near the top of *E*, and the tube *G* near the bottom of *B*. Pour water into *C* almoſt to the top of the pipe *F*; and ſtop the hole *C*. Then pour water into *A* which will run down into *B*. Then open the cock *D*, and the water will ſpout up to the height of *AB* above *D*. For the air in *B*, *F*, *E* is condenſed by the weight of the column of water *AB*; and its preſſure on the water in *C*, is equal to the weight of that column; and will therefore make the water ſpout to that height above the water in *C*, nearly. But the pipe leading to *D* muſt be turned curve. 219.

Ex. XXXVI.

AB is a *dart* or an *arrow*; at *A*, 3 or 4 feathers are placed nearly in planes paſſing through the arrow. If the feathers were exactly in this plane, the air could not ſtrike againſt the feathers, when the arrow is in motion. But ſince they are not ſet perfectly ſtrait, but always a little aſlant; whilſt the arrow moves forward, the air ſtrikes the ſlant ſides of the feathers; by which force the feathers are turned round, and with the feathers the arrow or reed. So there is generated a motion about the axis of the arrow; which motion will be ſwifter as they ſtand more aſlant. This motion is like the motion of the 220.

FIG. 220. ſails and axle of a windmill, turned round by the wind. The head *B* is made of lead or iron, and will therefore go foremoſt in the air; and the feathered end *A* the hindmoſt, as being lighter. An arrow will fly about 60 yards in a ſecond.

Ex. XXXVII.

221. *AB* is a veſſel which keeps its liquor till filled to a certain height; and if filled higher, lets it all run out. *EFG* is a crooked pipe, or crane open at both ends. If water be poured into the veſſel, it will continue in it till it riſes above *F*, and aſcend to the ſame height in the pipe *EF*. But riſing above *F*, the preſſure at *E* will make it run out through the pipe *EFG*, till the ſurface of the fluid deſcends as low as *E*. This is ſometimes called *Tantalus's cup*. The funnel *EFG* may be put in the handle of this cup, which will look neater.

Ex. XXXVIII.

222. *BC*, *CG* are two *bones* of an animal, moveable about the joint *FK*, by help of the muſcle *KD*. The joints of animals are either ſpherical or circular, and the cavity they move in is accordingly either ſpherical or circular. And the center of motion is in the center of the ſphere or circle, as at *C*. Let *W* be a weight hanging at *B*, and draw *CP*, *CK* perpendicular to *BW*, *KD*. Then if the weight *W* be ſuſpended by the ſtrength of the muſcle *KD*, it will be as *CK* : *PC* :: *W* : tenſion of the muſcle *KD*.

The bone *BC* is moved about the joint *FK*, by the ſtrength of the muſcle *KD*. For when the muſcle is contracted, the point *K* is moved towards *D*; and the end *B* towards *E*, about the immoveable center of motion *C*. The ſtrength of the muſcles is ſurprizingly great.

Borelli (in his Book, de Motu Animalium, Part I. *Prop.* XXII.) computes the force of the muſcles to bend the arm at the elbow; and ſays, a ſtrong young fellow can ſuſtain at arms end a weight of 28 lb. taking in the weight of the arm. And he finds the length of *CB* to *CK* to be in a greater proportion than that of 20 to 1. Whence he infers the ſtrength of theſe muſcles to be ſo great, as to bear a ſtretch at leaſt of 560lb.

It is evident that all animal bodies are machines. For what are the bones but leavers, moved by a certain power placed in the muſcles, which act as ſo many ropes, pulling at the bones, and moving them about the joints? Every joint repreſenting

the

FIG. 222. the fulcrum or center of motion. What are all the veſſels but tubes which contain fluids of different ſorts, deſtined for the uſe or motion of the ſeveral parts of the machine? And which by opening or ſhutting certain valves, let out or retain their contents as occaſion requires; or convey them to diſtant places, by other tubes communicating therewith. And therefore all theſe motions of an animal body are ſubject to the general laws of mechanics.

Ex. XXXIX.

223. The motion of a *man*, walking, running, *&c.* will eaſily be accounted for. Let us firſt ſuppoſe a man ſitting in a chair; he cannot riſe from his ſeat, till by thruſting his head and body forward, and his feet and legs backward, the line of direction, or the perpendicular from the center of gravity, paſs through his feet, as the baſe. Likewiſe when we ſtand upon our feet, the line of direction muſt fall between our feet; otherwiſe we cannot ſtand, but muſt fall down towards the ſide the center of gravity lies on. And when a man ſtands firm upon his feet, his legs make an iſoceles triangle, the center of gravity lying between them. And then he is not ſupported by the ſtrength of the muſcles, but by the bones of the legs and thighs, which then ſtand in a right line with one another.

When a man *AC* endeavours to walk, he firſt extends his hindmoſt leg and foot *S* almoſt to a right line, and at the ſame time bends a little the knee *H* of his fore-leg. Thus his hind leg is lengthened, and his fore-leg ſhortened; by this means his body is moved forward, till the center of gravity *V* falls beyond the fore-foot *B*; and then being ready to fall, he preſently prevents it, by taking up the hind-foot, and by bending the joints of the hyp, knee, and ancle, and ſuddenly tranſlating it forward to *T*, beyond the center of gravity: and thus he gains a new ſtation. After the ſame manner by extending the foot and leg *HB*, and thruſting forward the center of gravity beyond the foot *S*, and then tranſlating the foot *B* forward, he gains a third ſtation. And thus is walking continued at pleaſure.

His two feet do not go in one right line, but in two lines parallel to one another. Therefore a man walking has a libratory motion from one ſide to the other; and it is not poſſible to walk in a right line.

Walking on plain ground is eaſy, pleaſant, and performed with little labour. But in going up hill is very laborious, by reaſon of the great flexure of the joints required to aſcend, and their

FIG. 223. their ſuffering more ſtreſs from the weight of the body in that poſition. Deſcending down hill is, for the ſame reaſon, more laborious than walking on plain ground, but not ſo bad as aſcending.

The walking of birds is not unlike that of men; only their weight is intirely ſupported by the ſtrength of the muſcles; ſince their joints are always bent. Alſo their feet go in two parallel lines.

A man in walking always ſets down one foot before the other be taken up: and therefore at every ſtep he has both feet upon the ground. But in running he never ſets one down 'till the other be up. So that at each ſtep he has but one foot upon the ground, and all the intermediate time none. A good footman will run 400 yards in a minute.

Ex. XL.

224. When a *beaſt* ſtands, the line of gravity muſt fall within the quadrilateral made by his 4 feet. And when he walks, he has always 3 feet on the ground and one up. Suppoſe he firſt takes up the hind foot *C*. Before he does this, by extending his leg backwards, he thruſts forward his body and the center of gravity; then taking up the foot *C* he moves it forward to *F*. Then he immediately takes up the fore foot *B* on the ſame ſide, and carries it to *H*; then he takes up the hind foot *D*, and tranſlates it forward; and then the fore-foot at *A*; then *F* again, and ſo on.

When he trots, he takes up two together, and ſets down two together, diagonally oppoſite.

When he gallops, he takes up his feet one by one, and ſets them down one by one; though ſome animals ſtrike with the two fore feet nearly at once, and the two hind feet near at once; and have not above two feet on the ground at once. A good horſe will run half a mile in a minute.

Animals with ſix or more feet, take up the hindmoſt firſt; then the next, and then the next in order, to the foremoſt, all on one ſide; and after that, all the feet on the other ſide in the ſame order, beginning at the laſt. If they were to take up the foremoſt firſt, the animals would move backward.

Ex. XLI.

225. *AD* is a *bird* flying in the air, by help of the wings *F*, *T*; and the tail *C*. The ſtructure of their wings are ſuch, that in

ſtriking downward, they expand to their greateſt breadth, and become almoſt two planes, being ſomething hollow on the under ſide. And theſe planes are not then horizontal, but inclined, ſo that the back part *K* is higher than the fore part *DFG*. But in moving the wings upward, to fetch a new ſtroke, they go with the edge *DFG* foremoſt, and the wings contract and become hollow. Their bodies are ſpecifically lighter than men or beaſts. Their bones and feathers are extremely porous, hollow, and light. The muſcles, by which their wings are moved downwards, are exceeding large, being not leſs than a ſixth part of the weight of the whole body. When a bird is upon the ground and intends to fly, he takes a large leap; and ſtretching his wings right from his body, he ſtrikes them downwards with great force, by which they are put into an oblique poſition; and the reſiſtance of the air acting ſtrongly againſt them by the ſtroke, impels them, and the bird, in a direction perpendicular to their planes: which is in an oblique direction, or partly upwards, and partly horizontally forward: the part of the force tending upwards is deſtroyed by the weight of the bird; the horizontal force ſerves to carry him forward. The ſtroke being over, he moves his wings upwards, which being contracted, and turning their edges upward, they cut through the air without any reſiſtance, and being ſufficiently elevated, he takes a ſecond ſtroke downwards, and the impulſe of the air moves him forwards, as before. And ſo from one ſtroke to another, which are only like ſo many leaps taken in the air. When he has a mind to turn to the right or left, he ſtrikes ſtrongly with the oppoſite wing, which impels him to the contrary ſide. The tail acts like the rudder of a ſhip, except only that it moves them upwards or downwards inſtead of ſideways; becauſe its plane is horizontal. If a bird wants to riſe, he puts his tail in the poſition *LH*; or if he would fall, into the poſition *LI*. Whilſt it is in the horizontal poſition *LC*, it keeps him ſteady. A bird can by ſpreading his wings continue to move horizontally for ſome time, without ſtriking. For having acquired a ſufficient velocity, by keeping his wings parallel to the horizon, they meet with no reſiſtance; and when he begins to fall by his weight, he can eaſily ſteer himſelf upward by his tail; till his motion be almoſt ſpent, and then he muſt renew it by two or three more ſtrokes of his wings. When he alights, he expands his wings and tail full againſt the air, that they may meet with all the reſiſtance poſſible. The center of gravity of a bird is ſomething behind the wings; to remedy which, FIG. 225. which, they thruſt out their head and neck, in flying; which carries the center of gravity more forward.

It is impoſſible that ever men can fly by the ſtrength of their arms. For their pectoral muſcles are vaſtly too weak to ſupport ſuch a weight. For in a man they are not the 60th part of the reſt of the muſcles of the body: but in a bird they are more than all the others put together.

Some birds will fly 1000 yards in a minute.

Ex. XLII.

226. *AB* is a *fiſh* ſwimming; which he does by help of his fins and tail. A fiſh is nearly of the ſame ſpecific gravity as water; and moſt fiſh have a bladder *L*, which they can expand or contract, and ſo make themſelves lighter or heavier than water, in order to riſe or fall in it. The muſcular force by which the tail is moved is very great. The direct motion of a fiſh is by means of his tail *BCD*, moving from one ſide to the other, with a vibrating motion, which he performs thus. Suppoſe his tail in the poſition *FG*, being about to move it ſucceſſively to *H*, *I* and *K*; he turns the end *G* oblique to the water, which being moved ſwiftly through it in that poſition, the reſiſtance of the water acts obliquely againſt his tail, and moves him partly forward and partly laterally. The lateral motion is corrected the next ſtroke the contrary way; but the progreſſive motion is continued always forward. When his tail is arrived at *K*, he turns its obliquity the contrary way, that in moving back to *G*, it may ſtrike the water in the ſame manner as before. And thus he makes one ſtroke after another, and moves forward thereby as far as he pleaſes. The oblique poſition of his tail is moſtly owing to the elaſticity of his tail, which by bending, is put into that form by the reſiſtance of the water. They can exert a very great force with their tail; and which is neceſſary, to overcome the reſiſtance which their bodies meet with in the water. By help of the tail they alſo turn to one ſide, by ſtriking ſtrongly with it on that ſide, and keeping it bent, which then acts like the rudder of a ſhip. The fins of a fiſh ſerve to keep him upright, eſpecially the belly-fins *E*, which act like two feet; without them he would ſwim with his belly up; for his center of gravity lies near his back. His fins alſo help him to aſcend or deſcend, by expanding or contracting them, as he can with pleaſure, and ſo putting them in a proper poſition. His tail will alſo help him to riſe and fall, by inclining it obliquely, and turning it a little from an erect poſition,

tion, to one ſide. Fiſh can ſwim but ſlow, yet ſome of them will ſwim 70 or 80 yards in a minute; but they ſoon tire. FIG. 226.

Brutes can ſwim naturally, for they are ſpecifically lighter than water; and require to have but a ſmall part of their head out, for breathing. Alſo they naturally uſe their legs in ſwimming, after the ſame manner as they do when walking.

Birds ſwim very eaſily, being much lighter than water; and readily move themſelves along with their web feet.

Men cannot ſwim naturally, though they are ſpecifically lighter than water. For their heads are very large, and require to be almoſt all out of the water for breathing. And their way of ſtriking has no relation to that of walking. Men attain the art of ſwimming by practice and induſtry. And this art conſiſts in ſtriking alternately with the hands and feet in the water, which like oars will row him forward. When he ſtrikes with his hands, he neither keeps the palms parallel nor perpendicular to the horizon, but inclined. And his hands ſtriking the water obliquely, the reſiſtance of the water moves him partly upward, and partly forward. Whilſt his hands are ſtriking, he gradually draws up his feet; and when the ſtroke of his arms is over, he ſtrikes with his feet, by extending his legs, and thruſting the ſoles of his feet full againſt the water. And while he ſtrikes with his legs, he brings about his arms for a new ſtroke, and ſo on alternately. He muſt keep his body a little oblique, that he may more eaſily erect his head, and keep his mouth above water.

After the ſame manner may the motions, velocities, powers, and properties of any machine be explained and accounted for, by mechanical principles. I ſhall proceed to lay down a ſhort deſcription of ſeveral other machines, without being ſo particular in the calculation of their powers and forces. The mechaniſm of which being underſtood, will aſſiſt the invention of the practical mechanic, in contriving a machine for any uſe.

Ex. XLIII.

AB a machine to raiſe a weight, and ſtay it in any poſition. 227. *CD* a roller turned by the handle *E*. To the roller is fixed the racket wheel *F*. *GH* is a catch made of metal, moveable about *H*, and forked at the end *G*, where it falls into the teeth of the wheel *F*. As the roller is turned, the weight is raiſed by the rope *IKL*. And the catch *G*, ſlides freely over the teeth of the wheel, till the machine is ſtopped, and then the catch *G* falling

in

FIG. 227. in between the teeth, keeps the wheel fixed there, that it cannot turn back again with the weight.

Ex. XLIV.

228. *KI* is a machine to raiſe or depreſs the leaver *GH*, moveable about an axis at *H*, and to keep it at any poſition: *bf* a ſtring fixed at *f* and *a*, and going in the order *abcdef*, through the holes *b*, *c*, *d*, *e*, of the runner *IK*, which being put up or down, raiſes or lowers the beam *GH*, at pleaſure. And more or fewer holes may be made in the piece *IK*, as occaſion requires.

Ex. XLV.

229. *CD* another machine to ſtay a weight in any poſition. This is only a cylinder of wood, upon which is cut a channel for the rope to go in. If the weight *B* be lifted up, and *A* pulled down, then *B* will remain in any given poſition, by the friction of the cylinder and rope. And there may be taken as many turns of the rope about the cylinder, as there is occaſion for.

Ex. XLVI.

230. *C* is a clock weight carrying the two wheels *A* and *B*. *D* the counterpoiſe. *F* a pulley. *ADBFA* an endleſs cord. When the weight is down, draw the cord *G*, till the weight *C* riſe to the top; then the catch *e* keeps the wheel *A* from turning backwards. This may be ſerviceable for other uſes, beſides moving a clock.

Ex. XLVII.

231. *ADB* a machine for reckoning the number of ſtrokes or vibrations made. *DH* is a wheel moving about a fixed axis, upon the neck of which axis goes a braſs ſpring *L*, to keep the wheel from ſhaking. *AB* a piece of wood or metal, cut away between *I* and *K* to receive the wheel. The plane of the piece *AIKB* is perpendicular to the plane of the wheel. *FG* are two ſtaples, to guide the motion of the piece *AB* back and forward. When the piece *AB* is moved from *A* towards *B*, the edge at *I* catches the tooth *C*, and ſliding along the edge, moves the wheel about in direction *CD*; this brings the tooth *E* to the edge *K*. And when the piece *AB* is moved back from *B* to *A*, the edge at *K* ſliding down the tooth *E*, moves the

wheel from *E* towards *H*; which brings another tooth before the edge *I*; ſo that at every motion of *AB* back and forward, the wheel is moved the breadth of one tooth. And if the teeth be numbered, the index *M* will ſhew when the wheel has made one revolution. FIG. 231.

Ex. XLVIII.

ABED a machine moving one circle within another, concentrical to it. *ABC* repreſents a flat ring of braſs; and *abc* a ſmaller concentric ring lodged in a circular grove, turned within the larger; and kept in the grove by three ſmall plates of braſs *A, B, C*, fixed to the outward ring, and reaching over the edge of the inner one. Upon the inner ring is fixed a concentric arch of a wheel *de*, having teeth in it, which are driven round by the threads of an endleſs ſcrew *DF*, turning in a collar at *E*, and upon a point at *F*, both fixed to the outward ring. By this mechaniſm, any point of the circle *abc*, may be ſet to a given point of the circle *ABF*, by turning the ſcrew *DEF*. 232.

Ex. XLIX.

ABE is a *crane* for hoiſting goods up. *AF* is a double wheel, within which a man *A* walks, and by his weight raiſes the weight *W*, by help of the rope *FBEW*, which goes round the axis of the wheel at *F*. At *D* are two pullies, one vertical, and one horizontal; the vertical pulley facilitates the motion of the rope in hoiſting the weight; and the horizontal one ſerves for the rope to run on, when the crane *CDE* and weight *W* are drawn aſide, by the rope *GH*, in order to be lowered. *CDE* moves about the axis *BC*. At *E* is another pulley, for the rope to run on. If the rope *FBEI* go about a pulley at *I*, and be fixed with its end at *E*, the crane will lift twice the weight. 233.

Ex. L.

AB a *ſailing chariot*. This is driven by the wind, by help of the ſails *C, D*. *R* is the rudder. The wheels muſt be ſet at a greater diſtance, or the axle tree made longer, than in common chariots, to prevent overturning. Sailing chariots are proper for large planes and champagne countries; and are ſaid to be frequent in *China*. 234.

Ex. LI.

235. *BE* is a *ſmoak jack.* *AB* is a horizontal wheel, wherein the wings or ſails are inclined to the horizon. The ſmoak or rarified air moving up the chimney at *B*, ſtrikes theſe ſails, which being oblique, are therefore moved about the axis of the wheel, together with the pinion *C*, of 6 leaves. *C* carries the toothed wheel *D* of 120 teeth, all theſe are of iron. *E* a wooden wheel 4 or 5 inches diameter; this carries the chain or rope *F*, which turns the ſpit. The wheel *AB* muſt be placed in the ſtrait part of the chimney, where the motion of the air is ſwifteſt; and that the greateſt part of it may ſtrike upon the ſails. The force of this machine is ſo much greater, as the fire is greater. The ſails *B* are of tin, 6 or 8 in number, placed at an angle of $54\frac{1}{2}$ degrees.

Ex. LII.

236. An engine to make a *hammer ſtrike.* *W* the water wheel, 7 or 8 yards diameter. *D*, *D*, the floats. *BC* the axle, 3 or 4 feet diameter. *H* the hammer 3 or 4 hundred weight, moveable about the axis *OP*. *I*, *K* four coggs in the axis, lifting up the hammer as the axis goes round, that it may fall on the anvil *A*. *FG* a beam of wood acting as a ſpring, to give the greater force to the hammer. *MN* the courſe of the water down an inclined plane. *M* the place where it iſſues out. *LM* the perpendicular height of the water, 3 yards. All the machine except the water wheel is within the houſe.

237. A hammer may alſo be made to ſtrike thus; *A* is the hammer moveable about the point *C*. *G* the axle of a water wheel, in which axis are the pins *F*, *E*, *&c.* As the wheel and axle goes about from *F* towards *E*, the pins *F*, *E* thruſt down the end *B*, and raiſe the end *A* of the hammer. And when the end *B* goes off the pin, the hammer falls upon the anvil *D*.

Ex. LIII.

238. *I I* a crooked axis or elbow for the *ſuckers of pumps.* *IK* the peſtle or chain of the ſucker. Upon the axis is the lantern *EF*, which is turned by a great wheel, carried either by water, or men or horſes. The peſtles *IK* riſe and fall alternately, as the lantern *EF* goes about; and each gives one ſtroke of the pump for one turn of the lantern. Place pullies or rolls at *a*, *b*, *c*, *d*, for the chain *IK* to work againſt, when it goes out of its perpen-

dicular poſition; by the obliquity of the motion of the cranks *I*, *I*. FIG. 238.

Ex. LIV.

ABCD a particular combination of *pullies*. *T*, *T*, *T* are poſts to which the tackles are fixed. *S*, *S*, *S* are ſtays to keep them erect. If the power at *A* be 1, that at *B* is 3; at *C*, 9; and at *D* 27, where the weight is placed. 239.

Ex. LV.

A, *B* are two *bellows* going by water, and blowing alternately, but neither of them with a continual blaſt; *W* the water wheel. *DE* the direction of the water. *FG* the axis of the wheel; *a*, *a*, &c. 4 cogs of wood in the axis, forcing down the end of the bellows *A*. *bb*, &c. 4 cogs forcing down the end of the bellows *B*. *LM*, *NI* two rods of iron faſtened to the bellows and to the leaver *MN*, and moveable about the pins *M*, *N*. *SP* a piece of timber moveable about *S* and *P*. *OP* a beam ſerving for a ſpring, lying over the piece of timber *QR*. As the wheel and axle turns round, a cog *b* forces down the end of the bellows *B*, and makes it blow; this pulls down the end *N* and raiſes the end *M* of the leaver *MN*, which raiſes the bellows *A*. And when the cog *b* goes off, the bellows *B* ceaſe blowing; and a cog *a* forces down the bellows *A*, and makes it blow; and at the ſame time raiſes the bellows *B*. And thus the cogs *a*, *b* alternately force down the bellows *A*, *B*, and make them blow in their turns. *H* is the hearth or fire. 240.

A pair of bellows may be moved by water thus; *A* is a water wheel, carried by the water at *W*. *CD* a rod of iron going on the crooked axle-tree of the wheel; *DF* a leaver moveable about *E*. *FG* a chain going to the bellows *B*. *I* a weight. As the wheel goes about, the ends *D* and *F* of the leaver *DF*, riſe and fall; which motion raiſes the bellows, and the weight *I* carries them down again. 241.

Ex. LVI.

AB is a wheel with teeth, and a *roller* to draw up any weight. *H*, *H*, *H* the handles, which may be wrought by two or three men. 242.

But the eaſieſt and ſimpleſt rollers for common uſe are ſuch, as *C* and *D*. In theſe, as 30lb. is the weight to be raiſed, ſo muſt 243. 244.

FIG. muſt the radius of the axle be to the length of the handle, for a man to work it.

30. If a given weight P raiſes another weight W, on ſuch a machine as fig. 30. it will generate the greateſt motion poſſible in a given time; when the diameters AB, EF, and weight W are of ſuch quantities, that $W = \frac{\frac{2}{5}P \times AB}{EF}$, or when $EF = \frac{\frac{2}{5}AB \times P}{W}$. For then the motion will be greater, than if any one (H; AB, or EF) be altered, the reſt remaining the ſame.

36. And in ſuch a machine as fig. 36; the greateſt motion will be generated in W; if you make, as velocity of W : velocity of P :: as $\frac{2}{5}P$: to W.

Ex. LVII.

245. An engine to *drive piles*. A the rammer, drawn up by the rope BCD going over the pulley C. DN, DN ſeveral ſmall ropes for ſeveral men to pull at. M the pile. EF a brace and ladder to go up. The rammer A is bound at bottom with iron, leaſt it ſplit. And has two tenants on the backſide to keep it in the groves, made in the upright puncheons G, H, by which its motion is directed. The rammer is raiſed to the top, by men pulling at the ropes DN, and then letting go, all at once, it falls upon the head of the pile M.

Old piles are drawn out, by ſtriking gently upon their heads, whilſt they are pulled by a ſtrong rope ſtretched.

Ex. LVIII.

246. ACL a pair of *ſmith's bellows*. AL, BL, CL are three boards, the middle board BL divides the internal ſpace into two parts. In the middle board is a valve S opening into the upper part; and in the loweſt board is another valve T opening into the under part, The pipe P communicates only with the upper cavity. DE a leaver moveable about the axis GH. At I a weight is laid upon the upper board to make it fall. The bellows is fixed in the frame MK, by two iron pins, which are faſt in the middle board. And the pipe P lies upon the hearth. When the end E is pulled down by the rope EF, the end D is raiſed, and the rope or chain DR raiſes the lower board CL; this ſhuts the valve T and opens S, and the air is forced into the upper cavity, which raiſes the upper board, and blows through the pipe P. And when E is raiſed, the boards A and C deſcend, and the valve S ſhuts, and T opens. And the weight I forces the

FIG. 246. the air ſtill out of the pipe, whilſt more air enters in at the valve T; which, when C aſcends, is forced again through the valve S as before. And thus the bellows have a continual blaſt.

Ex. LIX.

247. An engine to raiſe water. $LMOI$ a great horizontal wheel. ABP the axis, P the pevot or ſpindle it turns upon. OQI the waves of the great wheel. QR a ſmall wheel perpendicular to the horizon, and placed under the edge of the great wheel: this wheel is moveable about the center C, in the end of the leaver EFC, which is moveable about the center D; EF the arch of a circle, whoſe plane is perpendicular to the horizon, and in the plane of the wheel QR. EG the chain of a pump.

Whilſt the great wheel is turned by the leaver NA, from O towards I, the wave Q preſſes down the wheel QR, and raiſes the end E, which draws up the water in the pump G. But when the deepeſt part of the wave is paſt the wheel QR, the wheel then riſes up into the hollow S, and then the chain EG deſcends, till the next wave raiſes it again. And thus every wave makes a ſtroke of the pump.

The wheel QR is placed there only to avoid friction, and ſo that a perpendicular to its plane may paſs through AB. If the number of waves be odd, and another pump wheel and leaver be placed diametrically oppoſite, on the other ſide of the great wheel, then theſe acting by turns will keep the motion uniform, and the power at N will always act equally.

Ex. LX.

248. BFG a *capſtain*, to draw great weights. BC the axis, which is driven about by men acting at A, A, by help of the leavers AB, AB. Here muſt only be 3 or 4 ſpires of the rope DCE folded about the axis BC; for the axis could not hold ſo much rope as there is ſometimes occaſion for. And to hinder the rope from ſlipping back, a man conſtantly pulls at E to keep it light. And the axis is made conical, or rather angular at the bottom C, to keep the rope from going any lower, whilſt the capſtain goes about.

Ex. LXI.

249. AL is a *jack* to lift great weights. E is a pinion upon the axis FG, GC a toothed wheel, and D a pinion upon the ſame

axis,

FIG. 249. axis, working in the teeth of the rack *AB*. The whole is incloſed in a ſtrong caſe *KL*, all of metal. The handle *GHI* goes on the axis *FG* on the backſide of the caſe.

When a weight is to be lifted, the forkt end *A* is put under the weight; then turning the handle *HI*, the pinion *E* moves the wheel *GC*, with the pinion *D*; and *D* raiſes the rack *AB*, with the weight.

Ex. LXII.

250. An engine to raiſe and let fall two weights with contrary motions ſucceſſively, whilſt the moving power acts always one way. *GH* a great horizontal wheel. *N*, *M* two lanterns, ſo placed on the axis *AB*, that the great wheel can only work one of them at once. When the cog wheel *GH* is turned by the leaver *LI*, it turns the lantern *M*, and raiſes the bucket *E*, whilſt *F* deſcends. Then *E* being raiſed, move forward the axis *AB*, that the lantern *M* may leave the wheel, and *N* come to it. Then the great wheel moving the ſame way as before, will now work upon *N*, and turn the axis the contrary way, and raiſe the bucket *F* whilſt *E* deſcends. Which done, move the axis back towards *A*, and you will again riſe the bucket *E*, and ſo on.

This may alſo be performed by placing the lanterns *M*, *N*, ſo that the great wheel may work them both at once; but making them moveable about the axis *AB*, then there muſt be a pin to faſten either of them to the axis; ſo that firſt one lantern, and then the other, being thus fixt to the axis, whilſt the other is looſe, the buckets *E*, *F* will aſcend and deſcend alternately.

Ex. LXIII.

251. A *mill for iron work*. *AB* the ſlitting mill, *CD* the plate mill. *SP* the clipping mill. *E*, *F* are two great water wheels. After the water is paſt the wheel *E*, moving in direction *QW*, it comes about to the wheel *F*, in direction *XY*. The water wheel *E*, with the lantern *G* on the ſame axis, carries the ſpur wheels *M* and *H*, with the cylinders *B* and *D*. And the wheel *F* with the lantern *I*, carries the wheels *N* and *K*, with the cylinders *A* and *C*. The cylinders *A* and *B*, as alſo *C* and *D* run contrary ways about. And the cylinders *A* and *B* are cut into teeth, for ſlitting iron bars. *C*, *D* are 8 inches diameter; *A* and *B* about 12. And theſe cylinders may be taken out and others put in, and may be brought nearer to, or farther from one

FIG. 251. one another, by help of ſcrews, which ſcrew up the ſockets where the axles run. The axles of *N*, *I*, *K* lie all in one horizontal plane. And ſo does *M*, *G*, *H*. But the cylinders *A*, *B*, and alſo *C*, *D*, lie one above another.

For making the plates; if a bar of iron be heated and made thin at the end, and that end put in between the cylinders *C*, *D*, whilſt the mill is going; the motion of the cylinders draws it through, on the other ſide, into a thin plate. Likewiſe a bar of iron, being heated and thinned at the end, and put in between the toothed cylinders *A*, *B*; it is drawn through on the other ſide, and ſlit into ſeveral pieces, or ſtrings. And then if there be occaſion, any of theſe ſtrings may be put through the plate mill with the ſame heat, and made into plates.

OPQ is the ſheers for clipping bars of cold iron into lengths. *V* a cog in the axis of the water wheel. *OP* one ſide of the ſheers made of ſteel, and moveable about *P*. The plane *LPR* is perpendicular to the horizon. When the mill goes about, the cog *V* raiſes the ſide *OP*, which as it riſes clips the bar *TQ* into two, by the edges *SP*, *RP*. All the engine, except the water wheels *E*, *F*, is within the houſe.

Ex. LXIV.

AFC a *windmill* to frighten birds from corn or fruit. This is made of wood. The ſails *F*, *F* a foot long, and their planes inclined to the axis *BC*, 45 or 50 degrees. The piece *B* goes upon the end of the axis *BC*, and is pinned faſt on, and the ſails and axis turn round together; and the axis goes through the board *AD*, and is kept from flying out of the hole, by the piece *B* pinned faſt. The whole machine is moveable about the perpendicular ſtaff *AG*, by which means the wind turns the mill about the axis *AG*, till the plane *AD* lies directly from the wind; and then the ſails face it. At *S* is a ſpring to knack as it goes about; and the like on the other ſide. 252.

Ex. LXV.

An *anemoſcope*, to ſhow the turnings of the wind. *CD* is a weather cock of thin metal, fixt faſt to the long perpendicular axis *DF*, which turns with the leaſt wind upon the foot *F*, and goes through the top of the houſe *RS*. To this axis is fixt the pinion *A*, which works in the crown wheel *B*, of an equal number of teeth. The crown wheel is fixt on the axis *PI*, on the end of which the index *NS* is fixt. The axis *PI* goes 253.

FIG. 253. through the wall *LM*, againſt the wall is placed the circle *NESW*, with the points of the compaſs round it. Then if the vane *CD* be ſet to the north, and at the ſame time the index *SN* fixt on the axis *PI*, to point at *S*. Then however the wind varies, it will turn the vane *CD*, and pinion *A*; and *A* turns the wheel *B* with the index *N*; ſo that the index will always be directed to the oppoſite point of the compaſs to the vane *DC*; or to the ſame as the wind is in.

Ex. LXVI.

254. *DEF* is a *rag-pump*, or *chain-pump*. *EF* the barrel, *CD* the roller. *GH* an endleſs chain, to which are fixt ſeveral leather buckets *I*, *I*, hollow on the upper ſide that aſcends. *AB* the handle. The uſe of this is to cleanſe foul waters from dirt and rubbiſh. The roller is ribbed to hinder the chain from ſlipping, in working. When the roller is turned, it draws up the chain through the pump, with whatever is in the water, and diſcharges it at the top. Inſtead of the roller *CD*, a wheel like a trundle may be uſed, called the rag wheel.

Ex. LXVII.

255. A *dyer's and fuller's mill.* *A* the great wheel carried about by horſes. This turns the trundles *B*, *C*, *D*; together with *E*, *F*, *G*. Then *E* turns the cog wheel *H*, with the axis *IK*, and the croſs pieces *L*, *L*; 1, 1, *&c.* are pullies or rollers. *MN*, *MN* wooden beaters, turning upon an axis paſſing through *N*, *N*. Whilſt the axis *IK* turns about, the end *b* ſlides along the pulley 1; and falling off, the part *M* ſtrikes againſt the cloth in the trough at *O*, *O*. The lantern *F* carries the cog-wheel *P*, and the cranks *Q*, *Q*; which work the pumps *T*, *T*, by help of the leavers *RS*, moveable about *a*. The trundle *G* carries the cog-wheels *V* and *W*, and *W* carries the trundle *X*, with the piſton *Y* that grinds the indigo in the veſſel *cd*; from whence it flows to the veſſel *Z*. The ends *m*, *m*, *&c.* of all the axles, run in pieces of timber going croſs the mill, and faſtened to one another and to the walls of the houſe.

Ex. LXVIII.

256. A machine to empty ſtanding waters. This is no more than a large pipe or ſyphon *ABC*, being extremely cloſe and tight that no air can get in.

If the pool of water *DE* is to be emptied over the hill *DHG*; let the pipe be placed with its mouth *A* within the water *DE*, and the mouth *C* within the water *FG*, if the pipe be very large. Then ſtop up *A* and *C*, and fill the pipe with water by the cock *B* at the top. Then ſtopping the cock *B* very cloſe, open *A* and *C*; and the water will flow through the pipe from *DE* into *FG*, which may run over at *F*, at a ſmall height above *C*, and go away. FIG. 256.

Note, the end *C* muſt always be lower than *A*, and the height of the top *B* above *DE* muſt not exceed 11 yards; for if it do, the water will not flow. If the pipe be very ſtrait, the end *C* need not be immerſed in the water; but if large it muſt; or elſe the air will inſinuate itſelf into the pipe at *C*, and hinder the flux of the water.

Ex. LXIX.

EFGH is a coal ginn. *E* the cog-wheel 11 feet diameter, and 72 cogs: this carries the trundle *F*, near 2 feet diameter, and 12 rounds, together with the roll *G*, 4 feet diameter. *AH* is the ſtart 20 feet long. The axis *AB* runs upon the kevy-ſtock *C*. There are two croſs trees *IK*, at the top, through which the axis *AB* goes. Theſe croſs trees are ſupported by four poſts *KL* at the four corners. When the coals are to be drawn up out of the pit, two horſes are yoaked at *H*, and go round in the path *OQD*, and draw the wheel about. And whilſt the loaded corf *N*, is drawn up to the top of the ſhaft *M*, by the rope going round the roll; the empty one at the other end of the rope, is deſcending to the bottom. And the loaded corf *N* being taken off, and an empty one put on, the horſes are turned, and made to draw the contrary way about, till the other corf comes to the top loaded: and ſo as one corf aſcends, the other deſcends alternately. A corf of coals weighs about 5 hundred weight, and contains about $4\frac{1}{2}$ buſhels. A pit is 40 or 50 fathom deep. And 50 fathom of the rope weighs about 3 hundred weight. 257.

Ex. LXX.

A *worm jack* for turning a ſpit. *ABC* the barrel round which the cord *QR* is wound. *KL* the main wheel of 60 teeth. *N* the worm wheel of about 30 teeth cut obliquely. *LM* the pinion of 15 or 16. *O* the worm or endleſs ſcrew, on which are two threads or worms going round, and making an angle with the 258.

axis

FIG. 258. axis of 60 or 75 degrees. *X* the ſtud, *Z* the loop of the worm ſpindle. *P* a heavy wheel or fly to make the motion uniform. *DG* the ſtruck wheel fixt to the axis *FD*. *S*, *S* ſeveral holes in the frame, to nail it to a board, which is to be nailed againſt a wall; the end *D* going through it. *HI* the handle, going upon the axis *ET*, to wind up the weight when down. *R* are fixt pullies, *V* moveable pullies with the weight. The axis *ET* is fixt in the barrel *AC*, and this axis being hollow, both it and the barrel turn round upon the axis *FD*, which is fixt to the wheel *KL*, turning in the order *BTA*; but cannot turn the contrary way, by reaſon of a catch nailed to the end *AB*, which lays hold of the croſs bars in the wheel *LK*.

The weight, by means of the cord *QR*, carries about the barrel *AB*, which by means of the catch, carries the wheel *KL*, which carries the nut *LM* and wheel *N*, which carries the worm *O* with the fly *P*. Alſo the wheel *LM* carries the axis *FD* with the wheel *DG*, which carries the cord or chain that goes about the ſpit head (a wheel like *DG*) which turns the ſpit. The more pullies at *R* and *V*, the longer the jack will go; but then the weight muſt be greater.

The catch lies between the end *AB* of the barrel and the wheel *KL*, and is thus deſcribed: *ff* the barrel, *n* the main ſpindle: *dr* a tumbler moving eaſy on the center pin *a*, faſtened to an iron plate, nailed to the barrel: *b* a collar of iron, turning a little ſtiff on the ſpindle; from this proceeds the tongue *bc*, paſſing through the hole *c* in the tumbler: *r* the catch of the tumbler. Now whilſt the barrel with the catch is turned about, in the order *efg*, upon the axis *n*; the collar is drawn about by the tongue *bc*, which tongue acting backwards, turns the tumbler about the center *a*, and depreſſes the catch *r*. But the barrel being turned the contrary way, the tongue then acts towards *d*; this depreſſes the end *d*, and raiſes the catch *r*, which then takes the croſs bars of the main wheel, and ſtops the barrel. This catch would alſo ſerve for a clock, and is better than a ſpring catch, becauſe it makes no noiſe in winding up.

Note, the jack need not be placed ſo, that the axis *FD* be parallel to the ſpit; but any way it can conveniently. For it is no matter whether the chain croſſes or not.

Ex. LXXI.

259. *DAF* the *hydroſtatical bellows*. *AB*, *EF*, two flat boards of oak; the ſides *AE*, *BF*, of leather, joined very cloſe to the top

and bottom, with ſtrong nails. *CD* a pipe ſcrewed into the piece of braſs, in the top at *C*. FIG. 259.

If a man blows in at the pipe *DC*, he will raiſe a great weight laid upon the board *AB*. Or if he ſtands upon the board *AB*, he may eaſily blow himſelf up, by bl wing ſtrongly into the pipe *DC*. If water be poured in at *D*, till the bellows and pipe be full, the preſſure upon *AB* within, will lift as much weight upon the top *AB*, as is equal to a cylinder of water, whoſe baſe is *AB* and height *CD*.

Ex. LXXII.

ADM is a *water-mill* for grinding corn. *A* the *water-wheel* 16 feet diameter, *BC* its axis. *D* the *cog-wheel*, with 48 cogs. *O* a *trundle*, with 9 rounds; *LI* its axis. *M*, *N* the *ſtones* 6 or 7 feet diameter. The lower ſtone *N* is the *lyer*, being fixt immoveable upon beams of wood; and the upper ſtone is the *runner*, and is faſtened to the *ſpindle LI*, by a piece of iron called the *rind*, fixt in the lower ſide of the ſtone, to go ſquare upon the ſpindle; between which and the ſtone, there is room left for the corn to fall through upon the lower ſtone. The ſpindle goes through the lower ſtone, and is made ſo tight with a wooden *buſh*, as to turn round in it eaſily. The upper ſtone, with the ſpindle *LI*, is ſupported on the end *I*, upon a horizontal beam of wood *FE*, called the *bridge*; the end *F* being fixt, and the end *E* lying upon the beam *HG*, fixt at *G*, called the *brayer* or *bearer*. The end *H* is ſupported by the *lifting-tree HK*, by help of a wedge at *K*. By this means, the upper ſtone may be raiſed or lowered. For if *KH* be raiſed with the leaver *Kk*, the end *E*, the axis *LI*, with the ſtone *M*, and the piece *GH* are all raiſed, and may be fixed there by the wedge *K*. Thus the ſtones may be ſet as near or far off as you will. The lower ſtone is broader than the upper ſtone, and is feathered, or cut into ſmall channels, to convey the flour out; and is encloſed with boards all around as *ab*, cloſe to the lower ſtone, and above the edge of the under one, to keep the meal in. And through one ſide of the boards is a hole called the *mill-eye*, through which the meal runs out into a *trough*. 260.

The ſurfaces of the mill-ſtones are not flat, but conical; the upper one an inch hollow, the under one ſwells up $\frac{3}{4}$ of an inch: ſo the two ſtones are wider about the middle, and come nearer and nearer towards the outſide; which gives room for the corn to go in, as far as $\frac{2}{3}$ of the radius, where it begins to be ground. The upper ſtone has a dancing motion up and down, FIG. 260. down, by the ſpringing of the bridge, which helps to grind the corn. The flour, as ſoon as made, is thrown to the outſide, by the circulation of the ſtone and the air, and driven out at the mill-eye. The quantity of flour ground, is nearly as the velocity and weight of the ſtone. The ſtone ought not to go round above once in a ſecond, for bread corn.

The corn is put into the *hopper S*, which falling down into the *ſhoe TV*, runs into the hole at the top of the ſtone *M*. The axis *LM* is made with 6 or 8 angles; which, as it turns about, ſhakes the end *V* of the ſhoe, and keeps the corn always running down. The axis *LM* may be taken off. *PQ* is the direction of the water, which acting againſt the floats *R*, carries about the water-wheel *A*, and cog-wheel *D*, which cog-wheel carries the lantern or trundle *O*, and the upper ſtone *M* that grinds the corn.

Sometimes one water-wheel *A*, carries 2 pair of ſtones, and then 2 cog-wheels as *D*, are put into the axis *BC*, which carry two trundles with the ſtones. Otherwiſe the cog-wheel *D* carries a trundle *O*, and ſpur-wheel; which ſpur-wheel carries 2 lanterns with the ſtones, one lantern on each ſide the wheel. Or ſometimes, the ſame cog-wheel *D* carries another lantern and cog-wheel, whoſe axis is parallel to the horizon; and this cog-wheel carries another lantern with the ſtones. And the trundle is ſuch, as to make the blue ſtones, or thoſe that grind wheat flour, go near twice as ſwift as the grey ſtones do. In theſe caſes, when one pair of ſtones is to ſtand ſtill, there is either a looſe rung to be taken out of its lantern, or elſe the bridge *EF* is ſhifted towards *H*, till the lantern *O* be clear of *D*.

The diameter of the water-wheel *A* muſt not be too large; for then it will move too ſlow; nor too little, for then it will want power. When a mill is in perfection, the velocity of the floats, wings, or hands *R*, upon the water-wheel muſt be $\frac{1}{3}$ the velocity of the ſtream.

The higher fall the water has, the leſs of it will ſerve to carry the mill. In an underſhot mill, where the water comes underneath the wheel, it is brought by a narrow channel called the *mill race*. The water is kept up in the mill dam, and let out by the *penſtock*, when the mill is to go: and the penſtock is raiſed or let down by help of a leaver. The penſtock being raiſed opens a paſſage to the water, 10 or 12 inches wide, through which it flows to the wheel. And when the mill is to ſtop, the penſtock is let down, and the orifice ſtopt.

When the water comes underneath the wheel, it is called an *underſhot mill*. But if it comes over the wheel (as in fig. 282.)

it is called an *overfal* or *overſhot mill*. This requires leſs water FIG.
than an underſhot mill; but there is not convenience in all 260.
places to make them. The water is brought to the wheel of
an overfal mill, by a trough; which is turned aſide, to throw
the water off the wheel, when the mill does not go.

A *breaſt mill* is that where the water is delivered into boxes, at about the height of the axis of the wheel, and moves the wheel by its weight. This requires more water than either of the other ſorts.

A good overfal mill will grind 2 ½ or 3 buſhels of corn in an hour; and in that time requires 100 hogſheads of water, having 10 or 12 feet fall.

Ex. LXXIII.

AB is a trap to catch vermin, made of boards. *GH* a piece 261.
of wood ſuſpended over the bar *IL*, by the leaver *DE*, moveable about *D*; and the thread *FE*, tied to the ſtart *CK*. *lmB* a piece of flat wood, moveable about *lm*, and lying on the bottom, whoſe end *B* comes through a hole in the ſide, in which is a catch to take hold of the end *K* of the ſtart, when the trap is ſet. When the vermin go into the trap, they tread upon the board *lmB*, on which a bait is laid, which puts down the end *B*, and the ſtart *CK* flies up. This gives liberty to the rod *DE* to riſe up; then the piece of wood *GH* falls down, and knocks them on the head. If two pieces of board were nailed on the ends *G*, *H*; to reach below the piece of wood *GH*, the trap would take the vermin alive.

ABE is another *trap*, the end *B* is wire; and the end *A* 262.
ſlides up and down in two groves in the ſides. When the trap is ſet, the end *A* is ſuſpended by the thread *CD*, tied to the rod *DI*, moveable about *O*, the end *I* being held by the crooked end of the wire *IS*, moveable about *R*, the end *RS* going within the trap. A bait is put on the end at *S*, and the end *E* of the trap being open, the vermin goes in and pulls at the bait *S*, this pulls the catch *I* from off the end of the leaver *ID*, which lets the end *A* fall down, and the vermin is taken.

Ex. LXXIV.

An engine for moving ſeveral *ſaws* for the ſawing of ſtones, 263.
&c. *ILLI* is a ſquare frame, perpendicular to the horizon, moving in direction *LL*, in gutters made in the fixed beams *AM*, *CB*; and running upon little wheels. *IL* two rods of iron fixed at

FIG. 263. at *I* and *L*. *op* two hands of iron running along theſe rods; to theſe are fixed the ſaws *S*, *S*. *HIK* is a triangle fixed to the axis of a great wheel. As the wheel and triangle go about from *H* towards *I*, the point *I* acting againſt the piece *G*, moves the frame towards *MB*, together with the ſaws *S*, *S*. When *I* is gone off, the angle *K* acts againſt the piece *F*, and moves the frame back again. Then *H* acting againſt *G*, moves it forward; and ſo the ſaws are moved back and forward, as long as the wheel turns round. As theſe ſaws work by the motion of the engine, the hands *op* deſcend. The parts *F* and *G* ought to be made curve; and little wheels may be applied at the points of the triangle *HIK*, to take away the friction againſt *F* and *G*. The axle of the wheel may be made to carry more triangles, and work more ſaws, if the power is ſtrong enough.

Inſtead of the triangle *HIK*, the frame may be moved by the two pieces *ab*, *cd*; going through the axis, acroſs to one another. So that *ab* may only act on *F*, and *cd* on *G*. *F* being only in the plane of *ab*'s motion, and *G* in that of *cd*. So that *F* never falls in the way of *cd*, nor *G* in the way of *ab*.

Ex. LXXV.

264. *A* is an *eolipile*. This is a hollow globe of braſs, with only a very ſmall hole at the mouth. Take it by the handle *B*, and ſet it on a fire till it is heated; then plunge it in cold water, and the air in it which was rarified, will be condenſed; and water will go into it, till it be about half full. Then if it be ſet on the fire, the water will turn into vapour by the heat, and will blow out at the mouth with great violence; and continue ſo till the water is ſpent.

Ex. LXXVI.

265. *ABD* is a *hygroſcope*. *BC* is an index hung by the (therm) ſtring *AB*, the point *B* hanging over the center of a circle, which is divided into equal parts. The ſtring *AB* twiſts and untwiſts by the moiſture or dryneſs of the air. By this means the index *BC* turns about, and ſhews the degrees of drought or moiſture, on the circumference *DC*.

Ex. LXXVII.

266. *A windmill*. *AHO* the upper room, *HOZ* the under one. *AB* the *axle-tree*, going quite through the mill. *STVW* the *ſails*

covered with canvaſs, ſet obliquely to the wind, and going about in the order *STVW*, their length about 6 yards, and breadth 2. *CD* the *cog-wheel*, of about 48 cogs, *a*, *a*, *a*; which carries the *lantern EF*, of 8 or 9 rounds, *c*, *c*; and its axis *GN*. *IK* the *upper ſtone* or *runner*; *LM* the *lower ſtone*. *QR* the *bridge*, ſupporting the axis or *ſpindle GN*. The bridge is ſupported by the beams *cd*, *XY*, wedged up at *c*, *d*, and *X*. *ZY* the *lifting tree* ſtanding upright: *ab*, *ef*, leavers whoſe centers of motion are *Z*, *u*: *fghi* a cord with a ſtone *i*, for a balance, going about the pins *gh*. The ſpindle *tN* is fixed to the upper ſtone *IK*, by a piece of iron, called the *rind*, fixed in the under ſide of the ſtone. The upper ſtone only turns about, and its whole weight reſts upon the bridge *QR*, and turns upon a hard ſtone fixed at *N*. The trundle *EF* and axis *Gt* may be taken away; for it fixes on the lower part at *t*, by a ſquare ſocket, and the top runs in the edge of the beam *w*. Putting down the end *f* of the leaver *fe*, raiſes *b*, which raiſes *ZY*, which raiſes *YX*, and this raiſes the bridge *QR*, with the axis *NG*, and the upper ſtone *IK*; and thus the ſtones are ſet at any diſtance. The lower ſtone is fixed immoveable upon ſtrong beams, is broader than the upper, upon which boards are placed round the upper at a ſmall diſtance, to confine the flour from flying away; and the flour is conveyed through the tunnel *no* down into a cheſt. *P* is the *hopper* into which the corn is put, which runs along the *ſhoe* or ſpout *r* into the hole *t*, and ſo falls between the ſtones where it is ground. The axis *Gt* is ſquare, which ſhaking the ſpout *r* as it goes about, makes the corn run out: *rs* a ſtring going about the pin *s*, which being turned about, moves the ſpout nearer or further from the axis, and ſo makes the corn run in faſter or ſlower, according to the wind. And when the wind is great, the ſails *S*, *T*, *V*, *W* are only part of them, or one ſide of them, covered; or perhaps only a half of two oppoſite ſails *T*, *W* are covered. Towards the end *B* of the axle-tree is placed another cog-wheel, trundle, and ſtones, with exactly the ſame apparatus as before. And the axle carries two pair of ſtones at once. And when only one pair is to grind, the trundle *EF* and axis *Gt* is taken out from the other: *xyl* is a *girth* of pliable wood, fixed at the end *x*; and the other end *l* tied to the leaver *km* moveable about *k*. And the end *m* being put down, draws the girth *xyl* cloſe to the cog-wheel, and by this means the motion of the mill is ſtopt at pleaſure: *pq* is a ladder going into the higher part of the mill. The corn is drawn to the top, by means of a rope going about the axis *AB*, when the mill is going. FIG. 266.

FIG. 266. In mills built of wood, the whole body of the mill turns round to the wind, on a tampin or perpendicular poſt. But in thoſe of ſtone, only the upper part turns; the roof is the ſurface of a cone; there is a wall plate of wood upon the top of the wall: in this a channel is cut quite round, in which are ſeveral braſs rollers. The roof has a wooden ring for its baſe, which exactly fits into this channel; and the roof is eaſily moved round upon the rollers, by help of a rope and windlaſs.

In the wooden mill, 1 is the mill houſe, which is turned about to the wind by a man, by help of the leaver or beam 2. 3 is a roller to hoiſt up the ſteps 4.

Concerning the poſition and force of the ſails, ſee Ex. 21, before.

Ex. LXXVIII.

267. *AB* a *force pump*. *C* the piſton fixed to the rod *EC*, moveable about *E*. *DF* the handle moveable about *D*. *a*, *d* two clacks or valves opening upwards. The piſton *C* muſt move freely up and down in the barrel, and exactly fill it, that no air get in. 'Tis made cloſe by circular pieces of leather cut to fit the barrel, and ſcrewed cloſe between pieces of braſs. This pump acts by preſſing down; for when the handle *F* is raiſed, it raiſes the piſton *EC*, and the water riſes from *H*, opens the valve *d*, and goes into the barrel, at the ſame time the valve *a* ſhuts. But when *F* is put down, the piſton *C* preſſing upon the water, ſhuts the valve *d*, and opens *a*, and forces the water, that has been raiſed, through the pipe *BG*. The piſton *C* muſt not be above 30 feet from the water in the well.

268. *LM* is *another force pump*, or a lifting pump. *N* the bucket. *a*, *b*, *c* valves opening upward. This pump is cloſe at the top *S*, and the ſmall rod of iron plays through a hole made tight with leather. This pump acts by forcing upwards; for when the handle *P* is put down, it lifts up the bucket *N*, the preſſure ſhuts the valve *b*, opens *c*, and forces the water in the barrel *NS* along the pipe *QR*. At the ſame time the valve *a* opens, and lets in more water from *M* into the barrel. And when *P* is raiſed, *N* deſcends, the valves *a*, *c* ſhut, and *b* opens, and lets more water paſs into the bucket *N*, through the upper part. And when the bucket *N* is drawn up again, the water is forced along the pipe *QR* as before. This pump is the ſame as a lifting pump, only there is added the valve *c*, which is not abſolutely neceſſary. No hole or leak muſt be ſuffered below the piſton or bucket; for air will get in, and ſpoil the working of the pump. And the bucket muſt always be within 30 feet of the water.

In theſe pumps the bore at *H* or *M*, through which the water riſes, ſhould not be too ſtrait; the wider it is, the faſter the water aſcends. Nor ſhould the pipe *BG* or *QR*, that diſcharges the water, be too ſtrait; for then the pump will work ſlower, and diſcharge leſs water in a minute, or require more force to work it. For the calculation of a pump, ſee Ex. XXII. FIG. 268.

There are ſeveral ſorts of *valves* uſed in pump work; as *T*, *V*, *W*, that at *T* being made of two pieces of flat leather, is called a *clack*. Theſe at *V*, *W* are made conical, or of any indented figure, and fit exactly into a hole of the ſame ſhape. At the bottom of the valve is put a pin acroſs it, to hinder its flying quite out of the hole.

Ex. LXXIX.

AB a *hydrometer*, to meaſure the denſities of liquors, eſpecially ſpirituous liquors. This is a hollow ball of glaſs, *B*, partly filled with quickſilver or ſhot; and hermetically ſealed at the top *A*, when made of a due weight, by trials. The ſmall tube *AB* is divided into equal parts, and graduated at equal diſtances. And theſe diviſions noted to which it ſinks in different fluids of the beſt ſorts; which points muſt be taken as ſtandards to compare others with. Then if the hydrometer be immerged in any fluid, and the point to which it ſinks in the ſurface be marked, it ſhews the denſity of it, and its goodneſs. For it ſinks deepeſt in the lighteſt liquor: and the lighteſt liquids are the beſt. 269.

Ex. LXXX.

AB is a *thermometer*, to meaſure the degrees of heat. *B* is a glaſs ball with a long neck *AB*. The ball and part of the neck is filled with ſpirit of wine tinged red with cochineal; and the end *A* is ſealed hermetically; in the doing of which, the end of the tube *A*, the ſpirit and included air, are heated, which rarifies the air and ſpirit; ſo that when the end *A* is ſealed, and the tube cools, the ſpirit contracts, and there is a vacuum made in the top of the tube. And therefore the ſpirit expanding and contracting by heat and cold, has liberty to riſe and fall in the tube. This ball and tube is encloſed in a frame, which is divided into degrees. Then as the top of the ſpirit riſes or falls, the diviſions will ſhew the degrees of heat or cold. Theſe diviſions are arbitrary, and therefore two thermometers will not go together, or ſhew the ſame degrees of healt and cold; except they be made to do ſo by graduating them both alike by obſervation. 270.

FIG. This is commonly put in the ſame caſe with the barometer, fig. 218.

270. There are other ſorts of thermometers. *CD* is a ball with a long neck open at the end *D*, partly filled with tinged ſpirit of wine, and put with the open end into the veſſel *D*, near the bottom; which veſſel is half full or more, of the ſame ſpirit. The top of the tube *CE* is air. So in warm weather, when the air in *C* is rarified by heat, it preſſes the ſpirit down into the bucket *D*, and as the point *E* deſcends, the diviſions being marked, ſhew the degree of heat; or when it aſcends, the degrees of cold. But this ſort is affected with the preſſure of the atmoſphere, and therefore is not ſo true.

Ex. LXXXI.

271. *DA* is an *artificial fountain.* *AE* a ſtrong cloſe veſſel of metal, *AB* a pipe reaching near the bottom of the veſſel, and ſoldered cloſe at *A*. *F*, *A* two cocks. If the cocks be opened and water poured in at *A*, till the veſſel be about half full. Then ſtop the cock *F*, and with a ſyringe inject the air at *A*, till it be ſufficiently condenſed within the veſſel. Then ſtop the cock at *A*, and take away the ſyringe. Then as ſoon as you open the cock at *A*, the compreſſion of the air at *C* will force the water up the tube *BA*, and ſpout up to the height *D*; and a little ball of cork may be kept ſuſpended at the top of the ſtream *D*.

But an artificial fountain is moſt eaſily made thus; take a ſtrong bottle *G*; and fill it half full of water. Cork it well, and through the cork, put a tube *HI* very cloſe, to reach near the bottom of the veſſel. Then blow ſtrongly in at *H*, till the air in the bottle be condenſed; then the water will ſpout out at *H* to a great height.

Any of theſe fountains placed in the ſun-ſhine, will ſhew all the colours of the rainbow; a black cloth being placed behind.

Ex. LXXXII.

272. *CpD* is *Archimedes*'s *water ſcrew.* This is a cylinder turning upon the axis *CD*. About this cylinder there is twiſted a pipe, or rather ſeveral pipes *no*, *pq* running ſpiral ways from end to end. This cylinder is placed higher at one end *D*, than at the other. And its uſe is to ſcrew up the water from the lower end to the higher. *AB* is a river running in direction *AB*. *a*, *b*, *c*, *d* ſeveral floats fixed to the cylinder. *EF* the ſurface of the water. Since the cylinder ſtands in an inclined poſition, the upper floats

a, *b* are ſet out of the water, and the under ones *c*, *d* within it. FIG. 272. So that the water acts only upon the under ones *c*, *d*; and turns about the cylinder in the order *a*, *b*, *c*, *d*. By this motion the water taken into the ſpiral tubes at the low end, is by the revolution of the cylinder, conveyed through theſe pipes, and diſcharged at the top into the veſſel *G*. If *AB* is a ſtanding water, there is no occaſion for the floats *a b c d*. And then the cylinder is to be turned by the handle at *D*. Inſtead of the pipe, a ſpiral channel may be cut round the cylinder, and covered cloſe with plates of lead. The cloſer theſe ſpiral tubes are, the more water is raiſed, but it requires more force. Alſo the more the cylinder leans, the more water it carries, but to a leſs height.

Ex. LXXXIII.

AL is a *rolling preſs*, for copper-plate printing. *DE*, *FG*, 273. two wooden rollers, of about 12 or 16 inches diameter, running upon the ends of two ſtrong iron axles, that go quite through them, and are fixed in them. To the axis of the upper one *DE*, is fixed the handle *BAC*. Theſe rollers run in braſs ſockets, and muſt run very true upon their axles, and may be brought nearer, or ſet further from one another, by wedges, in the frame at *P*, *R*. *HIK* is a flat table or plank, going in between the rollers, and ſliding freely upon the frame. *LM* the frame. *NO* a ſhelf to lay the paper upon. When the preſs is uſed, the upper roller is folded round with flannel, that every part of the print may take an equal impreſſion; and a paper bottom, ſpread upon the table *HIK*, where the plate is to lie, to prevent the indenting of the plank; then warming the plate well over a charcoal fire, and rubbing it with the ſort of ink proper for it, and laying it upon the paper bottom, on the table at *H*. Take the printing paper, and laying it carefully upon the plate, and turning the handle *CAB*, the motion of the roller *DE* turns the roller *FG*, and draws the table through between the rollers, together with the plate and paper; and the paper is printed.

Note, the paper muſt be thoroughly wetted in a trough; and after it has laid about a day or two, it is then to be paſſed through a ſcrew preſs, to ſqueeze the water out, and then it is fit for printing.

The ink made uſe of for printing copper-plates, is made of the ſtones of peaches and apricots, the bones of ſheeps feet and ivory, all burnt; this is called *Francfort* black. This muſt be well ground with nut oil, that has been well boiled; and then

FIG. 273. then it is fit for uſe. But the beſt ink is ſaid to be brought from *Holland.*

Ex. LXXXIV.

274. The *fire engine* to raiſe water. *LL* is a *great beam* or leaver, about 24 feet long, 2 feet deep at leaſt, and near 2 feet broad. It lies through the end wall *W* of the engine houſe, and moves round the center *a*, upon an iron axis. *CC* a hollow *cylinder* of iron, 40 inches diameter, or more, and 8 or 9 feet long; *P* the *piſton* ſuſtained by the chain *LP*. *F* the *fire-place* under ground; *BB* the *boiler* 12 feet diameter, which communicates with the cylinder, by the hole 2, and throat *pipe E*, 6 or 8 inches diameter. The boiler is of iron, and covered over cloſe with lead: in this, the water is boiled to raiſe a ſteam. 45 is the *regulator*, being a plate within the boiler, which opens and ſhuts the hole of communication 2; this is fixed on the axis 34 coming through the boiler, on which axis is fixed the horizontal piece *h*3, called the *ſpanner*; ſo that moving *h* back and forward, moves the plate 45 over the hole 2, and back again. *hl* is a horizontal rod of iron, moveable about the joint *h*. *xyedl* a piece of iron, with ſeveral claws, called the *wye*, moving about the axis *de*, in a fixed frame. The claw *sl* is cloven at *l*; and between the two parts, paſſes the end of *hl*, with two knobs to keep it in its place. *AA* is the *working beam*, in which is a ſlit, through which the claws *xy* paſs, and are kept there by the pin *q* going between them. *DDD* is a leaden pipe, called the *injection pipe*, carrying cold water from the *ciſtern S*, into the cylinder *CC*, and is turned up at the end within the cylinder. *f* the *injection cock*, to which is fixed the iron rod *fg*, lying horizontal. The end *g* goes through a ſlit, in the end of the piece *rg*, and on the end is a knob ſcrewed on, to keep it in. *pcbrg* a piece of iron with ſeveral claws called the *eff*, moveable about the axis *bc*. The claw *rp* goes through the ſlit in the beam *AA*, and is kept there by the two pins *o*, *n*: the claw *rg* goes over the piece *gf*: as the piece *gf* is moved back and forward, the injection cock *f* opens and ſhuts. 1, 1 *&c.* are ſeveral holes in the beam *AA*, that by ſhifting the pins, ſerve to ſet the pieces *p*, *x*, *y*, higher or lower, as occaſion requires. *N* is the *ſniffing clack*, ballanced by a weight, and opening outwards, to let out the air in the cylinder, at the deſcent of the piſton. In ſome engines a pipe goes from it to convey the ſteam out of the houſe. *G* is a leaden pipe, called the *ſinking pipe*, or *eduction pipe*, going from the cylinder to the *hot well H*; it is turned up at the end, and has a valve opening upwards; this carries away

the water thrown in by the cold water pipe, or injection pipe. FIG. 274. *t* is the *feeding pipe*, going from the hot well to the boiler, to ſupply it with water, by a cock opening at pleaſure. *i*, *i* are two *gage pipes*, with cocks, one reaching a little under the ſurface of the water in the boiler, the other a little ſhort of it. By opening theſe cocks, it is known when there is water enough in the boiler; for one cock will give ſteam, and the other water; they ſtand in a plate, which may be opened, for a man to go into the boiler, to clean or mend it. *m* is the *puppet clack*; from this a wire comes through a ſmall hole, to which is fixed a thread going over a pulley, with a ſmall weight at it; the weight on the clack *m* is about a pound for every ſquare inch. *YZ* the *ſteam pipe* going from the clack, out of the houſe. When the ſteam in the boiler is too ſtrong, it lifts up the puppet clack *m*, and goes into the ſteam pipe *YZ*, by which it is conveyed away; otherwiſe the boiler would burſt. *KK* a pipe carrying water from the ciſtern *S*, into the cylinder, to cover the piſton to a good depth. *I* a cock opening to any wideneſs, that the water may run in a due quantity; *M* a hole to let it out, through a pipe, into the hot-well *H*, when there is too much. *VVV* a *force pump*, with a bucket and clack, and two valves opening upwards. This pump is cloſe at the top *R*, and being wrought by the leaver *LL*, it brings water out of a pit, into the ciſtern *S*. *Q* the *pit* where water is to be raiſed. *X*, *X*, the *ſpears* which work in wooden pumps within the pit. The cylinder is ſupported by ſtrong beams as 7, 8, going through the engine houſe; 6, 6 is the firſt floor; 7, 7 the upper floor. At *O*, in the end of the beam *CL*, there are two pins, which ſtrike againſt two ſprings of wood, fixed to two timbers, lying on each ſide the great leaver *LL*; theſe pins ſerve to ſtop the beam, and hinder the piſton coming too low in the cylinder.

When the engine is to be ſet to work, the water in the boiler muſt be made to boil ſo long, till the ſteam is ſtrong enough; which is known by opening the cocks *i*, *i*. Then the hole 2 is opened, by moving the ſpanner *h*3 by hand; then the ſteam is let into the cylinder, which lets that end of the beam *LC* riſe up; this raiſes the working beam *AA*, moves the eff *prg*, which moves *gf*, and opens the cold water cock *f*: at the ſame time is moved the wye *xyl*, which draws *lh*, and ſhuts the hole 2. The cock *f* being open, the cold water ruſhing into the cylinder, is thrown up againſt the piſton, and deſcending in ſmall drops, condenſes the hot rarified ſteam, and makes a vacuum under the piſton. Conſequently the weight of

FIG. 274. of the atmoſphere, preſſing upon the piſton, brings down the end *LC*, which raiſes the other end *LQ*, which works the pumps *X*, *X*. As the end *LC* deſcends, the working plug *AA* deſcends, and moving the eff, *prgf*, and the wye, *xyslh*, ſhuts the cold water cock *f*, and opens the hole 2, and the ſteam goes into the cylinder, which takes off the preſſure of the atmoſphere; and the end *LQ* deſcends by the weight of the ſpears *X*, *X*; and the end *LC* aſcends as before, which opens *f*, and ſhuts 2. So by the motion of the beam *AA* up and down, the cock *f*, and hole 2, ſhut and open alternately: and by this means of condenſing and rarifying the ſteam by turns, within the cylinder, the leaver or beam *LL* conſtantly moves up and down; by which motion, the water is drawn up by the pumps, and delivered into troughs within the pit, and carried away by drifts or levels. At the ſame time, the motion of the beam *LL* works the pump *VRV*, and raiſes water into the ciſtern *S*.

When the engine is to ceaſe working, the pins *n*, *o* are taken out, and the cold water cock is kept cloſe ſhut, while the end *LC* is up.

The diameter of the pumps within the pit, is about 8 or 9 inches; and the bores of the pumps where the ſpears *X*, *X*, work, ſhould be made wide at the top; for if they be ſtrait, more time is required to make a ſtroke, and the barrels are in danger of burſting. Likewiſe if water to be raiſed from a great depth at one lift, the pumps will be in danger of burſting; therefore it is better to make 2 or 3 lifts, placing ciſterns to receive the water.

292. The ſpears or rods, that work in the pumps, conſiſting of ſeveral lengths are joined thus; each piece has a ſtud *a*, and a hole *b*; which are made to fit; and the ſtuds of one being put cloſe into the holes of the other, and the iron collar *g* drove upon them to the middle *d*, they are firmly fixt together.

274. There is never made a perfect vacuum in the cylinder; for as ſoon as the elaſtic force of the ſteam within is ſufficiently diminiſhed, the piſton begins to deſcend. The vacuum is ſuch, that about 8 lb. preſſes upon every ſquare inch of the piſton, or in ſome engines, not above 6 lb. This engine will make 13 or 14 ſtrokes in a minute, and makes a 6 foot ſtroke; but the larger the boiler is, the faſter ſhe will work. A cubic inch of water in this engine, will make 13340 cubic inches of ſteam; which therefore is 15 times rarer than common air. But its elaſtic force within the boiler is never $\frac{1}{10}$ ſtronger or weaker than common air; if ſtronger, it would force the water out of the feeding pipe.

This

FIG. 274. This engine will deliver 300 hogſheads of water in an hour, to the height of 60 fathom, and conſumes about 30 buſhels of coals in 12 hours.

293. In ſome engines there is a different contrivance to open and ſhut the regulator, which is performed thus; as the beam *QQ* aſcends, it raiſes *G*5, turns the wye 5*GCED*4 about the axis *AB*; and the weight *C* falling towards *B*, the end *E* ſtrikes a ſmart blow on the pin *L*, and drives the fork *FL* towards *L*; which draws the ſpanner *PO* towards *L*, and ſhuts the regulator. And when the beam *QQ* deſcends, a pin in it puts down the end 4, and turns the axis *AB*, and the weight *C* deſcending towards 5, throws the end *D* of the wye againſt *L*, which moves *PO*, and opens the regulator: the ſpanner *PO* ſliding upon the horizontal piece *O*. There is a cord *ron*, fixt at *r*, *n*, and the top of the wye *O*, to hinder it from going too far on each ſide.

Likewiſe, for opening and ſhutting the injection cock; inſtead of the pieces *rg*, *gf* of the eff (fig. 274.) ſome engines have quadrants of 2 wheels *H*, *I*, with teeth, which moving one the other, opens or ſhuts the cock *f*, of the injection pipe *K*.

In ſome engines there is a catch, held by a chain fixt to the great beam; and this catch holds the eff from falling back, and opening the cold water cock; till the riſing of the beam pulls the catch up by the chain, and then the eff falls.

A calculation of the cylinder and pumps of the fire engine.

274. If it be required to make an engine to draw any given number of hogſheads of water in an hour, from *f* fathom deep, to make any number of ſtrokes in a minute, by a 6 foot ſtroke. Find the ale gallons to be drawn at 1 ſtroke, which is eaſily found from the number of ſtrokes being given.

Let g = *number of ale gallons to be drawn at* 1 *ſtroke.*
p = *pump's diameter.* } in *inches.*
c = *cylinder's diameter.* }

Then $p = \sqrt{5g}$,

And ſuppoſing the preſſure of the atmoſphere on an inch of the piſton, to be 7 lb.

Then $c = p\sqrt{\frac{2.614f}{7}} = \sqrt{\frac{13.07fg}{7}}$.

Note, if inſtead of 7, you ſuppoſe the preſſure of the atmoſphere to be l pounds; and inſtead of a 6 foot ſtroke to make an r feet ſtroke; then $p = \sqrt{\frac{6}{r} \times 5g}$, and $c = \sqrt{\frac{13.07fg}{l}}$.

FIG.

Ex. LXXXV.

275. *AB* is the *water engine* to quench fire. *D*, *E* are two *pumps* 5 inches diameter, having each a clack *a*, *b* opening upwards. *CO* a large copper *air veſſel* 9 inches diameter. This veſſel ſtands upon a ſtrong plate *kw*, 5 or 6 inches above the bottom of the cheſt *NM*. *SY* is a braſs pipe coming through the end of the cheſt at *S*; and at *Y* it divides into two cavities going under the copper pot *CO*, to the two pumps *E*, *D*. The cavity *YW* leads to the pump *E*. And directly above this cavity at *W*, there is another cavity *y*, communicating with the pump *E*. And above the cavity *y* is placed the valve *r*, opening upwards into the copper pot *CO*, from this cavity. There are the like cavities belonging to the pump *D*; the firſt going to the valve *a*; the other from the pump to the valve *s* of the copper pot. Theſe cavities are made of hollow pieces of braſs ſcrewed faſt together. *Z* is a cock, through which are two paſſages, one along the pipe *SY*, and another at the ſide of the pipe into the cheſt *NM*. This cock, by turning the handle *ce*, opens one paſſage and ſhuts the other, as there is occaſion. *Xx* a leather pipe to be ſcrewed on the end *S*, to draw water out of a well or river.

PR is the *conduit pipe* reaching near the bottom of the veſſel *CO*, and ſoldered cloſe into the top of it. At *R* and *Q* are ſcrews, ſo that the pipe may be turned in any direction by the man that holds it. And at *V* a copper pipe muſt be ſcrewed on, or elſe a long leather one, which being flexible, is carried into rooms and entries. *HI* an iron axis, to which the iron leavers *FG*, *LK* are fixt. This axis moves in ſockets about *H*, *I*, which are ſcrewed hard down. *FK*, *GL* two wooden handles fixt to the leavers to work them by. *gh*, *pl* are two arches of iron fixt on the axis *HI*. *fd*, *mn*, the ſhanks of the piſtons, being two ſtrong rods of iron. *fg*, *ht*, *lq*, *mp* four iron chains fixt at *f*, *g*; and *h*, *t*; and *l*, *q*; and at *m*, *p*. At *f* and *m* are ſcrews to ſcrew the chains tight: theſe chains work the pumps. For when *FK* is put down, the chain *fg* pulls down the rod of the piſton *fd*. And when *FK* is raiſed, the chain *th* pulls it up again. And the ſame way the chains *lq*, *mp* raiſe and depreſs the piſton *mn*. In ſome engines there are two arches, like *hg*, *tp*, fixt near the end *I* of the axis, and chains at them; from the ends of which, as alſo from *t* and *q* two boards are ſuſpended. Theſe boards ſerve for treadles for men to ſtand upon, to help to work the engine.

The

FIG. 275. The veſſel *CO* and two pumps are incloſed in a cheſt *AN*, and the whole machine moveable on wheels. The fore axle-tree turning on a bolt in the middle, for the conveniency of turning to either ſide. But there are a great many forms of theſe engines. In ſome the leaver lies croſs over, in others lengthways; in ſome there is no chain work, but only pins for the piſtons to move upon.

When the engine is to play; if it is by the water in a river, *&c.* the pipe *Xx* muſt be ſcrewed on at *S*, and the end *x* put into the water. But if water is to be fetched, it muſt be poured into the cheſt *M*, which runs through the holes *T*, into the body of the engine *N*. Then turning the cock *ceZ* to open the proper communication; the handles *FK*, *GL*, muſt be moved up and down by men; by which means water is drawn into the pumps *E*, *D*, and forced into the veſſel *CO*, and out of the pipe *PR*. For the piſton *mm* being raiſed, the water is drawn along the cavity *ZYW*, through the valve *b* into the pump *E*; and when *mn* is depreſſed, the valve *b* ſhuts, and the water is forced into the cavity *y*, through the valve *r*, and into the pot *CO*; which cannot return for the ſhutting of the valve *r*, when the piſton *mn* riſes again. And the like for the other pump *D*. Since the piſton of one pump goes down whilſt the other goes up, the water is forced by turns into the veſſel *CO*, by theſe two pumps; ſo that there is always water going in. And the air confined at top of the veſſel at *C* being condenſed, will preſs the water up the pipe *PRQV*, and make it flow with a continual ſtream. If the water in *C* be compreſſed into half the ſpace, it will force the water to 30 feet high.

In ſome engines, there is another pipe as *PR*, coming through the copper pot, and through the ſide of the engine, and theſe two pipes may play both at once, if there is occaſion. And if not, the end of one is ſcrewed up.

Ex. LXXXVI.

A ſhip. This is the nobleſt machine that ever was invented. 276.
It is ſo compounded, and conſiſts of ſo many parts, that it would require a whole volume to deſcribe it. Some of the principal parts are theſe.

A the hull.
B the bow.
C the forecaſtle.
D the main deck.
E the ſtern.
F the ancient ſtaff and enſign.
G the poop-lantern.
H the rudder.

FIG. 276. *I* the bowſprit.
K the fore-maſt.
L the main-maſt.
M the mizen-maſt.
N top-maſts.
O top-gallant maſts.
P ſails
Q yards.
R vanes.
S the jack.
T the pendant.
V ſtays.
Vv main-ſtay, &c.
W ſhrowds.
X main-top-maſt back-ſtay, &c.
Y the crane-line.
Z the anchor, to which the cable is fixt.

1 Main-lifts.
2. Fore-braces.
3. Main-top-ſail-ſheets.
4. The fore-tacks.
5. Fore-top ſail-clew-lines.
6. Main-top-ſail-leetch-lines.
7. Fore-bunt-lines.
8. Mizen-bow-lines.
9. Main-top ſail-halliards.
And the like for the reſt of the ſails.

Moſt of theſe ropes are for hoiſting the ſails, and ſetting them in a proper poſition to the wind. For the wind always acts perpendicularly upon the plane of any ſail; and urges the ſhip in direction of that perpendicular. And by the help of the rudder *H*, ſhe is made to keep any direction required. For if the rudder be put about to any ſide; the water (as the ſhip moves along) will act violently againſt it, and drive the ſtern the contrary way, or her head the ſame way, as the rudder. A ſhip with a fair briſk wind will ſail 8 or 10 miles an hour.

That any one ſail may have the greateſt force to move a ſhip forward, it muſt be ſo placed between the point of the wind and the ſhip's way, that the *tan.* of the angle it makes with the wind, may be twice the *tan.* of the angle it makes with the ſhip's way.

When the rudder is ſet to an angle of $54\frac{1}{2}$ deg. with the keel, it has the greateſt force to turn the ſhip, and make her anſwer the helm.

301. Becauſe the figure of a ſhip is the cauſe of her going well or ill, and of making more or leſs way through the water; I ſhall here give the conſtruction of the fore part of a veſſel, that will move through the water with the leaſt poſſible reſiſtance.

Let *DdAcC* be the *water line*, or horizontal ſection of the water and the hull of a ſhip, *AB* 30 feet, *CD* the greateſt breadth 20 feet, *BC* 10 feet. *AeE* the ſtem and part of the keel. Then the following table ſhows the length of every ordinate, as *bc*, taken at the diſtance *Ab*, or 1, 2, 3, &c. feet from *A*; by which the curve *AcC* is determined.

Length

FIG. 301.

length of *AB* in feet	length of *bc* in feet	length of *AB* in feet	length of *bc* in feet
1	0.90	16	6.36
2	1.48	17	6.64
3	1.96	18	6.92
4	2.39	19	7.19
5	2.79	20	7.46
6	3.17	21	7.73
7	3.54	22	7.99
8	3.89	23	8.25
9	4.22	24	8.51
10	4 5	25	8.76
11	4.87	26	9 01
12	5.18	27	9.26
13	5.48	28	9.51
14	5.78	29	9 76
15	6.07	30	10.00

The practice is thus: having made $AB=30$ feet, and accordingly divided it into 30 equal parts; at the ſeveral points of diviſion, erect perpendiculars to *AB*, equal to the lengths given in the 2d col. of the table. The curve *AcC* drawn through the ends of all the ordinates, is the figure of the ſhip on each ſide.

The curve *AeE* which the ſtem and keel make, muſt be the ſame curve as *AcC*; if the depth *BE* is ſuppoſed equal to *BC*; and the ordinates, *be*, *BE* muſt all be drawn perpendicular to *AB*. But if the depth *BE* be taken greater or leſs than *BC*; then the ordinates muſt be taken greater or leſs in proportion.

Again if *CDE* be the ſection of the ſhip, made perpendicular to the axis *AB*, or horizontal plane *CAD*; and *ced* be any other ſection parallel to it; then whatever the curve *CED* is, all the ſections *ced* muſt be ſimilar to it.

If a ſhip is required to be built either greater or leſs than this; then it is only taking a greater or leſs length inſtead of a foot, and dividing it decimally, and uſing it inſtead of a foot, to meaſure off the lengths as in the table.

Likewiſe if it was required to have the breadth to be greater or leſs than is here aſſigned, whilſt the length remains the ſame; then it is only taking a proportionally greater or leſs line, inſtead of a foot, and ſetting off the ordinates *bc* by that.

FIG. 301. And thus the requiſites may be altered at pleaſure, ſtill retaining the general conſtruction.

If any ſhip carpenter thinks fit to build a ſhip according to this model, it will be found to move faſter through the water, than any other ſhip of the ſame length, breadth, and depth, and of a different form. The form of the curve is truly repreſented by the curve *AcC*.

But it muſt be obſerved, that the curve at *C*, the broadeſt part is not perpendicular to the ordinate *BC*, but makes an angle of about 76 degrees: to avoid this, it will be proper to produce *AB* a little further, and turn the ſide *AC*, at *C*, round in a curve, as quick as poſſible. Or elſe make the 2 or 3 laſt perpendicular ordinates, ſomething leſs than in the table, that the part of the curve at *C* may be in a paralleliſm with *AB*, as it ought; becauſe *C* is the broadeſt part.

But though the form here given is the moſt proper for ſailing faſt; yet perhaps it may not be ſo commodious as the common form, upon other accounts, as for the ſtowage of goods, *&c.* Yet privateers and ſhips of war made to purſue the enemy, ought to be built as near this form as they can conveniently. For it is a matter of great moment, either to have it in our power to come up with a ſhip we are able to take, or elſe to fly, and eſcape from one of ſuperior force.

That a ſhip may ſteer well, the water ought to come freely and directly to the rudder; and therefore ſhe muſt not be too ſhort from the midſhip to the ſtern; and towards the ſtern ſhe muſt riſe well, and be built very thin below, leſſening gradually to the ſtern-poſt. Likewiſe ſhe muſt draw conſiderably more water abaft than afore. To carry a good ſail, and alſo to avoid rowling, ſhe muſt be made pretty deep in the hold. As to the dead work, or upper part of the ſhip, that may be left to the fancy of the builder, or contrived to anſwer ſuch conveniences as may be wanted.

Ex. LXXXVII.

277. *AT* an *air pump*. *C*, *D* two braſs cylinders, 2 or 3 inches diameter, and a foot high, having two valves at the bottom opening upwards. *t*, *t* two piſtons working in the cylinders, having two valves alſo opening upwards. *FF* a handle going upon the axis of the wheel or lantern *E*, which wheel by the teeth, moves the racks *G*, *G*; and by them the piſtons, within the cylinders or pumps. *AB* a table or plate ſupported by the pillars *I*, *I*, *I*, *I*. *H* the receiver of glaſs, which by the hol-

FIG. 277. low pipe of braſs *nooo*, called the ſwan's neck, communicates with the cylinders by means of a hollow braſs pipe *PQ*; into which the ſwan's neck paſſes. *rrs* a mercurial gage, being a glaſs tube ſtanding in the bucket of mercury *s*, and communicating with the pipe *no*. *K* a cock under the table *AB* to let in air into the pipe *no*, and ſo into the receiver, when there is occaſion.

When the air is to be drawn out of the receiver *H*, a wet leather is placed over the plate, and upon that the receiver *H*. Then raiſing the right hand *F*, the piſton *t* of the barrel *D* is raiſed, which takes off the weight of the atmoſphere; conſequently the air paſſes out of the receiver *H*, through the ſwan's neck *no*, and through the hollow braſs *PQ*, through the valve into the cylinder *D*. Then the right hand *F* put down, the valve at the bottom of the cylinder *D* ſhuts, and the air paſſes through the valve at *t*: at the ſame time that the left hand being raiſed, draws the air from the receiver, through *noP*, through the valve into the cylinder *C*. So that by the motion of the handle *FF* up and down, the air is at length drawn out of the receiver *H*, by the pumps *C*, *D*. And the rarity of the air within the receiver, is known by the height of the mercury in the tube *rs*, which is known by the graduated frame. An abſolute vacuum can never be perfectly made. For when the ſpring of the air is ſo weak, as not to be able to lift up the valves at the bottom of the cylinders, no more air can be drawn out.

278. The handle *F* is lately made to turn always one way; thus *A* is a crank turned by the handle *F*. *NN* the leader or ſword going over the pin *I*, in the wheel *E* Whilſt the crank *A* is riſing, it raiſes the ſide *S* of the wheel, and when the crank deſcends, it thruſts down the ſame ſide *S* of the wheel *E*. So the racks are alternately raiſed and depreſſed as the crank goes about.

279. There are ſeveral ſorts of glaſſes made uſe of for the air pump As *A* a receiver open at top, covered with a braſs plate and oiled leather at *D*, and kept down by the croſs piece *EF*, ſcrewed down upon the pillars *BC*, which pillars are ſcrewed into the table *AB* of the air pump.

280. *H* a receiver open at top, with a plate and collar of wet leathers *K*, through which goes the ſlip wire *GI*, ſo tight as to let no air in. This ſerves to lift any thing up by the hook *I*.

MP is a transferrer *N* is a plate and leather, on which ſtands the receiver *M* *NP* a hollow tube going through the plate. *O* a cock to open or ſhut the paſſage. The cock *O* being open, and the air exhauſted by the pump, and then the

cock

FIG. cock being ſhut, the receiver and pipe may be taken away from
280. the air pump, the vacuum remaining in *M*.

L a receiver cloſe at the top; with infinite others of like ſort.

Ex. LXXXVIII.

281. *London-bridge water-works.* *AB* the axis of the water-wheel *CD*; which wheel is 20 feet diameter, and the axis 3 feet, and 19 feet long. *E*, *E* 26 floats $1\frac{1}{2}$ foot broad. *G* a ſpur wheel fixed to the axis *AB*, 8 feet diameter, 44 cogs of iron; this moves the trundle *H* $4\frac{1}{2}$ feet diameter, and 20 rounds; *HI* its iron axis. *IK* a quadruple crank of caſt iron 6 inches ſquare, each crank being a foot from the axis. The crank is faſtened to the axis at *I*, by help of a wedge going through both, which cauſes the crank to turn. *L*, *L* four iron ſpears belonging to the cranks, and fixed to the 4 leavers *MN*, 3 feet from the ends; which leavers are 24 feet long, moving on centers in a frame of wood. *P*, *P* four force pumps of caſt iron, wrought by four piſtons or rods, *NP*. Theſe pumps are 7 or 8 inches diameter, having valves opening upward. *O* a hollow trunk of caſt iron, to which the pumps are cloſe fixed. *Q* a ſucking pipe going into the water. *R*, *R* four hollow pipes, 7 inches diameter, and cloſe fixed to the lower part of the pumps; theſe pipes are cloſe ſcrewed to the hollow iron trunk *S*, into which 4 valves open. *T* a pipe communicating with the trunk *S*, through which the water is forced to any height. There are alſo four forcers placed at the ends *M*, *M* of the leavers *M*, *N*, and working in four more pumps, to which belong other trunks and pipes, *O*, *Q*, *R*, *S*. At *B* the other end of the axis, there is placed exactly the ſame work as at *A*; ſo that the great wheel *CD* works 16 pumps. There is alſo a machine made of cog-wheels and trundles, contrived to raiſe the great wheel as the tide riſes. The great wheel will go at any depth of water; and as the tide turns, the wheels go the ſame way with it; but ſtand ſtill at high and low water.

As the great wheel is carried about by the tide, it carries round the ſpur wheel *G*, which carries the trundle *H* with the cranks *IK*, which by the ſwords *L*, move the leavers *MN*. When the end *M* is pulled down, *N* is raiſed with the piſton *NP* in the pump *P*, by which means the water is drawn out of the river through the pipe *Q*, into the pump *P*; and when *NP* deſcends, the valve ſhuts, and the water is forced through the pipe *R*, through the trunk *S*, and along the pipe *T* into the town. And when *N* riſes up again by the motion of the cranks, the valve in *S* ſhuts, and that in the pump opens, and more water riſes through the pipe *Q* into the pump *P*. And as the cranks ſtand every way, there is always water riſing in ſome of the pumps; and ſome always forcing through *R*, *S*, *T*. When the tide is ſtrongeſt, the great wheel goes 6 times round in a minute. This engine is ſaid to raiſe 30 or 40 thouſand hogſheads of water in a day. FIG. 281.

Ex. LXXXIX.

The pile engine for Weſtminſter-bridge. *A* the great cog-wheel fixed to the great ſhaft *D*. *MO*, a trundle and fly turned by the cog-wheel; this is to prevent the horſes from falling when the ram is diſcharged. *B* the drum or barrel on which the great rope is wound. *C* a leſs barrel on which the rope *L* is wound, carrying the weight *N*. The uſe of this is to hinder the follower from falling too faſt. The barrels *BC* are moveable about the great axis *D*. The cog-wheel and barrel *B* are fixed together by the bolt *F*, going through the cog-wheel into the barrel. *EI* is a leaver moveable about 1, going through the great ſhaft *D*; this lifts up the bolt *F*, the end *E* being made heavier by a weight; by which means it locks the barrel *B* to the great wheel *A*. *KI* the forcing bar going into the hollow axis of the great ſhaft *D*; this reſts upon the leaver *EI*. *XY* a great leaver moveable about 3, the end *X* being heavier, which with the end *Y* preſſes down the bar *KI*, thruſts down the end of the leaver at *F*, and lets the bolt *F* deſcend, to unlock the barrel *C*. *Z* a rope fixed at *X*, and going up through the guides at *R*. *GK* a crooked leaver moveable about 2, the roller at the end *G* being preſſed with the great rope, forces the end *K* againſt the catch at *K*, and hinders the bar *KI* from aſcending. When the rope *H* ſlackens, the ſpring 7 forces the end *K* from the catch, and the bar *KI* aſcends. *H* the great rope going round the barrel *B*, over the pulley *P*, up to the top and over the pulley *Q*, then down to the follower, where it is fixed. *T* the ram that drives the piles. *S* the follower, in which is fixed the tongs *W*, moveable about the center. *VV* the guides between which the ram falls. At the inſide of the guides at *R*, where they are faſtened together, there are two inclined planes. At the bottom of the follower is a ſlit, to receive the handle 6 of the ram *T*, to be taken up by the tongs *W*. *a*, *b*, *c*, *d* timbers for horſes to draw at, in direction *abcd*. 283.

As the horſes go round, the great rope *H* is wound about the barrel *B*; and the follower *S*, and ram *T* are drawn up, till the

FIG. 283. tongs come between the inclined planes, which ſqueezing the ends 4 4 together, opens the end 5, and lets the ram fall down. Then the follower *S* taking hold of the rope *Z*, raiſes the end *X*, and depreſſes the end *Y* of the leaver *XY*, which thruſts down the bar *KI*, which thruſts down the end *FI* of the leaver *FI*, with the bolt *F*, and unlocks the barrel *B*, which turning about the axis, the follower *S* deſcends by its weight, till it comes to the ram *T*; and the end 5 of the tongs ſlips over the handle 6 of the ram. Then the rope *H* ſlackens, and the ſpring 7 forces the end *K* from off the catch at top of the bar *KI*, and lets the bar riſe, and the weight *E* raiſes the bolt *F*, and locks the barrel *B* to the wheel *A*; and the horſes ſtill going about, the end 5 of the tongs takes hold of the handle 6, and the ram *T* is taken up as before.

All this machine is placed upon a boat, which ſwims upon the water; and ſo is eaſily conveyed to any place deſired.

Ex. XC.

284. *GH* a *blowing wheel.* *AB*, *CD* an iron croſs; to this is fixed the circle of iron *EF*. To theſe are fixed 12 leaves *I*, *I*, *I*, which reach no nearer the center than the iron circle. 1, 1, 1 are holes, through which the air paſſes into the cavities between the leaves. There is the ſame croſs and iron circle on the other ſide, but without any hole. Through the center of both ſides is put an iron axis and fixed there, and on the further end a handle is put to turn it by. This wheel is encloſed in a caſe, which juſt touches the edges of all the leaves. But the rim or out edge *KK* is at a ſmall diſtance from the ends of the leaves. On this ſide or flat of the caſe, there is a hole left againſt the holes 1, 1, to let the air through; the other flat is cloſe. *LM* is the ſucking pipe, being a tube fixed upon the caſe, ſo as to communicate with the cavities, by the holes 1, 1. *G* is the blowing pipe, and is another wooden tube communicating with the inſide of the caſe. The axis turns upon two concave pieces of metal fixed to the caſe, the handle being on the back ſide of the figure. *abcd* is a thin ring of wood faſtening the leaves all together; and the like on the other ſide. On theſe rings are put two circles of blanketing to go cloſe to the caſe, and alſo upon the iron circle *EF*.

The frame being fixed, and the handle turned about in the order *BCAD*. The motion of the leaves moves the air very ſwiftly to the outſide, which being confined by the rim, is forced in a tangent along the tube *G*; whilſt new air aſcends along the ſucking pipe *LM*, paſſes through the hole in the frame, and through the holes 1, 1, into the cavities between the leaves; and ſo thrown out of the wheel, through the blowing pipe *G*. FIG. 284.

If the pipe *LM* be continued to the place where any foul air is, it may ſoon be thrown out through the tube *G*, and diſperſed abroad. Or if the tube *LM* communicate with the freſh air, and *G* with any cloſe room, freſh air may preſently be injected into the room.

Ex. XCI.

AB an *artificial fountain* to play with either end up. *A* and *B* two cavities; *FO*, *KB* two open pipes, fixed to the baſons at *K* and *O*. *GHI* and *CDE* two curve tubes open at both ends. When the fountain ſtands on the end *A*, pour water in at *O*. Then turning the fountain like an hour-glaſs upon the end *B*, the water will deſcend through the pipe *CDE*, and ſpout out at *E*. The air paſſing up the pipe *OF* to give it liberty. The water falling down upon the baſon *EK*, runs through the pipe *KB*, into the cavity *B*. And the fountain being turned, the water will deſcend through *GHI*, and ſpout out at *I* as before. And ſo being turned, it will play a-freſh as often as you will. 285.

Note, while the jet *E* is playing, if the end *O* of the pipe *FO* be ſtopped with the finger, the jet will ceaſe playing; which being taken off, it will begin again; and ſo may be made to play or ſtop at pleaſure.

Ex. XCII.

AF a *water barometer*. *AD* is a ſmall tube open at both ends, cemented in the neck of the bottle *CE*. Then the bottle being a little warmed to drive out ſome of the air; the end *A* is immerged in water tinged with cochineal, which goes into the bottle as it cools. Then it is ſet upright, and the water may be made to ſtand at any point *B*, by ſucking or blowing at *A*. This is a very ſenſible barometer; for if it be removed to any higher place, a very ſmall decreaſe in the air's gravity, will make the water riſe ſenſibly in the tube. This may be made uſe of to find the level of places. But it is ſubject to this inconvenience, that it is a thermometer as well as a barometer, the leaſt alteration of heat raiſing the water in the tube. To prevent which, it muſt be encloſed in a veſſel of ſand; and then the air included in the bottle, will retain the ſame degree of heat, at leaſt for a ſmall time. 286.

FIG.

Ex. XCIII.

287. *ADOF* is a *jet d'eau*. *AB* the reſervoir where the water is kept. *CDIO* the pipe of conduct, which conveys water from the reſervoir. *O* the cock, or adjutage, being a ſmall hole in a thin horizontal plate, fixed upon the end of the pipe, through which the water flows. *OF* the jet of water, ſpouting up through the hole *O*, which deſcends again in the ſtreams *FE* and *FH*. *OF* the height of the jet. *AG* the horizontal height of the water in the reſervoir. If the part *LIO* of the pipe of conduct be buried under the ſurface of the water *KH*, and be inviſible; the jet will ſeem to riſe out of the water *KH*, as in many artificial fountains.

The adjutage is ſometimes made conical, but the beſt ſort for ſpouting higheſt, is a thin plate with a hole in it. The bore of the adjutage ought to increaſe with the height of the reſervoir, and the larger the adjutage, the higher the jet will go, provided the pipe of conduct be large enough to ſupply it with water. Pipes of conduct ought not to be made with elbows, but to turn off gradually in a curve as *DIO*. The diameters of pipes of conduct ought at leaſt to be 5 or 6 times the diameters of the adjutage, or elſe it will not ſpout ſo high. If a reſervoir be 50 feet high, and the adjutage half an inch; the pipe of conduct ſhould, at leaſt, be 3 inches; or if the adjutage be an inch, which is better, the pipe of conduct muſt be 6 inches. And in theſe caſes the jet will riſe to the greateſt height it can have. In general the diameter of the adjutage ought to be nearly as the ſquare root of the height of the reſervoir. And if you would have the velocity in the pipe of conduct to be the ſame at all heights of the reſervoir, that the friction may not increaſe too much; then the ſquare of the diameter of the pipe of conduct muſt be as the cube of the diameter of the adjutage. When water is carried a great way through pipes, the friction of the pipes will diminiſh its velocity, and the jet will not riſe ſo high.

A jet never riſes to the full height of the reſervoir. If the height be 5 feet 1 inch, the jet will only riſe to 5 feet; thus the jet *OF* wants the ſpace *FG* of the height of the reſervoir. And the defect *FG* is as the ſquare of the height of the reſervoir *OG*. But ſmaller jets fall ſhort more than in that proportion, being more retarded by the reſiſtance of the air. The greateſt jets never riſe 300 feet high; for the velocity is ſo great, that the water is diſſipated into ſmall drops, by the reſiſtance of the air. If a ball a ball of cork or light wood be laid at *F*, it will be ſuſpended by the pillar of water, and play there without falling. FIG. 287.

Ex. XCIV.

AGE is a *compound ſteelyard*, for weighing vaſt weights. *IG*, *CK* two leavers moveable about *B* and *E*. *LE*, *MB* two fixed pieces. *AC* a croſs bar, ſupporting the end *C*, and moveable about the pins *A* and *C*. The center of gravity of *IG* and *AC* is in *B*; and of *CK* and the hook *DN*, in *E*. *H* the weight to be weighed; *F* the counterpoiſe moveable along the graduated leaver *BG*. The machine is hung upon the hooks at *L*, *M*. Here the power *F* is to the weight *H*, as $AB \times DE$ to $CE \times BF$. 288.

Ex. XCV.

ABC is a *horſe mill* to grind corn; *C* the *ſpur-wheel* having 72 cogs; *B* the *lanthern* of 7 rounds; *A* the *hopper*, *E* the *ſhoe*; *F*, *G* the two *mill-ſtones*. *H* a leaver or *arm* 8 feet long, going into the axis *D* of the great wheel; *I* the traces to which one or two horſes are yoaked. As the horſe goes about in the path 1 2 3, he draws the arm *H*, which turns the great wheel *C*, and this drives the trundle *B* with the upper ſtone *F*, which grinds the corn; the corn is put into the hopper *A*, and falling into the ſhoe *E*, runs through a hole at top of the ſtone *F*, and in between the ſtones where it is ground. *KL* is the upper floor. The whole is within a houſe. 294.

Ex. XCVI.

AB a *lifting ſtock*, ſet perpendicular; its uſe is to raiſe a great weight. *LO* is a ſlit going through it, in which there moves the leaver *CD*. *II*, *KK* two ſets of holes; into which goes the pins *G*, *H*. When the weight *W* is to be raiſed, it is hung on the hook and chain at the end *D* of the leaver. And the pin *G* being put into the firſt hole *I*, and the end *C* being put down, the other end with the weight is raiſed, and then the pin *H* is put into the ſecond hole *K*, under the leaver; then the end *C* being raiſed to *E*, the pin *G* is put into the ſecond hole *I*. Then *E* being put down to *C*, and the end *F* raiſed, the pin *H* is put into the third hole *K*. Thus the leaver and pins being thus ſhifted from hole to hole, the weight *W* is by degrees raiſed up. 295.

FIG.

Ex. XCVII.

296. A *bob gin*, for raiſing water. *AB* a large water-wheel carried by the water *W*. *C* and *D* two cranks, upon the axis, on each ſide the wheel, lying contrary ways. *EF*, *GH*, two pieces of timber moving about on the cranks *C* and *D*, and alſo moveable at the joints *F*, *H*, upon two pins. *FI*, *HK* two beams, moving on the axes *L* and *M*. *I*, *K* two arches with chains fixed to them, by means of which the pumps *O*, *N* are wrought. When the water-wheel goes about, one crank as *C* pulls down the bar *EF*, together with the end *F* of the beam *FI*, and at the ſame time raiſes the end *I*, which draws the water up in the pump *O*. In the mean time the other crank *D* is raiſing the end *H*, and depreſſing the end *K*. When by motion of the wheel, the crank *C* begins to aſcend, the end *I* begins to deſcend, and the end *K* to aſcend. So that one beam goes up whilſt the other goes down, and there is always one pump working.

Ex. XCVIII.

297. A *gunpowder-mill*. *AP* the water-wheel; *B* its axis. *RPS* the water-courſe. *E* a ſpur-wheel carrying the two drums *C*, *D*, and the rollers *CF*, *DH*, to which they are fixed. *a*, *a*, *&c.* 10 or 12 pins, or cogs fixed in either roller, equally on all ſides. *b*, *b*, *&c.* 10 or 12 peſtils, 10 feet long, and 4 or 5 inches broad, armed with iron at the bottom; in theſe peſtils are pins fixed to anſwer the pins *a*, *a*; which lift them up as the rollers turn round. *m*, *m*, *&c.* are wooden mortars, into which the peſtils fall, each mortar will hold about 20 lb. of paſte. *OQ*, *IK*, *LN* are timbers through which the peſtils work, and ſerve to keep them direct.

The materials being put into the mortars *m*, *m*; as the mill goes about, the pins in the rollers take up the peſtils *b*, *b* by their pins, and when theſe pins go off, the peſtils fall into the mortars *m*, *m*, and beat the ingredients to a paſte. And as theſe cogs are placed on all ſides the circumference of the rollers, there will be always ſome peſtils riſing, and others falling in a regular order.

Ex. XCIX.

298. A *crane* or engine to raiſe a great weight, and keep it in any poſition. *AB* a double wheel for a man to walk in; *CD* a ſpur-

wheel upon the ſame axis. *E*, *F*, *G* are three wheels alſo fixed upon one axis, of which *G* is of wood, and *E* is moved by *CD*. FIG. 298. *HI* is a catch moving on the pin *I*, this falls in between the teeth of the wheel *F*. *KLM* a half ring of iron, in which is a grove, going upon the edge of the wooden wheel *G*. *NM* a piece of timber fixed to the ring at *M*, and to the leaver *PN*, and is moveable about the pins *M*, *N*. The leaver *PN* is moveable about the center *O*. *QR* a wooden rod, reaching to the catch *IH*. *PST* a ſtring fixed to the leaver at *P*. *VXW* the rope going about the axis of the great wheel to raiſe the weight.

When the great wheel *AB* goes round, together with *CD*, the rope *VXW* raiſes the weight *W*. The wheel *CD* drives *E*, together with *F* and *G*; and the end of the catch *IH* ſlides freely over the teeth of the wheel *F*; and the motion being ſtopped, the catch *IH* acting againſt the teeth of *F*, hinders the wheel *F* from turning back, and ſo keeps the weight *W* ſuſpended. But if you pull at the ſtring *T*, it raiſes the leaver *PO*, and thruſting the rod *QR* againſt the catch, raiſes it out of the teeth of the wheel *F*, and lets the weight *W* deſcend. But leſt it deſcend too faſt, the leaver *PO* is to be raiſed higher, by pulling at the ſtring *TS*, and this depreſſes the end *ON* of the leaver, and draws down the piece *NM*, together with the ring *KLM*, which ring being drawn cloſe againſt the wheel *G*, ſtops the motion, or regulates it at pleaſure.

Ex. C.

299. *An engine for drawing water.* *A* the cog-wheel, 10 feet diameter; *B* its axis, running in the frame *FFFF*, and on the foot *Z*. *C* a trundle 3 feet diameter. *K* its axis 15 or 20 feet long, running in the ſtocks *G*, *G*. *D*, *D* two cranks of iron on oppoſite ſides of the axis, and 2 feet long. *OP*, *QR* two beams moving upon an axis in the frame *SSSS*. *PD*, *RD* two rods of wood or iron, reaching from the beams to the cranks, moveable about *R* and *P*; and turning round on the cranks *D*, *D*. *I*, *I* two rods of iron, fixed to two chains that go over the arches *O*, *Q*; and to two piſtons that work in the pumps *x*, *y*. *E* the tiller to which the horſes are yoaked; 1 2 3 4 the path in which the horſes go round. *H*, *H*, the ſurface of the earth. The wheel *A*, and trundle *C* are in a pit; the axis *K* under ground; and the cranks *D*, *D*, are in a pit.

When the horſes, walking in the ring 1 2 3 4, draw about the cog-wheel, by the tiller *E*; this turns the trundle *C*, with the

FIG. 299. cranks *D*, *D*; and the rod *PD* being drawn down, pulls down the end of the beam *P*; and raiſes the other end *O*, with the rod *Ix*; and draws the water out of the pump *x*. In the mean time, the other crank raiſes the rod *DR*, with the end *R* of the beam; and the other end *Q*, with the rod *I* deſcends; and the piſton goes down into the pump *y*. But as the wheel *A* goes about, the rod *RD* is pulled down, and *QI* riſes up, and draws water out of the pump *y*, whilſt *OI* and the piſton deſcends in the pump *x*. Thus whilſt one piſton goes up, the other goes down, and there is always one pump diſcharging water.

Inſtead of two cranks, one may have three or four cranks, at equal diſtances round the axis, and theſe will move three or four beams *QR*, and work three or four pumps. But beams of timber ſhould be put between every two working beams, *OP*, *QR*, for the axles to run in.

Ex. CI.

300. *AEL a twiſting mill to make thread or worſted.* *B* a cog-wheel 3 feet diameter, of 33 or 34 teeth. *C* the drum of 4, 6, or 8 rounds, going on the ſquare end of the axis of the cog-wheel. *D* a ſpur-wheel, 2 feet diameter, and 30 or 32 teeth; this is fixed to the reel *E*. The reel conſiſts of 4 long pieces of wood, 6 or 7 feet long, 3 of which are fixed in the cog-wheel *D*, and are alſo fixed to one another by croſs bars going through the axis of the reel; the fourth long piece of wood which compoſes the reel, is not fixed in the cog-wheel, but may be ſet nearer or further from the axis, by help of the pins 1, 1, 1. *F* a drum of 12 rounds, carried by the cog-wheel *B*; theſe rounds are fixed into the barrel *G*, of 1 foot and 6 or 8 inches diameter. *MNOP* a fixed frame 6 or 7 feet long, and 4 feet broad. 22, 22, &c. are whorles, carried round by the leathern belt *IKLIH*. Theſe whorles run in iron ſockets at the bottom of the frame, and are kept in their places by the ſnecks 3, 3, fixed to the upper ſide of the bottom part of the frame; upon the ſpindles of theſe whorles are put the bobbings, with the thread or worſted. The ſpindle and whorle is repreſented at *a*, the bobbing at *b*, the bobbing with the worſted on it at *c*. The length of the whorle and ſpindle is 10 or 11 inches, length of the bobbing 6 or 7 inches; diameter of the whorle where the belt runs, about an inch; diameter of the bobbing at top $1\frac{1}{2}$ inch; at the ſmalleſt part $\frac{2}{3}$ of an inch; theſe are for worſted. The whorles may be taken out of the ſnecks at pleaſure, and they are kept in theſe ſnecks by a feather put acroſs the ſlit through two holes. The bobbings

they uſe for thread are repreſented at *d*; *e* is a piece of lead which goes upon the top of the ſpindle to keep down the bobbing: *f*, *g* are two wires fixed in it, for the thread to run through, from the bobbing to the reel, the diameter of the whorle about half an inch. The number of ſnecks, ſpindles, and bobbings on one ſide of the engine is 20 or 24, that is 40 or 50 in all. 4, 4, *&c.* are wires in the upper part of the frame, for the thread to run through from the bobbings, the number of wires are equal to the number of ſpindles. Alſo in the horizontal beam *QR* are the ſame number of wires, 5, 5, 5, *&c.* to direct the thread to the reel. *n*, *n* are rollers for the edge of the belt to move over. 6, 6 are two hanks upon the reel. When the belt grows ſlack by ſtretching, the frame *MN* is drove back, by means of a wedge *S*, and ſo kept at a greater diſtance from the roller *G*. FIG. 300.

The trundle *C* may be taken off, and others of more or fewer rungs put on, as occaſion requires, by lifting the end *T*, of the axis of the keel, out of its ſocket; and the finer the thread, the fewer rungs it muſt have. The circumference of the reel *DE* for worſted is 4 feet, 4 inches; for thread is 5 feet, 5 inches.

When the thread or worſted is wound upon the bobbings, by the help of a wheel, they are put upon the ſpindles, as *c*; and then put within the belt *IKL* under the ſnecks 2, 2; then the handle *A* being turned, carries the cog-wheel *B* about, which drives the drum *F*, and the barrel *G*; the barrel *G* moves the belt in direction *IKL* about the frame *MN*, which reſting on the whorles 2, 2, moves them, and the whorles and bobbings very ſwiftly about. In the mean time, the drum *C*, is turned round by the axis of the cog-wheel *B*, and *C* carries about the ſpur-wheel *D* and the reel, with a ſlow motion. So the threads being put through the wires 4, 5, and fixt to the reel, theſe threads will be wrapt about the reel, and make the hanks 6, 6; as many as there are bobbings. When the hanks are of ſufficient bigneſs, they muſt be taken off the reel, which is done by pulling out the pins 1, 1, and then one ſide of the reel will fall in, and the hanks ſlackt, and may be taken off one after another, by lifting the end *V* of the axis out of its ſocket.

The double yarn, *&c.* is to be wound tapering on the bobbings, as at *c*, making it broadeſt at the low end, otherwiſe it will not come freely off the bobbings, without breaking.

The frame work conſiſts of perpendicular beams fixt in others lying horizontal, as deſcribed in the figure, the breadth from *A* to *V* being 9 or 10 feet. The lower part of the frame

FIG. 300. *MN* conſiſts of two elliptical pieces, cut out of boards, and ſet at about a hand's breadth diſtance one above the other, with pieces of wood between. In the lower (which is broader than the other) are the ſockets, in which the bottom part of the ſpindle of the whorles move; in the upper, the ſnecks are fixt. The part *OP*, in which are the wires, is an elliptical piece like the under ones, and fixt thereto by 4 perpendicular pieces or pillars of wood. All the reſt will be plain from the figure.

Ex. CII.

303. *AEKF is a clock.* The different forms and conſtructions of clocks are almoſt as various as the faces of thoſe that make them. The following is a common 8 days clock. *KF* is the moving part; *AE* the ſtriking part.

The work contained between 2 braſs plates is as follows; *F* the *firſt* or *great wheel* of 96 teeth; *G* the *ſecond wheel* of 60 teeth, its pinion *g* of 8 leaves; *H* the *third wheel* of 56 teeth, its pinion *h* of 8 leaves; *I* the *balance wheel* of 30 teeth, its pinion *i* of 7 leaves; and *K* the *balance*. Likewiſe *A* the *great wheel* of 78 teeth; *B* the *pin wheel* of 48 teeth, *b* its pinion of 8 leaves; *C* the *hoop wheel* of 48 teeth, *c* its pinion of 6 leaves; *D* the *warning wheel* of 48 teeth, and its pinion *d* of 6 leaves; *E* the *fly*, *e* its pinion of 6 leaves.

The ends *R*, *R*, of the *arbors* of the wheels *A*, *F* come thro' the face of the clock, and theſe arbors are fixt in the *barrels* *P*, *P*, of 6 or 7 inches circumference; and on theſe barrels, the therm *ſtrings Tt* are wound, which go round two *pullies* with the weights, that carry the wheels about. Theſe two barrels are moveable round about within the wheels, but are kept from turning back, by the *catch S* and its *ſpring*, and the *racket wheels* *Q* fixt to the barrel. The weights are wound up by help of the *winch* or handle 11. In the rim of the wheel *B* are 8 *pins*, which as the wheel goes round, thruſt back the end 5 of the *hammer O*, and when it goes off the pin, the *ſpring* 7 makes the hammer *O* ſtrike againſt the *bell N*.

302. 303. The wheel *C* has a *hoop* upon its rim, which is cut away in one place, to let the end 2 of the *detent* fall in. In the rim of the wheel *D*, there is a pin which ſtops againſt the end *x* of the arm *wx*, and hinders the wheels turning about.

On the axis *op* is fixt the two *pieces vs*, and the detent 1 2. On the axis *qr* is fixt two *pieces wx* and the *lifter* 3; and on the end *r* of that axis, which comes through the fore plate, is put the *lifter* 10, and pinned faſt on.

The *arbor* of the wheel *G* comes through the *face*; upon this arbor between the face and *fore plate*, is put the wheel *z* of 20 teeth, its arbor being hollow, and under the wheel is put the braſs ſpring *l*, with the concave ſide upward, this ſpring having a ſquare hole in it, to fit the ſhoulder of the arbor of *G*. The wheel *Z* of 40 teeth turns upon a fixt pin or axis, and is driven by the wheel *z*. The dial wheel *f* of 48 teeth, is put with its hollow *ſocket* upon the arbor or *ſocket* of *z*, then the face being put on, their ends come through it, and the *hour hand k* is put upon the ſquare end of *f*, and the *minute pointer W*, upon the end of *z*, the wheel *z* being thruſt down to bend the ſpring, and then a pin put in to keep it there; the pinion of *Z*, called the *pinion of report*, has 8 teeth, and drives the wheel *f* and the hour hand. Now the ſpring *l* keeps the wheel *z* pretty tight upon the axis of *G*, ſo that *G* will carry it about along with it. And if the minute pointer be thruſt about, it will force about the wheel *z*, and alſo *Z*, and likewiſe *f* with the hour pointer. FIG. 302. 303.

The arbor of the wheel *A* goes through the *back plate*; upon it, behind the plate is put the wheel *V* (or *pinion of report*) of 28 teeth, and pinned there. The double wheel *XY* is carried by *V*, and turns upon a pin fixt on the back of the plate. The wheel *X* has alſo 28 teeth, and the *count wheel Y* is divided into 11 parts of unequal lengths, according to the ſtrokes the clock is to ſtrike at every hour; part of this wheel is repreſented at *s*. A ſlender ſpring is put on with this wheel to keep it tight. This part may be made more ſimple, by leaving out the wheels *V*, *X*; and putting *Y* upon the axis of *A* inſtead of *V*; but it muſt be put on the contrary way.

The arbor of the balance wheel *I* comes through the fore plate, almoſt to the face, and through a hole in the face is put the hollow ſocket of the *ſecond pointer* 12; and this ſhows the ſeconds by a ſmall circle divided into 60 parts. And the face is alſo divided into two circles, ſhowing hours and minutes.

The *pendulum* hangs on the fixt piece of braſs *M*, by a button at top, and a thin piece of braſs going into a ſlit at *M*, and a flat piece of braſs goes into the *fork L*, ſo that if the pendulum moves, it muſt move the rod *KL*, and balance *K* along with it.

The *pallats* 8, 9 of the balance *K*, are ſo formed, that the under ſide of 8, and upper ſide of 9, where the teeth of the wheel *I* act, are poliſhed planes; and made ſloping, ſo that a tooth ſliding along the under ſide of the pallat 8, will force

the

FIG. 302. 303. the balance *K* to the left hand; and a tooth ſliding along the upper ſide of the pallat 9, will force it to the right.

The work is put together, by ſetting the teeth together that are markt in the wheel *B* and in the pinion *c*, and likewiſe in the wheel *C* and pinion *d*. Then the minute pointer is put on the arbor of *z*, mark to mark; and the hour pointer the ſame way on the arbor of *f*. And the wheels *z*, *Z*, *f* are ſet to one another according to their marks.

The weights hanging upon the wheels *A*, *F*; and the pendulum made to vibrate, the wheel *F* drives *G*, which drives *H*, which drives *I*; then whilſt the pendulum vibrates to the right, a tooth ſlips off the pallat 9, and in its return to the left, a tooth ſlips off the pallat 8, then on the right another goes off 9, and ſo on alternately; and the weight cauſing the teeth to act againſt the pallats of the balance, keeps the pendulum in motion; and the wheel *I* goes round in a minute.

As the wheel *G* goes round it carries about *z*, with the minute pointer once round in an hour; *z* drives *Z*, which drives *f* once round in 12 hours. Whilſt the wheel *z* goes round, the pin *m* raiſes the lifter 10, which lifts up the piece 3, and the arm *wx*; the piece 3 raiſes the detent 1 2, together with *vs*; the end 2 of the detent being raiſed above the hoop, the wheel *C* moves about, and by the oblique figure the end of the detent 2, it raiſes the end of the detent higher, and alſo raiſes *s* out of the notch of the count weel. Then the wheel *D* turns round, till the pin in the rim ſtops at the end *x*, which hinders the motion. But as the wheel *z* goes further about, the lifter 10 falls down off the pin, together with the piece *wx*, and latch 3, which ſuffers the wheel *D*, and the reſt to turn round; and the pin wheel cauſes the hammer to ſtrike ſo often, till the end *s* falls into a notch of the count wheel, and then the detent 2 falls into the vacancy in the hoop, and locks the work; which continues ſo till the next hour, that the piece 10 is raiſed again; and then ſhe ſtrikes as before; the wheel *C* goes round every ſtroke of the clock; but ſhe ſtrikes 1 more every ſucceeding hour, becauſe the teeth between the notches are made longer and longer in the count wheel; and it turns round once in 12 hours.

General rules in all clocks.

In the ſtriking part, the pin wheel being divided by the pinion of the hoop wheel, the quotient ſhews the number of pins in the pin wheel.

If 78 be divided by the number of pins, the quotient ſhews the revolutions that the pin wheel makes for one revolution of the count wheel. FIG. 302. 303.

The hoop wheel divided by the pinion of the warning wheel, muſt be a whole number.

In the moving part, the *train* is the number of beats the clock makes in an hour; which is 3600, if ſhe beats ſeconds: in this caſe the balance wheel muſt have 30 teeth.

If *G* turns round once in an hour and ſhews minutes; then the quotient of *G* divided by the pinion of *H*, multiplied by the quotient of *H* divided by the pinion of *I*, and that multiplied by twice the teeth in *I*, muſt be equal to the train. And if ſhe beats ſeconds, then the product of the two quotients muſt be 60.

If alſo *G* ſhows the hours, then the quotient of *f* divided by the pinion of *Z*, multiplied by the quotient of *Z* divided by *z*, muſt be 12.

From the great wheel to the balance, the wheels drive the pinions; but to the dial wheel, the pinions drive the wheels; the former quickens, the latter leſſens the motion.

Any wheel being divided by the pinion that works in it, ſhews how many turns that pinion hath to one turn of the wheel. As if the pinion be 5 and the wheel 60, it is ſet down thus,

5) 60 (12 times. Or thus $\frac{60}{5} = 12$ times.

The teeth of ſeveral wheels and pinions that work in one another, are ſet down thus,

4) $\underline{36}$ (9 times
8) $\underline{80}$ (10 times
6) $\underline{54}$ (9 times
5) 40 (8 times

Or thus $\frac{36}{4} \times \frac{80}{8} \times \frac{54}{6} \times \frac{40}{5}$.

In the former way, the number on the left hand of any wheel is the pinion that it drives; and the number over it is the pinion on the ſame axis. In the latter way the ſeveral fractional quanties repreſent the quotient.

Any wheel and the pinion it drives, will have the ſame motion with another wheel and pinion, when their quotients are equal.

FIG. 302. 303. equal. Thus a wheel of 36 drives a pinion of 4, all the ſame as a wheel of 45 does a pinion of 5; or a wheel of 90 a pinion of 10.

In any motion you may uſe one wheel and one pinion, or elſe ſeveral wheels and ſeveral pinions, provided they all give the ſame motion. Therefore when a number is too big to be cut in one wheel, you may divide it into two or more quotients.

In a wheel and pinion that work in one another, their diameters muſt be as the number of teeth in each. And the diameter muſt be meaſured, not to the extremity, but to the middle of the tooth, or where they act.

303. The excellency of clock-work, conſiſts in forming the teeth truly, and to fit the notches exactly without ſhaking, and to play freely; the teeth muſt be cut into the form of cycloids, which reſembles the ſhape of a bay leaf.

A clock goes exacter as the pendulum is longer, and the bob pretty heavy, and to make but ſmall vibrations; and for more exactneſs, to play between two cycloidal cheeks; and the longer the arms $K8$, $K9$, the eaſier the clock goes. The length of a ſecond pendulum is $39\frac{2}{10}$ inches. See the theory of pendulums in Prop. XL. XLI. LVIII.

The pallats 8, 9, are here formed after the common way: but there is another way of forming them. From the center of motion θ, deſcribe two ſmall arches $\alpha\beta$, and $\delta\varepsilon$. Theſe ſmall lines or planes $\alpha\beta$, and $\delta\varepsilon$, and alſo the working ſide of the tooth $\lambda\mu$, muſt all range to κ the center of the balance wheel. And the ends of the pallats $\alpha\gamma$, and $\delta\eta$, muſt range a little to the right hand of the center θ. Then the teeth of the balance wheel will fall alternately on the ſides $\alpha\beta$, and $\delta\varepsilon$. And any tooth, whilſt it acts againſt $\alpha\beta$, or $\delta\varepsilon$, will have no effect in moving the pendulum; but lies *dead*, till it makes its *eſcape* off the angle α or δ; and then in moving along the plane $\alpha\gamma$, or $\delta\eta$, it forces the pendulum to the right or left.

But the conſtruction will be better thus; take $\theta\tau$, $\theta\phi$ each equal to $\frac{1}{3}\alpha\delta$. From the center τ deſcribe the arches $\alpha\beta$, $\delta\varepsilon$; and let the ends $\alpha\gamma$, $\delta\eta$, range to ϕ. Or perhaps it may anſwer the end as well, to deſcribe $\alpha\beta$, and $\delta\varepsilon$, from the center θ; and let the acting ſide of the tooth range (not to κ, but) to υ the outſide of the circle deſcribed with the radius $\kappa\upsilon = \frac{1}{3}\kappa\mu$.

The inconvenience of any of theſe conſtructions, is, that the pallats are too thick, and can hardly find room to fall in between the teeth of the balance wheel.

Ex.

Ex. CIII.

FIG. 304.

ABC is a *cutting engine* to cut the teeth of clock wheels. *AC* an iron plate $2\frac{1}{2}$ feet long, and 3 or 4 inches broad. *EE* another plate fixt 4 or 5 inches lower. *G* a *ſlider*, ſliding along a groove in the end *C*: this is made of ſeveral plates of iron fixt to one another with ſcrews, and fitting cloſely to the edges of the plate, and to the ſides of the groove, and likewiſe to the upper and under ſide of the plate; this is to cauſe it to move truely along the groove when forced forward or backward, by the ſcrew at *I* and its handle; for this ſcrews through *C*, and turns round in a collar in the end *G*. The end of this ſlider turns up perpendicular; to this is fixt the part *F* by a pin *K*, which goes ſquare into this part, and through a round hole in *F*; ſo that the part *F* can turn about the ſcrew pin *K*, and may be fixt by turning the nut 2 with the key 9, which nut ſcrews upon the end of the pin.

B is a braſs wheel of 96 teeth, carrying the pinion *D* of 12 leaves; theſe move between the cheeks *LL*, *MM*; which are joined by the croſs bars *N* and *P*; theſe cheeks and their machinery turn round on the axis *LM*, in the part *F*. *f* is the cutting wheel, whoſe edge is nothing but a file to cut the teeth, as it goes about; this goes upon the arbor of the pinion *D*. There are a great number of theſe cutting wheels, of different ſhape and bigneſs; which may be taken off the arbor and others put on; theſe parts are deſcribed at *a*, *b*, *c*, *d*; *a* the pinion on its arbor, *b* the cutting wheel going upon the arbor which is octagonal, and fits it exactly, having the ſides marked that are put to each other, *c* a hollow piece which goes on the ſame axis; and the nut *d* ſcrews on the end of the arbor, to keep *c* and the wheel *b* faſt on. The ends of the arbor are hardened ſteel, and pointed; and this arbor runs between the cheeks *LM*, through which cheeks there goes 2 ſcrews, with holes to receive the points of the arbor; and theſe ſcrews are ſet to a proper diſtance, by ſcrewing them in or out by help of a key *p*, going on ſquare upon the end, and then the ſcrews are lockt there, by the nuts *O*, *O*. *rs* is a ſpring, fixt with one end to the under-ſide of the croſs-bar *N*, and the other end *s* lying upon the plate *AI*, and this ſpring raiſes the part *LLMM*, when the notch is cut. *tu* is a ſcrew pin going through the bar *P*, its end *u* reſts upon the plate *AI*, and hinders the wheels from deſcending lower. 1, 1, are two ſcrew pins, which ſcrew through *LM*, by help of the key *p*, and go with their points

FIG. 304. points into *F*, which has two holes to receive them; theſe pins are lockt to *L*, *M* by turning the two nuts, which alſo ſcrew upon the pins. Theſe ſcrew pins, nuts, and cheeks, all turn round together in the holes in *F*.

H is the *dividing plate*, being a braſs circular plate 15 or 16 inches diameter. This plate is fixt to a hollow braſs axle *Q* an inch in diameter; and this axle goes through the two plates *AI*, *EE*; and both the wheel and its axle turn about together; the lower plate cannot be ſeen, but is repreſented at *R*. Near the edge of this plate, there are 24 concentric circles, each divided by points into a certain number of equal parts, *viz.* 366, 365, 360, 118, 100, 96, 92, 90, 88, 84, 80, 78, 76, 72, 70, 68, 64, 62, 60, 58, 56, 54, 52, 48. The uſe of theſe is to divide a revolution into any number of equal parts, according to theſe different circles.

ei is an arbor going through the hollow axle *QR*, with the ſhoulder *h* againſt the top of that axis, then the nut *n* is ſcrewed upon the end *i*, to keep it faſt. *m* is the wheel to be cut into teeth; there is a hole made in the center, juſt to fit the part *ge*, which being put on, and the piece *l* above it, they are then ſcrewed hard down with the nut *k* going on the end *e*. Then if the wheel *H* be turned round, it carries about with it the wheel *m*. There are ſeveral arbors *ei*, for fitting different wheels *m*.

wy is a moveable index; it turns about a nail, as a center in the end *w*, there being a ſlit in it, to let the bottom of the ſcrew *x* paſs through as it moves. *y* is moveable back and forward, and may be fixt any way by the two ſcrews. *z* is a ſteel point, which moves along the circumference of any circle you require, from one point to another. *T* is the winch to turn the wheel *B*; 2 is the handle to pull down the machinery near the plate.

To uſe this machine. An arbor *ei* proper for the wheel *m*, which is to be cut, being put through the axis *Q*, and ſcrewed faſt, as appears at *R*; and then the wheel *m* and the parts *k*, *l* put on above *Q*, and ſcrewed faſt. Looſen the ſcrew *x*; and, moving the index *wy* till the ſteel point *z* fall in the circle, containing the ſame number of parts, the wheel *m* is to be divided into, there ſcrew it faſt with the ſcrew *x*. Then putting on the cutting wheel *f* proper for the work, turn the handle and ſcrew *I*, and drive the machinery with the wheel *f* towards *Q*, till the edge of *f* lie juſt over the edge of the wheel *m* to be cut; there fix it by the handle *V*; and turn the wheel *H* till *z* falls into ſome point of the circle; then take hold of the handle *S*, and pull it down, till *f* falls againſt the edge of *m*; then hold-

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ing it there with one hand, turn the winch *T* with the other; which carries about *B*, and this drives *D* with the cutter *f*, and this motion cuts a notch in the edge of *m*, and when it is deep enough, the pin *tu* (properly ſet) ſtops at the plate *AC*, and hinders it from going further. Then let go *S*, and the ſpring *rs* raiſes up the wheels, &c. This done, pull about the wheel *H*, till *z* fall in the next point of diviſion; then draw down *S*, and turn the machine as before, till you have made another cut deep enough. And thus you muſt proceed, till *z* has gone thro' all the points of diviſion in the circle, and then your wheel is cut into its proper number of teeth. FIG. 304.

When the number of teeth wanted to be cut anſwers to none of the circles, take ſuch a circle as can be divided by your number, and if the quotient be 2, 3, 4, &c. then you muſt ſet *z* to every 2d, 3d, 4th point, &c. ſkipping the reſt. As if you want 21 teeth, take the circle 84, which divided by 21 gives 4; ſo that you muſt ſet *z* to every 4th tooth only, and ſo cut it.

A crown wheel may be cut the ſame way; but then the center of the wheel *f* muſt be brought over the edge of the wheel to be cut, and there fixed. Alſo oblique teeth may be cut in a wheel after the ſame manner; but you muſt firſt eaſe the ſcrew *K*, and then turn the cutting frame about *K* as an axis, till the cutter *f* have a proper degree of obliquity, and there ſcrew faſt the pin *K*, by the nut 2, and proceed as before.

After the teeth are cut with this engine, they are ſtill to be wrought into their proper form, with files ſuitable for the buſineſs; and this the workman muſt do by hand.

Ex. CIV.

EH is a *glazier's vice*, for drawing window lead. *PG*, *QH* two axles, running in the frame *KL*, *ML*. *C*, *D* two wheels of iron caſe-hardened, $1\frac{1}{2}$ inch broad, and of the thickneſs of a pane of glaſs; theſe wheels are fixed to the axles, and run very near one another, not being above $\frac{1}{16}$ of an inch diſtant; acroſs their edges are ſeveral nicks cut, the better to draw the lead through. *E*, *F*, two pinions of 12 leaves each, turning one another, and going upon the ends of the axles, which are ſquare, and kept faſt there by the nuts *P*, *Q*, which are ſcrewed faſt on with a key. *A*, *B* two cheeks of iron, caſe-hardened, and fixed on each ſide to the frame with ſcrews; theſe are cut with an opening where the two wheels meet, and ſet ſo near the wheels, as to leave a ſpace equal to the thickneſs of the lead; ſo that between 305.

the

FIG. 305. the wheels and the cheeks there is left a hole, of the form repreſented at *N*, which is the ſhape of the lead when cut through. The frame *KLML* is held together by croſs bars going through the ſides and ſcrewed on: and a cover is put over the machine to keep out duſt; and it is ſcrewed faſt down to a bench, by ſcrew nails *LL*.

When it is uſed, the lead to be drawn is firſt caſt in moulds, into pieces a foot long, with a gutter on each ſide. Take one of theſe pieces, and ſharpen one end a little with a knife, and put it into the hole between the wheels; then turning the handle *I*, the lead will be drawn through, of the form deſigned.

Ex. CV.

306. *AC* a *water-mill* for grinding corn, without either trundle or cog-wheel. *BC* is the arbor, or axis of the mill; this is a cylindrical piece of wood, about two feet diameter; *GHIKLMN* is a leaf or wing of wood, whoſe breadth is about the radius of the arbor; this runs ſpiral-wiſe round the arbor from bottom to top, aſcending in an angle of about 35 degrees; it muſt every where ſtand upright on the ſurface of the arbor. Inſtead of one you may uſe two of theſe ſpiral leaves, eſpecially if they be narrow. This arbor and its ſpiral leaf, turns round upon a pivot *P* at the bottom; and at the top *B*, it has a ſpindle which goes through a plank, and is fixed to the upper mill-ſtone *D*, which turns round with it; ſo that the arbor has little or no friction. *QRST* is a hollow cylinder, made of ſtone or brick, to encloſe the arbor and its leaf; and whoſe inſide is walled as near as poſſible, juſt to ſuffer the leaf to turn round without touching; ſo that no water can eſcape between the leaf and the wall; and conſequently it can only run down the declivity of the leaf; its top is repreſented by the circle *QBTV*. *RWS* is an arch to let the water out at the bottom, to run away; and big enough to go through to repair the engine. *F* is the trough that brings the water; *D*, *E* the two mill-ſtones. *A* the hopper and ſhoe. The arbor and its leaf may be cut altogether out of the ſolid trunk of a tree; or elſe the leaf may be made of pieces of boards, nailed to ſeveral ſupporters of wood, which are to be let every where into holes made in the body of the arbor, ſo as they may ſtand perpendicular to its ſurface; and all ſet in a ſpiral. And the ſpiral is made on this conſideration; that for every 10 inches in the circumference of the axis, you muſt riſe 7 inches in the length. But at the top *G*, it will be better

to riſe faſter, ſo as to have its ſurface almoſt perpendicular to the ſtream. FIG. 306.

When the mill is to go, the corn is put into the hopper at *A*, which runs down the ſhoe, through the mill-ſtone *D*. And the ſpout *F* being opened, the water falls upon the oblique leaf *GHIK*, and by its force turns the axis *BC* about, and with it the ſtone *D*, and grinds the corn.

Ex. CVI.

307. *DBF* is the *arch of a bridge*, which ſhall ſuſtain itſelf, and all the parts of it in equilibrio. Such an arch will be ſtronger than any other, becauſe an arch that can ſuſtain itſelf, will more eaſily ſuſtain an additional weight, than an arch that cannot ſuſtain itſelf, but only by the coheſion of the mortar. This arch *DBF* is a ſemicircle, whoſe center is *R*, and vertex *B*; and the wall *ATta* muſt be ſo built, that the height *AT*, in any place *A*, muſt be as the cube of the ſecant of the arch *BA*, which will cauſe it to run upwards towards *D*, in the form of the curve *tST*. But as this form is not commodious for a bridge, the conſtruction may be performed thus. In any place of the arch, as *A* let the ſuperincumbent part *AT* be built of heavier materials, than at *B*; in proportion of the cube of the ſecant of the arch *BA*; for the parts near *B*; but in ſomething leſs proportion in the parts towards *A* and *D*. And the right line *GSg* being drawn, will nearly terminate the top of the wall. But as materials cannot well be procured for this purpoſe, the following way may be uſed.

308. With the radius *BR* deſcribe the arch *DBd*, of 90 degrees; *DB*, *Bd* being each 45. And if *BR* conſiſt of 100 parts, make *BS*, 16. Draw the right line *GSg*, perpendicular to *SBR*: and the arch *DBd* ſhall ſupport the wall *DGgd* in equilibrio in all its parts. If the arches *DB* and *Bd* be made each 60 degrees, and the height *BS*, 7 parts; and the right line *GSg* drawn; then the arch *DBd* will equally ſupport the wall *DGgd* in all parts: but then the materials made uſe of about the places *a*, *a*, ought to be only about half the weight of thoſe at *B* and *D*. And theſe are the principal caſes in which a circle is ſerviceable, for making an arch ſtand in equilibrio.

309. Another equilibrial arch is from the catenary. Make the *latus rectum BS*, 100 equal parts; *BR*, *AR*, *RF* each 159; deſcribe the catenary *ABF*, through the points *A*, *B*, and *F*. Then *ABF* will be an arch which will ſupport the wall *AGgf* in equilibrio, in every point of it. The fault of this arch is, that by reaſon

of

FIG. 309. of the height *BS*, there is too much weight of wall upon it, which will endanger the ſinking the piers, except the foundation be very good; and it likewiſe raiſes the bridge too high.

310. Another arch of equilibration is this; make *SB*, *BR*, *AR* of any lengths at pleaſure; draw the right line *GS* parallel to the horizon; and to the aſſymtote *GS*, draw the logarithmic curve *AB*: which may be done thus; draw *AG* perpendicular to *GS*; divide *SG* into any number of equal parts, and as many points of diviſion as you have, find ſo many mean proportionals between *SB* and *GA*; ſet theſe from the reſpective points in *SG* downwards, in lines drawn through theſe points parallel to *SR*; and theſe will give ſo many points, through which the curve *BA* is to be drawn; and the curve *bF* is drawn the ſame way; between theſe, the pier *BD* is placed with a tower upon it. The only fault this arch has, is, that the water way is diminiſhed by the pier *BD*; and as many arches, ſo many ſupernumerary piers there will be.

311. I ſhall now ſhew how to deſcribe an arch clear of all theſe inconveniencies. Make *BR*, *AR*, *RF*, each equal to 30 feet; *BS*, $3\frac{1}{2}$ feet. Draw *AG*, *Fg* parallel to *RS*. Divide *SG*, *Sg* into 30 equal parts or 30 feet; through all the points of diviſion, draw lines parallel to *SR*, as *TC*. Then upon each of theſe lines, ſet off from *SG* downwards; the number of feet you find in the following table, reſpectively, as *TC*; then *C* will be in the arch. Do the ſame for the ſide *Sg*. Then the curve *FBCA* drawn through all theſe points *C*, will be the arch required. The curve is eaſily drawn through theſe points, by help of a bow held to every three points; or rather to four or five points at once; which may eaſily be done by two or three perſons holding it.

FIG. 311.

value of *ST* in feet.	value of *TC* in feet and dec. parts.	value of *ST*.	value of *TC*.	value of *ST*.	value of *TC*.
0	3·500				
1	3·517	11	5·754	21	14.014
2	3·568	12	6.231	22	15 417
3	3·653	13	6.769	23	16.970
4	3·774	14	7·372	24	18.687
5	3·931	15	8.047	25	20.585
6	4·127	16	8.799	26	22.682
7	4·362	17	9.636	27	24·999
8	4·639	18	10.567	28	27·557
9	4 961	19	11.600	29	30 381
10	5·332	20	12·745	30	33·500

If the thickneſs of an arch at top, *BS* be ſuppoſed to be 3 feet, 4 feet, or 5 feet, *&c.* It will require a different curve to be conſtructed; but this ſeems to be ſtrong enough for the bigneſs of the arch; eſpecially, if built of ſtrong, ſound ſtone. Here 3½ feet is aſſigned for the thickneſs of the arch; but it muſt be made 2 or 3 inches leſs, on the account of the parapet wall, for this adds weight to the whole. Alſo if the top *GS* is not exactly horizontal, but is 2 or 3 feet lower at *G* than at *S*; the thickneſs *BS* ought to be 2 or 3 inches leſs upon that account; or if higher at *G*, two or three inches more: but theſe niceties make no ſenſible difference in practice.

This curve differs from the catenary (in fig. 309.) For at the vertex *B* it is leſs curve than the catenary; and towards *A* it is more curve. The curvature at *B* in this arch is very near that of a circle, whoſe radius is *BR*. And the curvature increaſes from the vertex *B*, and is at leaſt about *C*.

At the points *A*, *F*, where the arch ſprings, it riſes at an angle of 73° : 1′, above the horizon.

If an arch is required to be either greater or leſs than this, it is no more than taking any other equal parts inſtead of feet; and ſetting off all the lines by theſe equal parts.

In this ſcheme I have drawn a circle to ſhew the difference between that and this arch. The like I have done in fig. 309, and 310. Whence it appears, that a circle circumſcribes all theſe arches of equilibration; and conſequently a circle is too curve at the lower parts, or at the hanch of the arch.

FIG. 311. If any architects or builders of churches or bridges, ſhall pleaſe to make uſe of this curve here conſtructed (fig. 311.) for the form of an arch; they will find it the ſtrongeſt arch poſſible to be made, for theſe given dimenſions. And where many thouſand pounds are laid out in building a ſingle bridge, it is certainly worth the pains to ſeek after the form of an arch, which ſhall be the ſtrongeſt poſſible, for ſupporting ſo great a weight. And it is very ſurprizing that no body has attempted it. Inſtead of that, all people have contented themſelves with conſtructing circular arches; not knowing that different preſſures againſt the arch, in different places, require different curvatures, which does not anſwer in a circle where the curvature is all alike. A circle, it is true, is very eaſily deſcribed, and that may be one reaſon for making uſe of it: but ſurely, the deſcription of the curve here given, is very eaſy by the foregoing table, and can create no difficulty at all. If there be any difficulty in the practice, it is only in cutting the ſtones, of a true curvature, to fit the arch exactly in all places; but this is eaſily managed with a little care, by taking proper dimenſions; obſerving that every joint muſt be perpendicular to the curve in that point.

A circle, or any other curve, where the curvature is not properly adapted to the weight ſuſtained, is not capable of ſuſtaining ſo vaſt a weight; but muſt in time give way, and fall to ruin; except the mortar happen to be ſo ſtrong as to keep it together. On the contrary, the arch here deſcribed, ſuſtaining every where a quantity of preſſure proportional to its ſtrength, will never give way, ſo long as the piers, which are its baſes, ſtand good; but by virtue of its figure, will ſtand firm and unſhaken, as long as the materials the arch is made of, will laſt.

As to the piers, their thickneſs may be $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{5}$ the width of the arch, according to the firmneſs of the ground they are to ſtand on. They muſt be conſiderably broader than the bridge, reaching out on each ſide into the water, being built with ſharp edges to divide the ſtream. At the bottom they muſt be well fenced with ſterlings for their ſecurity. The outermoſt pier muſt be built far backwards, to ſuſtain the oblique preſſure of the arch, which has nothing elſe to butt againſt; otherwiſe the pier or buttreſs will yield to the preſſure of the arch, and the arch will break.

Another conſtruction.

In the former conſtruction, I made the height *BS* to be only $3\frac{1}{2}$ feet. But as that may be reckoned too weak for an arch of 60 feet wide, like Weſtminſter bridge, where the height above the arches is 8 or 10 feet. Therefore I have

here given a new table for conſtructing the arch of a ſtrong FIG. 311.
bridge, calculated upon the ſame principles as the former, being 7 feet, all the other dimenſions remaining the ſame. This arch riſes from the pier at an angle of 70° 20′. In the former I ſet off all the points of the curve from the line *SG*; in this I ſet them off from the line *BEE*, which is a tangent to the top of the arch at *B*.

The conſtruction is thus. Having drawn the line *EBH* through the top *B* of the arch parallel to the baſe *AF*, take from the table col. 1 any length, and ſet it from *B* in the line *BE*, as to *L*, and from *R* to *l*, and draw the line *LI*. Then from col. 2. of the table, take the correſpondent length of *LC*, and ſet it from *L* to *C*, in the line *LI*; then *C* is a point in the curve. And thus all the other points of the arch muſt be found, and then a curve drawn regularly through them all, gives the form of the arch.

The TABLE.

value of *BL* in feet.	value of *LC* in feet.	*BL*	*LC*	*BL*	*LC*
1	0.021	16	6.251	25½	19.252
2	.086	17	7.173	26	20.263
3	.194	18	8.183	26½	21.316
4	.346	19	9.285	27	22.412
5	.543	20	10.488	27½	23.553
6	.787	20½	11.129	28	24.741
7	1.078	21	11.798	28½	25.978
8	1.419	21½	12.495	29	27.266
9	1.811	22	13.223	29½	28.605
10	2.258	22½	13.981	30	30.000
11	2.761	23	14.772		
12	3.324	23½	15.596		
13	3.951	24	16.455		
14	4.644	24½	17.349		
15	5.409	25	18.281		

Ex. CVII.

QCF is the *weighing engine*, at the turnpikes, for weighing 312.
road waggons. *CD* is a ſtrong beam of wood moving about the center *I*. *EF* a ſteel yard, moveable about the center *H*, and

FIG. 312. and ſuſpended at *D*, by the iron hook *DH*. *PA* ſeveral iron chains ſuſpended at a hook, moveable about the center *P*; *PH* is about 3 or 4 inches, *HF* about 10 or 12 feet. The 4 chains at *A* are to put round the waggon. *F* a leaden weight fixed at the end *F*, whoſe weight is about $1\frac{2}{3}$ hundred weight. *G* a moveable weight of $\frac{5}{6}$ of a hundred weight, this is moved along the graduated beam *HF*, at pleaſure. *KNL* a ſcaffold to walk on. *CS* is a chain hanging at *C*, and fixed to the braſs pulley *S*. Round this pulley goes the rope *MSR*, whoſe end *M* is fixed to the croſs bar *QQ*, of the frame *QQTT*. In this frame, the wheels and axles 1, 2, 3 move round; being turned by the handle *B*, fixed to an iron wheel or fly. Theſe wheels and trundles are iron, the trundles contain 11 teeth, the wheels about 60. The rope *R* is wound about the wooden axle 3, being 5 or 6 inches diameter. At the end of the axle oppoſite to *B*, is another handle to be uſed upon occaſion. The frame *QT* is fixed faſt in the ground, that it may not be pulled up. The beam *CD* and ſteelyard *EF*, move between the cheeks *KZ* and *NX*, which ſerve to guide them, and likewiſe ſtrengthen the frame they move in, which frame is tied together with ſeveral braces, as *NO*, *VO*, *&c.*

When any waggon is to be weighed, the 4 chains *A* are hooked round it, and a man turns the handle *B*; which, by turning the wheels, winds the rope about the axis 3, which pulls down the end *C*, which raiſes the end *D*, of the leaver *CD*. The end *D* raiſes the ſteelyard *EF*, with the chains *A*, and the waggon; and a man upon the ſcaffold *NL*, moves the weight *G* till it be in equilibrio; and the diviſions of the beam ſhew how much the waggon is above 60 hundred weight.

In ſome engines the beam *CD* is wanting; inſtead of which there are two blocks and pullies, the upper one faſtened to a croſs beam *ZX*; the lower is hooked to the piece *DH*; and the rope goes from the top block, to the axis 3; but in this caſe, the axis of the wheels are parallel to the ſide of the machine, and not perpendicular, as they are in the former; and then there is but one wheel and pinion, each of iron, the wheel of 110 or 120 teeth, and pinion 11.

Laſtly, in ſome machines that likewiſe want the beam *CD*; the wheels and axles 1, 2, 3 are placed in the top of the machine, above *ZX*; where being turned round, they raiſe the beam *EF*, either by a rope going from it, or by blocks and pullies.

Ex.

Ex. CVIII.

FIG. 313.

HRK is a *large organ.* *IIH* the *ſound board*; this is compoſed of two boards; the *upper board*, or *cover HHH*; and the *under one III*, which is far thicker than the upper one. Each of theſe is made of ſeveral planks laid edge-ways together, and joined very cloſe. In the under ſide of the under board, there are ſeveral channels made, running in direction *LL*, *MM*, *&c.* continued ſo far, as is the number of ſtops in the organ; and coming almoſt to the edge *HK*. Theſe channels are covered over very cloſe, with leather or parchment, all the way, except a hole which is commonly at the fore end next *HK*, upon which a valve or puff is placed. Theſe channels are called *partitions.* When this flap or valve is ſhut, it keeps out the air, and admits it when open. On the upper ſide of the under board, there are likewiſe cut ſeveral broad, ſquare gutters, or channels, lying croſs the former, but not ſo deep as to reach them; theſe lie in direction *LN*, *PQ*, *&c.* And to fit theſe channels, there are as many wooden *ſliders* or *regiſters*, *f*, *f*, *f*, *&c.* running the whole length; and theſe may be drawn in or out, at pleaſure. The number of theſe is the ſame as the number of ſtops in the organ.

IKKK the *wind cheſt*; this is a ſquare box, fixed cloſe to the under ſide of the under board, and made air tight, ſo that no air can get out, but what goes through the valves, along the partitions.

V, *V* are the *valves* or *puffs* which open into the wind cheſt; and are all encloſed into it, and may be placed in any part of it, as occaſion requires. One of theſe valves with the ſpring that ſhuts it, and wire that opens it, is repreſented apart, on the left hand.

C, *D*, *E*, *F*, *&c.* are the *keys* on which the fingers are laid, when the organ plays. Theſe keys lie over the horizontal bar of wood *W*; in which are ſtuck as many wire pins *z*, *z*, on which the keys are put; and the keys move up and down upon this bar, as a center. 3 is another bar, againſt which the keys fall when put down; on this alſo are ſeveral wires, going through the keys to guide them; and on this bar a liſt is faſtened, to hinder the knocking of the keys againſt it.

Now the keys are made to communicate with the valves ſeveral ways, as I ſhall now deſcribe. *s*, *s*, *s*, are the *key rollers*, moving on the pivots *t*, *t*; theſe rollers lie horizontally one above another, and one at the end of another; of ſuch a length, as to reach from the valve to the key. *a*, *a*, *a*, arms

or

FIG. 313. or leavers fixed to the key rollers; *w*, *w* the *valve wires* fixed to the arms *a*, *a*, and to the valves *V*, and going through the holes *b*, *b*, in the bottom of the wind cheſt. *b*, *b*, *b* arms fixed likewiſe to the key rollers. *d*, *d*, *d* the *key wires*, fixed to the arms *b*, *b*; and to the keys *C*, *D*, *E*. Now putting down the end of any of the keys *C*, *D*, *E*; it pulls down the arm *b*, by the wire *d*, which turns the roller *s* about, with the arm *a*, which pulls down the wire *w*, which opens the valve; which is ſhut by the ſpring, as ſoon as the key is let go. In this conſtruction, there muſt be a worm ſpring faſtened to the key, and to the bar *W*, on the further ſide, to keep the end 5 of the key down.

Another method of opening the valves is this. *xy*, *xy*, are ſlender leavers moveable upon the centers 1, 1. 5*x*, 5*x* are wires going from the far ends of the keys, to the ends *x* of the leavers. *yV*, *yV*, other wires reaching from the ends *y*, of the leavers, through the holes *b*, to the valves *V*. So that putting down the key, *C*, *D*, *&c.* raiſes the end 5, which thruſts up the end *x* of the leaver, by the wire 5*x*; this depreſſes the end *y* of the leaver; which pulls down the wire *yV*, and opens the valve *V*.

A third way of opening the valves is this. At the end of the key 6, is a leaver 8, 9, moving upon the center 7. This with the key makes a compound leaver. From the end 9, there is a wire goes to the valve. Now putting down the end 6 of the key, raiſes the end 8, which depreſſes the end 9 of the leaver 8 9, and pulls down the wire, and opens the valve. I have only drawn one of theſe in the ſcheme, and but a few of the others, to avoid confuſion.

R, *R* are the *rollers* to move the ſliders, by help of the arms *cf*, *cf*, which are fixed horizontally in theſe rollers. *ke*, *ke* are leavers alſo fixed in the rollers. *le*, *le* are the handles, which lie horizontally, and paſs through the holes *l*, *l*, and are faſtened to the leaver *ke*, being moveable about a joint at *e*.

Now any handle *lp* being drawn out, pulls the end *e* towards *l*, which turns *Rk* about, along with the arm *cf*; and the end *f* pulls out the ſlider *fg*. And when *p* is thruſt in, the arm *cf* likewiſe thruſts in the ſlider *fg*.

Upon all the ſeveral rows of holes which appear on the top of the upper board, are ſet upright ſo many rows of pipes; *X* is a *flute pipe* of wood, *Z* a *flute pipe* of metal, *Y* a *trumpet pipe* of metal. The pipes paſs through holes made in boards, placed above the upper board, to keep them from falling.

FIG. 313.

The pipes are made to communicate with the wind cheſt after this manner. When any ſlider *fg* is drawn out, holes are bored through the upper board, through the ſlider, and through the under board, into the partition below: ſo that any pipes, placed upon theſe holes, will then communicate with the partition; which, by its valve, communicates with the wind cheſt. But when the ſlider is thruſt in, the holes in the ſlider do not ſtand againſt the holes, in the upper and under boards; and the communication is ſtopt, ſo that no wind can get to the pipe.

qT, *qT* are the *bellows*, which muſt be two at leaſt. *q*, *q* the *wings*; *O*, *O* the *handles*, moving upon the fixt axes *nn*, *nn*. Each of theſe bellows conſiſts of two boards; the under board is fixt immoveable. In this there is a *valve r* opening inwards, and a tube leading to it, called the *conveying tube*. There is alſo a hole in this under board, from which a tube leads to the *portvent*, which is a ſquare tube 24, riſing upwards, and is inſerted into the under ſide of the wind cheſt at 2. And in the tube leading to the portvent, there is a valve which opens towards the portvent; which ſuffers the air to go up the portvent, but none to return. All the bellows are conſtructed after the ſame manner. Now the handle *O* being put down, raiſes the upper board *T*, and the air enters through the valve *r*; and when the handle is let go, the weight of the upper board *T*, (which carries 3 or 4 lb. to every ſquare foot) continually deſcending, drives the air through the portvent to the ſound board. And as one pair of bellows, at leaſt, is always deſcending, ſince they work alternately, there will be a conſtant blaſt through the portvent.

In chamber organs, there is but one pair of bellows, which conſiſts of 3 boards, in nature of a ſmith's bellows; and ſo has a continual blaſt.

All the inner work is hid from ſight, by the face of the inſtrument ſtanding upon 36.

As many partitions *LL*, *MM*, *&c.* as there are in the ſound board; ſo many valves *V*, *V*, rollers, *s*, *s*, or elſe ſo many leavers *xy*, or 89, and their wires, and that is juſt as many as there are keys *A*, *B*, *C*, *D*, *&c.* And there are generally 61 keys, with flats and ſharps; reaching from *G* to *G*, the compaſs of 5 octaves. But the ſcheme could not contain them all. Likewiſe there are as many handles *l*, *l*, *&c.* rollers, *R*, *R*, *&c.* ſliders *f*, *f*, *&c.* as there are different ſtops upon the organ. And it muſt be obſerved, that, between the ſliders *f*, *f*, *&c.* there are as many ſliders, on the right hand; and the ſame number of handles and rollers, which cannot be expreſſed in this

FIG. 313. this ſcheme. And other rows of pipes placed between *LN*, *PQ*, *&c.* And towards the middle of the organ, the leaſt pipes are placed, and the leaſt partitions; the greateſt being on the outſide.

There are many ſtops in ſome organs, but generally 10 or 12 on each hand; theſe are ſome of them, diapaſon, principal, fifteenth, twelfth, tearce, cornet, trumpet, french horn, vox-humana, flute, baſſoon, cremona, *&c.* and a contrivance to ſwell the notes of ſome of the ſtops.

When this noble inſtrument is to be played upon, put down the handle *O* of the bellows, this raiſes the upper board *T*, and cauſes the air to enter in at the valve *r*. Then that handle being let go, the other handle *O* is put down; during this time, the board *T* of the firſt, deſcending, and ſhutting the valve *r*, drives the air through the other valve, up the portvent into the wind cheſt. Then drawing out any handle, as the flute ſtop *pl*, this draws out the ſlider *fg*, and all the pipes in the ſet *LN* are ready to play, as ſoon as the keys *C*, *D*, *E*, *&c.* are put down. Therefore putting down the key *D*, by laying the finger upon it, opens the correſpondent valve *mV*, and the air enters thro' it, into the pipe *X*, and makes it ſound. In the ſame manner, any other pipe, in the ſet *LN*, will ſound, when its key is put down. But no pipe in any other ſet *PQ* will ſpeak, (becauſe the communication is ſtopt) till its ſlider *f* is drawn out by the correſponding handle *l*.

Pipes are made either of wood or metal; ſome have mouths like flutes, others have reeds. The ſmalleſt pipes are made of tin, or of tin and lead. The ſound of wooden and leaden pipes is ſoft. Short pipes are open, and the long ones are ſtopt; the mouths of large ſquare wooden pipes are ſtopt with valves of leather. Metal pipes have a little ear on each ſide of the mouth, to tune them, by bending it a little in or out. Whatever note any open pipe ſounds, when the mouth is ſtopt it will ſound an octave lower: and a pipe of twice its capacity will ſound an octave lower.

314. It will not, I think, be foreign to my deſign, if I give a ſhort account of the method of tuning organs or harpſicords. But I muſt firſt premiſe ſomething concerning the ſcale of muſic. It is known from undoubted experiments, that if *AL* be a ſtring of a muſical inſtrument, and if the ſtring be ſtopt ſucceſſively at *Y*, *s*, *q*, *p*, *o*; and $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ of it be made to vibrate, it will ſound an eighth, a fifth, a fourth, a greater third, and a leſſer third, reſpectively.

Now as the difference between the fourth and fifth is accounted a whole tone, whereof there are 6 to make up the octave; therefore we ſhall have $\frac{3}{4}-\frac{2}{3}$, or $\frac{1}{12}$ for the difference of the ſtrings, that are to ſound a note one above the other, whereof the greater is $\frac{3}{4}$. Conſequently if the ſtring is 1, that difference would be $\frac{1}{9}$. Therefore $\frac{8}{9}$ of any ſtring, will ſound a note higher; and $\frac{8}{9}$ of this ſecond, would ſound 2 notes higher than the firſt; and $\frac{8}{9}$ of this third, would ſound 3 notes higher than the firſt, &c. and this being 6 times repeated to make up an octave, we ſhall have $\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}=\frac{1}{2}$, as it would be if this note was exact, but that product is leſs, being but .4933; and therefore $\frac{8}{9}$ is too ſmall, and $\frac{1}{9}$ too great for a tone. And 6 of theſe notes do not exactly make up an octave. FIG. 314.

After the ſame way, if we take the difference between the fourth and leſſer third for a whole tone ($\frac{5}{6}-\frac{3}{4}$), we ſhall get $\frac{9}{10}$ of the ſtring for a whole note, but this will be found to be too great, being .5314 inſtead of .5; therefore $\frac{1}{10}$ is too ſmall for a note.

If we try by half notes, we ſhall ſtill be no better. The leſſer third, the greater third, and the fourth differ by half a note; of which there ought to be twelve in the octave. In the former caſe we get $\frac{24}{25}$ for the length of the ſtring, in the latter $\frac{15}{16}$; and $\frac{1}{25}$ or $\frac{1}{16}$ for the length of half a note. The firſt is far too little, and the latter as much too big.

As none of theſe notes or half notes will make up an octave; ſo neither will any number of thirds, fourths, or fifths, make one or more octaves. A leſſer third contains 3, a greater third 4, a fourth 5, and a fifth 7 half notes. Therefore 4 leſſer thirds, or 3 greater thirds make an octave; and 12 fourths ſhould make 5 octaves; and 12 fifths, 7 octaves. But if this was ſo, then we ſhould have $\overline{\frac{5}{6}}^{4}=\frac{1}{2}$, $\overline{\frac{4}{5}}^{3}=\frac{1}{2}$, and $\overline{\frac{3}{4}}^{12}=\overline{\frac{1}{2}}^{5}$ (that is $\overline{\frac{3}{4}}^{\frac{12}{5}}=\frac{1}{2}$), and $\overline{\frac{2}{3}}^{\frac{12}{7}}=\frac{1}{2}$. But never a one of theſe is ſo. And hence we may conclude, that no ſcale, made up of theſe notes, or half notes, or any combinations of them, or of thirds or fifths, &c. can be perfectly exact.

Now to contrive a ſcale to anſwer as near as poſſible all the requiſites. Let *AL*, *Am*, *An*, *Ao*, &c. to *AY* or $\frac{1}{2}$ *AL*, be 13 geometrical proportionals. Then theſe ſtrings *AL*, *Am*, *An*, &c. will ſound all the half notes in the octave, gradually aſcending. Therefore if *AL* be put $=1$, $AY=\frac{1}{2}$; *Am*, *An*, *Ao*, &c. being ſo many mean proportionals between 1 and ^{1}, will be found, as ſet down in the following table.

Cords

FIG. 314.

cords.	*ſtrings*	*notes equally aſcending.*	*pure concords.*	*errors*
ground	*AL*	1.00000	1.00000	0
b ſecond	*Am*	.94387		
※ ſecond	*An*	.89090		
leſ. third	*Ao*	.84090	.83333	*b* $\frac{1}{13}$
gr. third	*Ap*	.79370	.80000	※ $\frac{1}{15}$
fourth.	*Aq*	.74915	.75000	※ $\frac{1}{150}$
※ fourth	*Ar*	.70711	. . . . .	. . . . .
fifth	*As*	.66742	.66666	*b* $\frac{1}{100}$
leſ. ſixth	*At*	.62996	.62500	*b* $\frac{1}{15}$
gr. ſixth	*Au*	.59460	.60000	※ $\frac{1}{13}$
b ſeventh	*Aw*	.56123		
※ ſeventh	*Ax*	.52973		
eight	*AY*	.50000	.50000	0

In this table, the 3d column ſhews the lengths of the vibrating ſtring, when the ſcale aſcends by equal degrees of ſound, or when all the half notes are equal.

The 4th col. ſhews the length of the ſtring to ſound the pure concords.

The laſt col. relates only to the concords: and ſhews the error of the 3d col. expreſſing what part of a whole tone it is; and whether it is below (expreſſed by *b*) or above (by ※). By this col. we can judge how the ſcale in the 3d col. will perform. And theſe errors are found by comparing the 3d and 4th col. together. As ſuppoſe you would know what it is in the fifth, we ſhall have $66742-66666=76$, and $70711-62996=7715$, which repreſents a tone in that place. Then $\frac{76}{7715}$ or $\frac{1}{100}$ is the error, which is but the hundredth part of a whole tone. And as the number in the 3d col. is greater, it ſhews, that by this ſcale, the fifth is flatter than it ought to be. And ſo are the reſt of the errors found out, and examined.

Now its evident, that the error in a fifth or a fourth is quite inſenſible in practice; but the thirds and ſixths ſuffer the moſt, being in ſome but the 13th part of a note, which perhaps may be ſenſible to a good ear; but then it will not be ſo perceptible in a third, as it would be in a fifth; becauſe a third is leſs perfect than a fifth. And the ſweeter the cord, the more eaſily is an imperfection diſcovered. Now although theſe trifling errors will take away ſomething from the ſweetneſs of the harmony,

mony, and will hinder the ſcale from being abſolutely perfect; yet there is no remedy for it, but what is worſe than the diſeaſe. FIG. 314.

As to the tuning this inſtrument, it is plain that the notes ought not to be tuned by perfect fifths, for the upper note will always be the hundredth part of a note too high. And ſince one muſt take 12 fifths, before he can come at the ſame note again, whence he ſet off; there will at laſt be an error of $\frac{12}{100}$ or $\frac{1}{8}$ of a note, which is very diſcoverable in a fifth. The method therefore to be taken, is, to make the upper note a very little flatter than a perfect fifth; by firſt tuning it perfect, and then lowering it a ſmall matter, but not ſo much as to offend the ear. And after you have thus gone through the octave, if you find the laſt note either too high or too low, begin a new, and alter them all a little, according to your judgment, till the laſt does agree: but this judgment is to be attained principally by practice. Upon the firſt octave being rightly tuned, all the reſt depend, and therefore one ought to be very exact in it; for in all the reſt, there is nothing to do but to take the eights above and below. And you ought to begin to tune about the middle of the inſtrument.

Thoſe that tune by thirds, ought to take the upper note of the greater third, as ſharp as the ear will bear. And leſſer thirds ſhould be taken as flat as they may.

Moſt people in tuning, take ſome of the fifths perfect, and leave others imperfect; which they call bearing notes. But this is attended with great inconvenience: for the muſick ought to be ſo ſet, that no fifth ought to fall on any of theſe bearing notes, which inſtead of being a perfect concord, will be no better than a diſcord; ſince the error in theſe bearing notes is very great. For if there is but one in an octave, its error is $\frac{1}{8}$ of a note, if two of them, and both alike, $\frac{1}{16}$ of a note. Now if theſe people be ſo nice as to diſtinguiſh $\frac{1}{100}$ part of a note, much more would they be offended at $\frac{1}{8}$ or $\frac{1}{16}$. And always to avoid taking the fifths upon theſe notes, when they come naturally in the way, would be cramping, and even ſpoiling the muſic. And another diſadvantage would ariſe, that a piece of muſic could not be tranſpoſed upon any key at pleaſure, whatever need there might be for it: but muſt be tied down to a very few. And if this method could cure in any meaſure, the errors of the fifths; yet it would not at all mend thoſe of the thirds, which are far greater. And if any one third ſhould happen to be bettered, it is certain that others will be made as much worſe, and will be turned into diſcords.

Some that like not the *equiharmonic* or *iſotonic ſcale*, above deſcribed, would compoſe a ſcale of ſeveral ſorts of tones and ſemi-tones, as $\frac{1}{9}$, $\frac{1}{10}$, and $\frac{1}{16}$, $\frac{1}{17}$, $\frac{1}{18}$, $\frac{1}{19}$, $\frac{1}{20}$, *&c.* but what end can this anſwer? it is very eaſy to ſhew, that if ſome cords may be taken perfect, others will be miſerably bad, and degenerate into diſcords. But this ſcheme ſeems to be built only upon the conſideration of abſtract numbers; for it regards only this, how to make ſeveral fractional quantities reſolve themſelves into others more ſimple, by multiplication; which is a thing of no manner of uſe in muſic.

Others, to avoid the badneſs of the cords which fall in ſome places, have invented quarter notes; which makes the muſic extremely hard to play; and is, beſides, far from anſwering the end propoſed. Therefore upon the whole, I cannot but think the ſcale above deſcribed, to be the beſt for practice. For ſo ſmall an error as $\frac{1}{100}$ of a note in a fifth, and every where the ſame, cannot be ſenſible, or do much hurt to the muſic. And I will venture to ſay, that the very alteration of the weather, in 24 hours time, by heat or cold, drought or moiſture, will have ſuch an effect upon either ſtrings or pipes, as to cauſe a greater difference than this amounts to. There are imperfections in every thing, which we cannot quite take away; all we can do, is to make them as little as poſſible.

More examples of the conſtructions of engines might here be added. But as there is ſuch an infinite variety in the world, it would be an endleſs taſk to deſcribe all the kinds of them. Therefore I think it needleſs to produce any more, eſpecially, ſince their conſtruction and uſe depend upon the ſame principles as theſe already deſcribed. And if the reader does but thoroughly underſtand the powers and forces of theſe before mentioned; he cannot be at a loſs to find out the powers, forces, or motions, of any other machine, though never ſo compounded.

AN ALPHABETICAL INDEX OF THE TERMS uſed in MECHANICS.

A.

AIR PUMP, a machine to draw the air out of a glaſs. *Fig.* 277.

Ajutage, the ſpout for a *jet d'eau* in a fountain.

Amplitude, the diſtance a ball is ſhot to.

Anemoſcope, a machine for ſhewing the point of the wind. See *Fig.* 253.

Angle of application, is the angle which the line of direction of a power makes with the leaver it acts upon.

—— *of inclination*, is the angle an inclined plane makes with the horizon.

—— *of traction*, the angle which the direction of a power makes with an inclined plane.

Aqueduct, an artificial river, or tube to convey water.

Arbor, the axle or ſpindle of a wheel. *Fig.* 185. *ef.*

Arch, a hollow wall made of a circular form, to ſupport any building.

Areometer, an inſtrument to meaſure the weight of liquors.

Arm, any piece of timber or metal, that projects horizontally from ſome part of a machine.

Axle, or *Axis*, the line or ſpindle about which a wheel turns round. *Fig.* 185. *ef.*

Axis in peritrochio, a machine for raiſing weights, conſiſting of a wheel, fixed upon a cylinder for its axis.

B.

Balance, a machine to weigh bodies in; one of the mechanic powers. *Fig.* 188.

Balance wheel, the fly or pendulum of a watch.

Barometer, a machine to ſhew the weight of the air or atmoſphere. See *Fig.* 218.

Baroſcope, the ſame as barometer: a weather-glaſs.

Barrel of a wheel, is the axle, or cylindrical body, about which the rope goes.

—— *of a pump*, is the hollow part of the pump where the piſton works.

Barrs, ſtrait pieces of timber or metal, that run croſs from one part of a machine to another.

Baſe, the foot of a pillar.

Baſil, that angle, the edge of a tool is ground to.

Batten, a piece of timber three or four inches broad, and an inch thick.

To *Batter*, to lean backward.

Bauk, a long piece of timber.

Beak, the crooked end of a piece of iron, to hold any thing faſt.

Beam, a large piece of timber lying acroſs any place.

Beetle, a wooden inſtrument, or mallet, for driving piles.

Bevil, any angle that is not right. An oblique angle.

Block, a lump of wood.

Blocks, pieces of wood in which the ſheevers or pullies run, and through which the ropes go. *Fig.* 42. *A*, *B*.

Bolts, large iron pins.

Bond, the faſtening ſeveral pieces of timber together, either by mortiſe and tenant, dove-tailing, *&c.*

Brace, a piece of timber fixed obliquely into others, to ſtay them from moving any way.

Brackets, the cheeks of the carriage of a mortar. A cramping iron to ſtay timber work; alſo ſtays ſet under a ſhelf to ſupport it.

Brads, nails having no broad heads.

Bridge, any horizontal beam, *&c.* that is to ſupport ſomething.

Butments, thoſe ſupports on which the feet of arches ſtand.

Buttreſs, a piece of ſtrong wall that ſtands on the outſide of another wall to ſupport it.

C.

Capſtand, a machine on board a ſhip, to hoiſt the maſts, or raiſe any weight. *Fig.* 248.

Caſcade, a fall of water.

Cataract, is a precipice, or violent fall of water in a river, thro' high rocks, cauſing the water to fall with a great noiſe and force.

Catch, ſome ſmall part of a machine; which, in its motion, hooks or lays hold of ſome other part to ſtop it.

Center of motion, the point about which a body moves.

—— *of gravity*, the point upon which a body being ſuſpended, it will reſt in any poſition.

—— *of magnitude*, a point equidiſtant from the oppoſite extremes of a body. The middle.

—— *of percuſſion*, the point of a vibrating body that gives the greateſt ſtroke.

Center pin, a pin about which, as a center, a body moves.

Chain pump, a pump having ſeveral buckets fixed to an endleſs chain, which goes through it, and is moved round upon an axle. *Fig.* 254.

Chaps, two ſides of a machine which take hold of any thing.

Cheeks, two upright parts of a machine, anſwering to one another in poſition and uſe.

Chronoſcope, a pendulum to meaſure time.

Clack, a ſort of valve which is flat, like a board; ſerving to ſtop a fluid from running out. *Fig.* 268 *T*, a flap.

Clampt, when the edges of two pieces of boards are joined together, ſo as the grain of one may lie croſs the grain of the other.

Claſp, a ſort of buckle to faſten any thing.

Claſp nails, thoſe with little heads to ſink into the wood.

Claws, ſlender crooked pieces of metal in a machine, which ſerve to move or hold any thing. Long teeth.

To *Clench* or *clinch*, to double back the end of a nail, that it may not draw out again. To rivet.

Clench nails, nails that may be clinched.

Cock, a braſs ſpout to let a fluid run out, or ſtop it by turning.

Cogs, the wooden teeth of a great wheel. *Fig.* 185, *a*, *a*, *a*.

Cog-wheel, a large wheel made of timber, where the teeth ſtand perpendicular to the plane of the wheel. *Fig.* 185, *CD*.

Collar, a ring of metal that goes about any thing, near the top, in which it turns round.

Column, the ſhaft or trunk of a pillar.

Contrate-wheel, a wheel in a clock, where the teeth are parallel to the axis, and ſtand on the under ſide of the rim.

Corbel, a piece of timber, or ſtone, ſet under another piece, to diſcharge the weight.

Crab, a ſmall capſtand with three claws, to be placed on the ground, moveable from one plane to another. This is called a flying capſtand.

Crane, a machine for hoiſting goods out of a ſhip, or for raiſing timber or ſtone. *Fig.* 233.

Crank, that part of an iron axis which is turned ſquare with an elbow. *Fig.* 238. *I I.*

Croſs-tree, a horizontal beam fixed acroſs another.

Crow, a ſtrong ſquare bar of iron, forked at the end, to remove heavy timber, *&c.* by uſing it with the hands.

Crown-wheel, in a clock or watch, is that next the balance, its teeth ſtand in the upper ſide of the rim, and not in the edge. *Fig.* 166. *FG.*

Cupolo, a hollow arched tower, in form of a hemiſphere, or of a bowl turned upſide down.

D.

Denſity, is a greater or leſs quantity of matter contained in a given ſpace.

Detents, are thoſe ſtops, which being lifted up, the clock ſtrikes; and falling down, ſhe ſtops.

Dog nails, nails uſed for faſtening hinges.

Dome, a round vaulted roof or tower. A cupola.

Dormant, a great beam lying croſs a houſe. A ſummer.

Dormer, a window in the roof of a houſe.

Dove-tailing, letting one piece of timber into another, with a joint in form of a dove's tail, being broader at the end, that it may not draw out again.

Drum, the lantern or trundle, which is carried by a great wheel. *Fig.* 266. *EF.*

Drum head, a timber head, or lump of timber, in form of a drum.

E.

Edging, the outſide or border.

Endleſs chain, a chain with the ends joined together; by which any part of a machine is wrought.

Endleſs ſcrew, a ſcrew working in the teeth of a wheel; which may be turned about for ever. *Fig.* 193. *E.*

Engine, a mechanical inſtrument compoſed of wheels, leavers, ſcrews, *&c.*

Eolipyle, a hollow globe of metal, filled with water, and put in the fire; the heat and vapour ruſhes out at a ſmall hole, with a great noiſe. *Fig.* 264.

Equilibrium, the equality of weight, of two or more bodies, *&c.* keeping one another at reſt.

Eye, a hole in ſome part of a machine, through which any thing is put.

F.

F.

Face, the foot or foreſide of a machine, or of ſome principal part of it.

Fang, ſome ſmall piece of metal like a long tooth, that by its motion moves ſome other part.

Fellies, pieces of wood on the outſide of a wheel, which make the rim.

Ferril, a ſort of hoop.

Floats, the flat boards ſet perpendicular on the edge of a water-wheel; by which the water drives the wheel about. *Fig.* 236. *D*, *D*.

Fly, that part in a clock, *&c.* that regulates the motion, and makes it uniform. *Fig.* 168, 169, 170.

Force, any thing that acts upon a body to put it in motion.

Force pump, a pump that diſcharges water by preſſing it upwards. *Fig.* 267, 268.

Frame, the outwork of any machine, or what holds all the reſt together.

Free, clear of all impediment.

Friction, the reſiſtance that bodies have by rubbing againſt one another.

Fulcrum, that which ſupports a leaver in moving any heavy body.

G.

Gain, the levelling ſhoulder of a joiſt or other timber.

Gin, a machine to raiſe great weights. *Fig.* 257.

Gravity, the weight of bodies.

——— *ſpecific*, by this one body weighs more or leſs than another of the ſame bulk.

——— *relative*, is the weight of a body in a fluid, or on an inclined plane.

Groove, a channel cut in wood or ſtone.

Gudgeons, the eyes in the ſtern of a ſhip, on which the rudder hangs. The center pins of an axle.

Gyration, a whirling round.

H.

Hand, an index or pointer.

Handle, the part of an inſtrument to take hold on with the hand.

Hand ſpike, a wooden leaver to be uſed with the hand, in moving any thing.

Head, the top part of any thing.

Hem, the edge of ſome cloth turned down and ſewed.

Hinge, an iron joint on which a door turns, *&c.*

To *Hitch*, to catch hold on, with a hook or turn of a rope.

To *Hoiſe*, or *hoiſt*, to heave up, or raiſe by force.

Hook pins, taper iron pins with a hook head, by which they are ſtruck out again. They ſerve to pin the frame of a roof or floor together, till wrought off.

Hoop, a circular ring to put about any thing, to keep it faſt.

Hydraulics, the art of making engines for water works.

Hydrometer, an inſtrument to meaſure the denſity of fluids. *Fig.* 269.

Hydroſtatics, a ſcience teaching the weights, preſſures, motions, and properties of fluids.

Hydroſtatical balance, an inſtrument for finding the ſpecific gravity of bodies.

Hygrometer, *Hygroſcope*, } an inſtrument for meaſuring the moiſture and dryneſs of the air. *Fig.* 265.

I.

Jack, an engine to lift up a loaded cart, or the roof of a houſe, *&c.* See *Fig.* 249. Alſo an engine to roaſt meat. *Fig.* 258.

Jack pump, a chain pump.

Jaums, door poſts, or window poſts, *&c.*

Jet d' eau, the pipe of a fountain, which ſpouts up water into the air. *Fig.* 287.

Impetus, any blow or force wherewith one body ſtrikes or impels another.

Joint, the place where one part is fixed into another, or moves about it.

Joiſts, thoſe pieces of timber framed into the dormant, in a houſe, and on which the boards of the floor are laid.

K.

Keys, ſtones in the top of an arch, to bind the ſweeps together.

Kenk, a ſnarle, or little turn upon a rope, that it cannot run.

Knee, a crooked or angular branch of timber.

L.

Lantern, that part which is moved by the cogs of a great wheel, acting againſt the ſpindles or rounds. The drum. *Fig.* 266. *EF.*

Latch, that which faſtens a door, *&c.* A *ſneck.*

Leaver, or *lever*, a bar of iron or wood to raiſe a weight: one of the mechanic powers.

Leaves, the teeth of a pinion. *Fig.* 185. *c, c, c.*

Ledge, a flat border or plain, adjoining to a thing.

Level, an inſtrument to place any thing horizontal.

Linch pin, a pin that keeps a wheel from coming off its axle.

Lip, a thin edge turned hollow.

Loop, a piece of metal having a hole in the end, which goes over ſomething. A nooſe in a rope that will ſlip.

M.

Machine, a mechanical inſtrument for moving bodies.

Mechanics, a ſcience that teaches the principles of motion, and conſtruction of engines, to move great weights.

Mechanic powers, are theſe ſix; the balance, leaver, wheel, pulley, ſcrew, and wedge: and according to ſome, the inclined plane.

Mitre, an angle of 45 degrees, or half a right one. And *half mitre* is a quarter of a right angle.

Momentum, quantity of motion; or the force or power a body in motion has to move another.

Mortiſe, a ſquare hole cut in a piece of ſtuff, to receive the tennant.

Motion, is the ſucceſſive change of place of a body; or its paſſing from one place to another.

Moving force, any active force or power that moves a body.

Mouth, the part or parts of a machine, that take hold of any thing. The entrance into any cavity.

N.

Nave, or *Naff*, the block in the middle of a wheel.

Neck, a part near the end, cut ſmall.

Notch, a dent, nick, or ſlit made in any thing.

Nut, the pinion of a wheel. *Fig.* 185. *AB.* A ſmall piece of metal going upon the end of a ſcrew nail.

O.

Oſcillation, the vibration or ſwinging of a pendulum.

P.

Paddles, a ſort of oars. The laddle boards on the edge of a water-wheel.

Pedeſtal, the baſe or bottom of a pillar.

Peers, or *Piers*, a ſort of buttreſſes, for ſupport and ſtrength.

Peg, a pin to go into a hole.

Pendulum, a weight hung by a ſtring or wire, ſwinging back and forward, to meaſure time.

Penſtock, the ſluice or door, that opens or ſhuts the paſſage of the water to a water-wheel.

Percuſſion, the ſtriking of one body againſt another.

Peſtil, a long piece of wood or metal, which riſes up and falls down again to beat or bruiſe ſomething.

Pevets, or *Pivots*, the ends of the ſpindle of the wheel in a clock or any machine, which play in the

Pevet holes, the holes in which the ends of a ſpindle or axle of a wheel turn. *Fig.* 185, *e*, *f*.

Picket, a ſtake pointed with iron, to drive into the ground.

Pillar, a perpendicular column ſupporting one end of an arch, *&c.*

Pinion, a little wheel at one end of the ſpindle, conſiſting but of a few leaves or teeth. *Fig.* 185, *AB*.

Piſton, a round piece of wood moving up and down within the body of a pump, to draw up the water.

Plate, a piece of timber, on which ſome heavy work is framed, as wall plate, *&c.* A flat piece of metal.

Pneumatics, a ſcience teaching the properties of the air.

Pole, a long ſtaff, or ſlender piece of wood.

Poſt, a perpendicular or upright beam of wood.

Power, the force applied to an engine to raiſe any weight. Or any force acting upon a body to move it.

To *Project*, to jet out or hang over.

Projectiles, balls or any heavy body thrown into the air.

Prop, a ſtay or ſupport for any thing, to bear it up.

Pulley, a ſmall wheel with a channel in the edge of it, moving about an axis fixed in a block; the channel is to receive a rope that goes over it. One of the mechanic powers.

Pump, an engine to raiſe water. *Fig.* 204.

Punchins, ſhort pieces of timber, placed upright to ſupport ſome weight.

R.

Rag-wheel, the barrel or wheel in a chain pump.

Rails, ſmall pieces of wood joined into others; thoſe pieces into which the pannels of doors, *&c.* are fitted.

Rammer, an inſtrument for driving ſtones or piles into the ground, or beating the earth.

Random, the diſtance to which a ball is ſhot.

Range, the direction a ball is ſhot in, from a piece of ordnance.

To *Range*, to run ſtrait, or directly in a line.

To *Reeve*, to paſs a rope through any hole.

Return, the ſide that turns off from any piece of ſtreight work.

Ribs, ſlender pieces of timber, ſerving for ſtrength and ſupport.

Riglets, little flat, thin, ſquare pieces of wood.

Rim, the circular part or outſide of a wheel.

To *Rivet*, to batter down the end of a nail, that it draw not out again.

Rod, a long ſlender piece of wood or metal.

Roll, or *roller*, an engine turned by a handle, to raiſe weights, *Fig*. 243.

Rounds, the ſtaves or ſpindles in a lantern, againſt which the teeth of a great wheel work. *Fig*. 266, *c*, *c*. The ſteps in a ladder, *&c.*

Ruler, a thin ſtrait piece of wood.

Rungs, ſpindles or rounds. *Fig*, 266, *c*, *c*.

Runner, a flat circular ring, between the nave and linpin of a wheel. Alſo a ſort of rope on board a ſhip, to hoiſt with.

S.

Sails, large pieces of canvaſs, by which ſhips, windmills, *&c.* are carried, by help of the wind.

Scantlin, ſtuff cut to a proper ſize.

Screw, one of the mechanic powers. The tap with the thread is the *male ſcrew*, the hollow that receives it, is the *female ſcrew*.

Scribing, drawing an irregular line upon one piece of ſtuff, parallel to the irregular ſide of another, with a pair of compaſſes opened to a due diſtance; and carried along the ſide of it. Then the wood in the firſt piece being cut away, theſe two pieces will fit each other.

To *Seaze*, to bind or faſten a rope, *&c.*

Shaft or *ſhank*, any long part of an inſtrument, eſpecially that is held with the hands.

Sheers, two poles ſet up an end ſloping, and tied together at top; and ſecured by a rope from falling. Their uſe is to raiſe any weight by help of a block and tackle at top.

Sheevers, pullies, the little wheels that run in blocks, by a rope going over them.

Shelf, a board, *&c.* fixt horizontally.

Shoulder, a part of timber or metal, cut thicker than the reſt, in order to ſupport ſomething.

Shrouds, the ledges on the edge of a guttered wheel.

Sills, *ſells*, or *groundſils*, pieces of timber that lie on the ground, into which others are fixed Sole trees.

Siphon, a crooked glaſs tube, for drawing off liquors. *Fig.* 216.

Sleepers, pieces of timber laid as a foundation and ſupport for others that are to lie upon them.

Slings, theſe are made of a rope ſpliced with an eye at either end, to go over a caſk or ſome heavy thing, which is to be hoiſted.

Snatch block, a great block with a ſheever in it, and a notch cut through one of the cheeks of it; to hitch the rope into the pulley, for readineſs.

Socket, a hollow piece of metal, in which any thing moves.

Sole, the bottom of the gutter or channel, in a guttered wheel.

Sole tree, the loweſt piece of timber which lies flat on the ground, into which the upper works are framed. The groundſel.

Spaniſh burton, a ſort of tackle to hoiſt goods, like *Fig.* 197.

Spear, a long pointed iron, or piece of timber.

Specific gravity, is that whereby one body weighs more or leſs than another of the ſame magnitude.

Spike, a pointed iron, or piece of wood.

Spindle, the axle of a wheel. *Fig.* 185. *ef.*

Spires, the turns of a rope about a cylinder or roller.

To *Splice*, to join two ropes together by working the ſtrands into one another.

Spokes, pieces of wood running from the center of a wheel to the circumference, like rays.

Spring, an inſtrument made of ſteel, that being bent, it continually exerts a great force, that it may unbend itſelf again. Springing plates are ſometimes made of braſs.

Spur, a ſort of prop, ſet aſlope to thruſt.

Spurs, long wooden teeth ſtanding in the edge of a large timber wheel. *Fig.* 192, *a*, *a*, *a*.

Spur wheel, a wooden wheel where the teeth ſtand in the edge of the rim. *Fig.* 192. *CD.*

Staff, a ſtick or ſmall piece of wood.

Statics, a part of mechanics, teaching the motions and properties of heavy bodies.

Stay, a piece of timber, or other thing fixed as a prop or ſupport to ſome heavy body.

Steelyard, an inſtrument to weigh bodies, conſiſting of a long beam and a moveable weight, *Fig.* 190.

Stock, the wooden part of a thing. and into which it is fixt.

Stopple or *ſtopper*; a plug, that fits into a hole.

Stops, any ſmall pieces in a moving machine, that ſerve to ſtop the motion.

Strain, the ſtreſs or violence any thing ſuffers by a weight or force acting againſt it.

Stroaks or *ſtraiks*, the iron going round the circumference of carriage wheels.

Stud, a knob, or little button. A ſolid piece of metal fixt to a plate.

Stuff, any wood that joiners work upon.

Swivel, a metal ring that turns about any way.

Syphon, the ſame as ſiphon. *A crane.* *Fig.* 216.

Syringe, an inſtrument for injecting liquors into any place.

T.

Tackles, blocks with pullies and ropes in them, to heave up goods. *Fig.* 196, 197.

Tenon, the ſquare end of a piece of wood, made to fit into a mortiſe hole.

Thermometer, } an inſtrument to ſhew the degrees of heat and
Thermoſcope, } cold. *Fig.* 270.

Thread, the ſpiral ridge that goes winding round a ſcrew.

Thruſt, the action againſt a body to puſh it forward.

Tight, ſtiff, cloſe.

Tongue, a thin ſlender piece of metal in a machine.

Tool, an inſtrument to work with.

Tooth, the indented part on the edge of a wheel that moves ſome other wheel. Or what ſerves to cut, or take hold on.

Tranſom, an overthwart beam in a building.

Triangle, an engine ſtanding on three legs, to raiſe weights with. *Fig.* 195.

Trundle, the part which is carried about by the teeth of a wooden wheel. The lantern or drum. *Fig.* 266. *EF.*

Trunk, a hollow tube or box.

Tumbler, a part in a machine that rolls about upon an axis, and plays back and forward.

Tumbrel, a roller, or cylindrical beam of wood.

Tympanum, a kind of wheel placed on an axle, and has ſtaves or rounds inſtead of teeth, and is carried about by a great wooden wheel. A trundle or drum. *Fig.* 266. *EF.*

V.

Valve, a piece of wood, *&c.* ſo fitted into a hole, that it opens and lets a fluid paſs through one way; and ſhuts and ſtops it the other. *Fig.* 268. *V. W.* *A ſucking valve*, is that where the water follows the piſton. *A forcing valve* when it is driven through before it.

Vane, a ſail, or fan; generally to ſhow the point of the wind.

Velocity, an affection of motion, and is that by which a body paſſes over a certain ſpace in a given time. Swiftneſs, or celerity.

Vibration, the moving or ſwinging of a pendulum back and forward.

Vis inertiæ, a property of body, by which it reſiſts any impreſſed force, and endeavours to continue in the ſame ſtate.

W.

Wallower, a trundle upon a horizontal axis. *Fig.* 257, *F.*

Waterpoiſe, an inſtrument to try the ſtrength of liquors. A hydrometer.

Web, the thin broad part of an inſtrument, as the web of a key, *&c.*

Wedge, an inſtrument to cleave wood. One of the mechanic powers.

Weight, the tendency of bodies downward. The matter raiſed by an engine.

Wheel, a machine conſiſting of an axis and a circular rim, with teeth in it, and then it is called a toothed wheel.

—— *Smooth*, a wheel without teeth, turned by a rope.

Wheel and axle, a machine to raiſe weights. One of the mechanic powers, *Fig.* 30.

Winch, an inſtrument with a crooked handle, to turn any thing about with.

Winder, a winch or handle to wind about.

Windlaſs, a machine to raiſe great weights. On board a ſhip, it ſerves to hoiſt the anchor. It is an horizontal roller, turned round by handſpikes.

Windmill, a mill to grind corn, moved by the wind. *Fig.* 266.

Wing, a thin broad part that covers ſomething, or hangs over it. Alſo what helps to give due motion to any thing, as the hands in a water wheel; a part of a ſail, *&c.*

Worm, a ſpiral thread running round a cylinder, forming a ſort of ſcrew.

A LIST OF THE PRINCIPAL MACHINES deſcribed in this BOOK.

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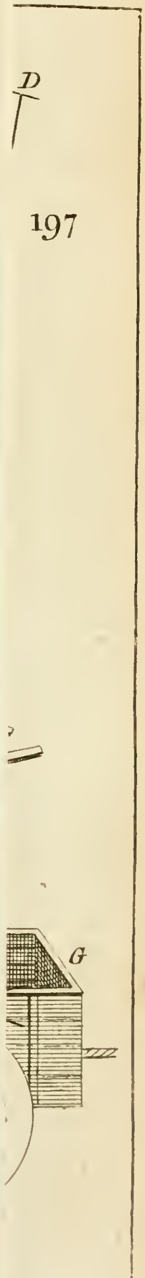

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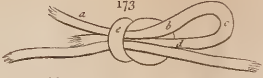

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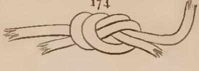

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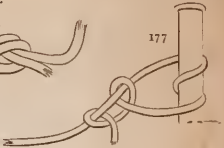

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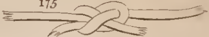

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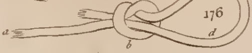

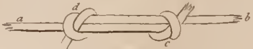

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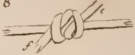

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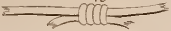

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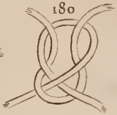

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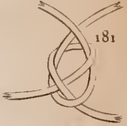

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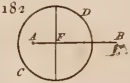

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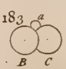

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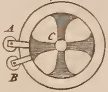

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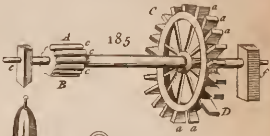

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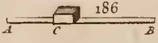

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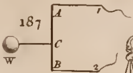

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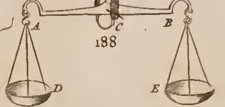

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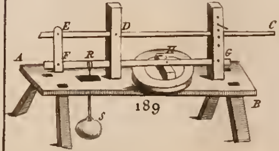

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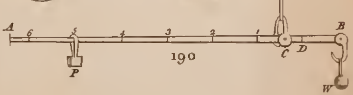

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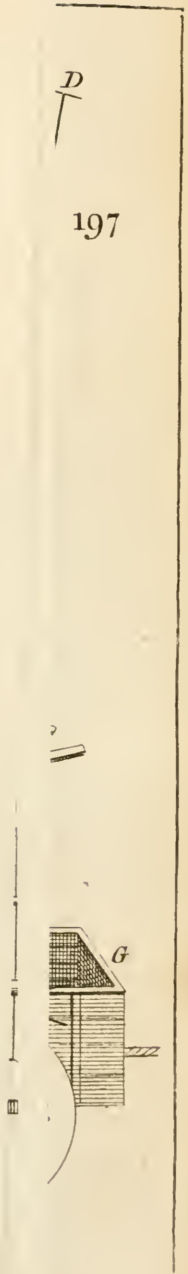

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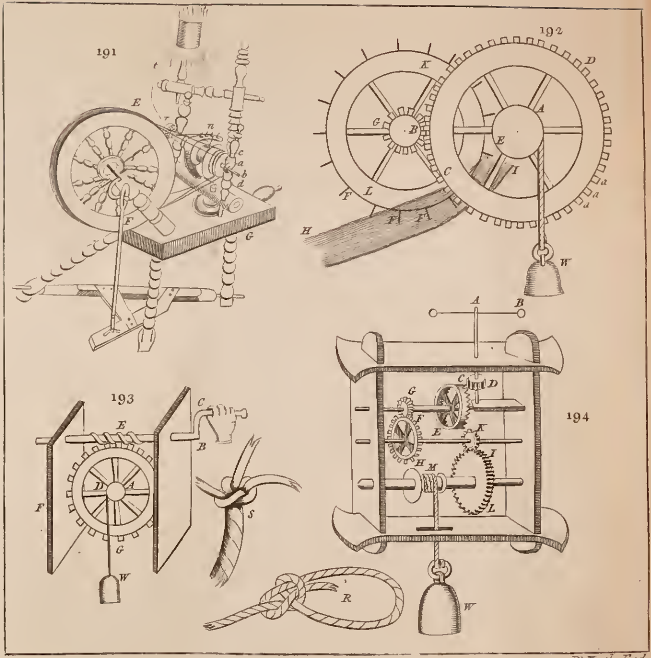
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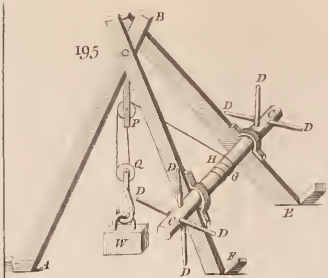

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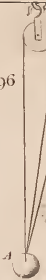

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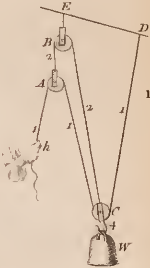

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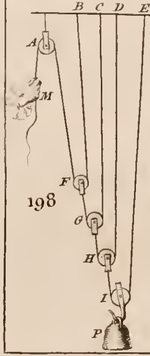

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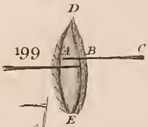

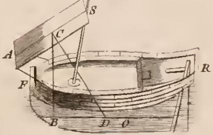

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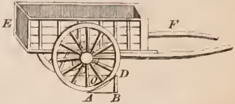

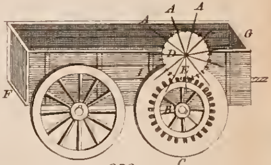

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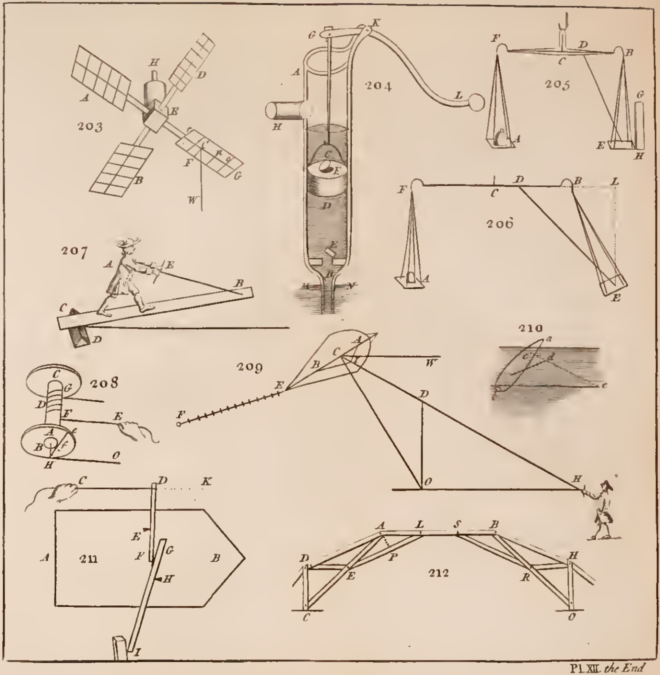

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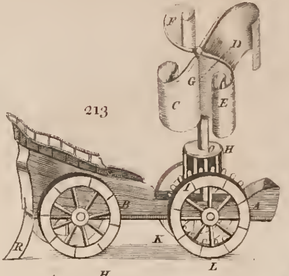

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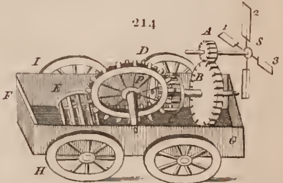

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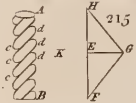

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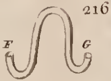

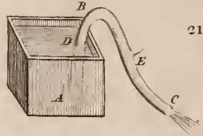

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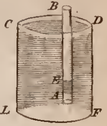

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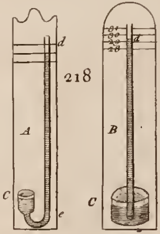

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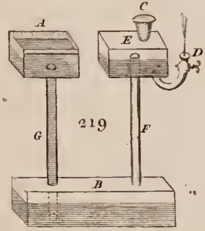

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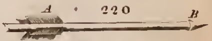

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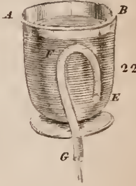

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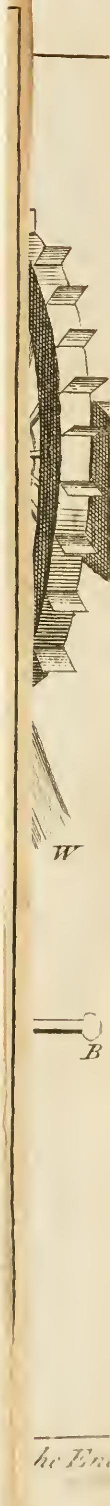

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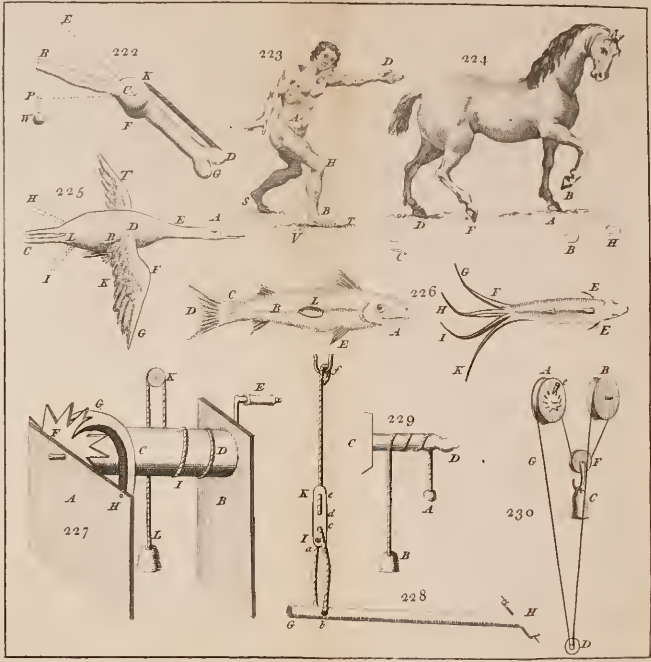

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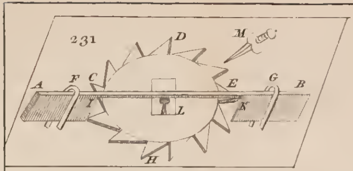
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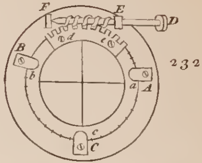
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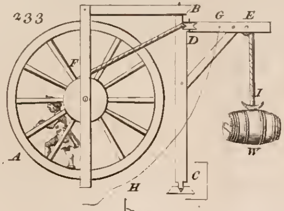
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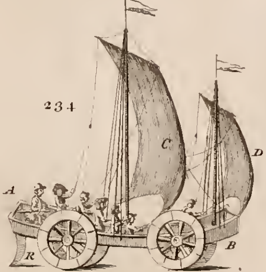
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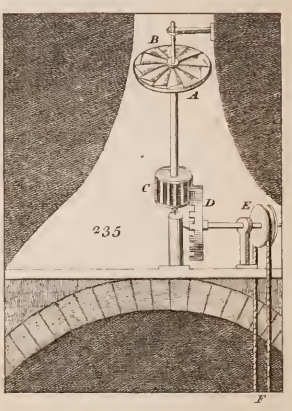
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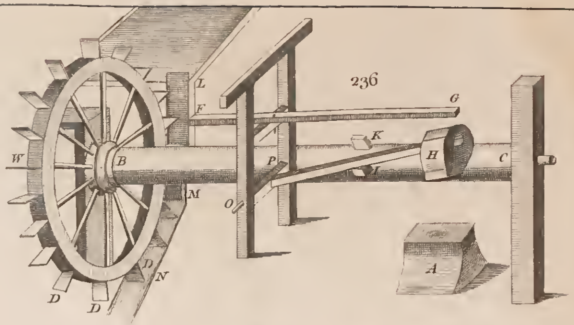

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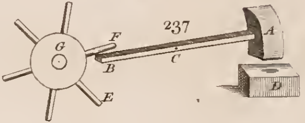

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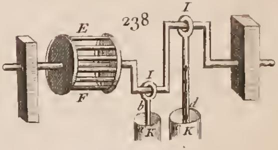

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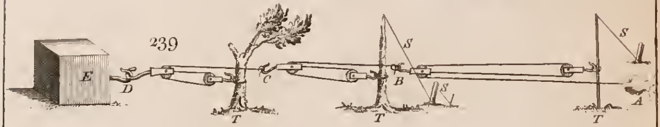

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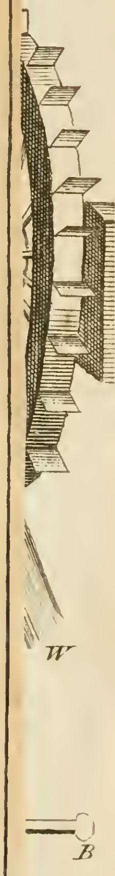

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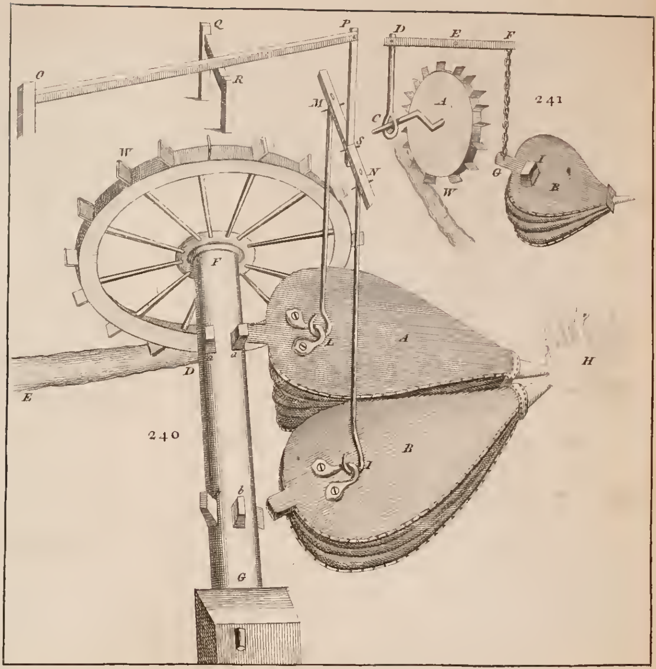

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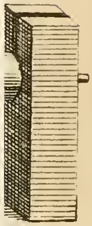

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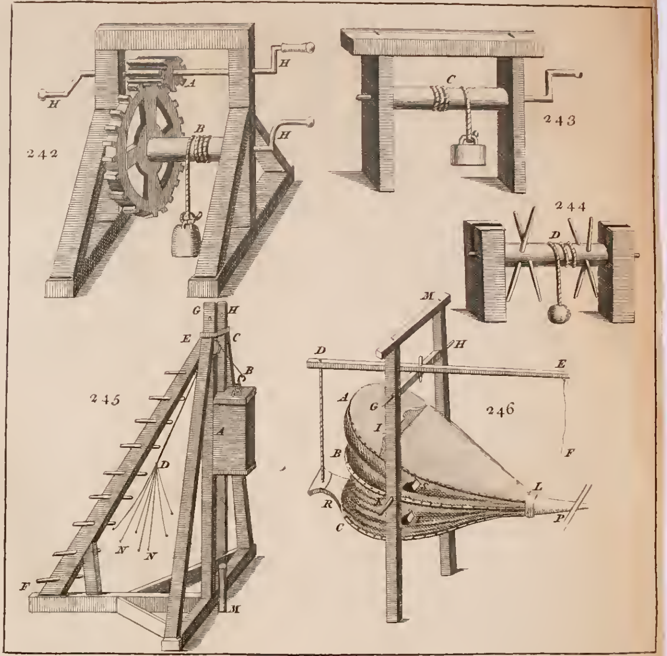

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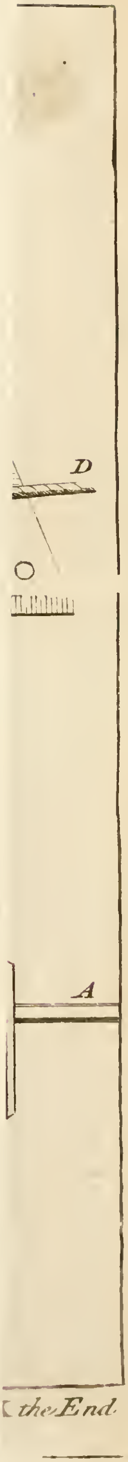

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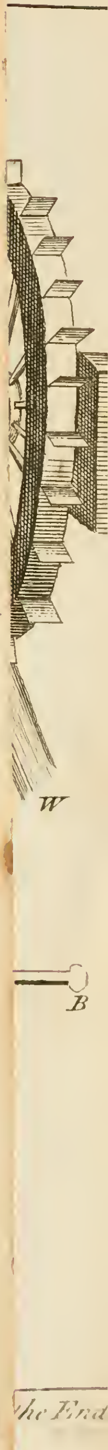

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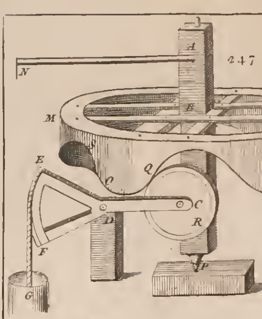

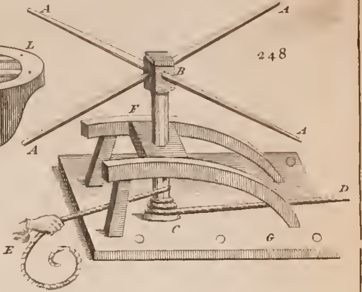

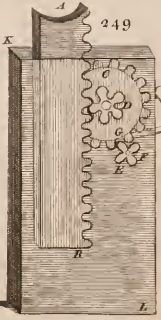

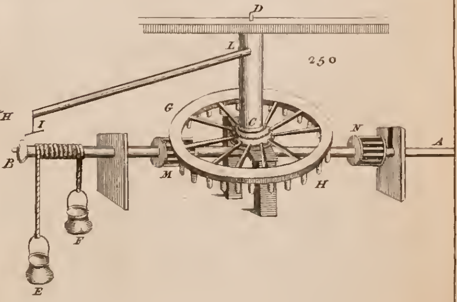

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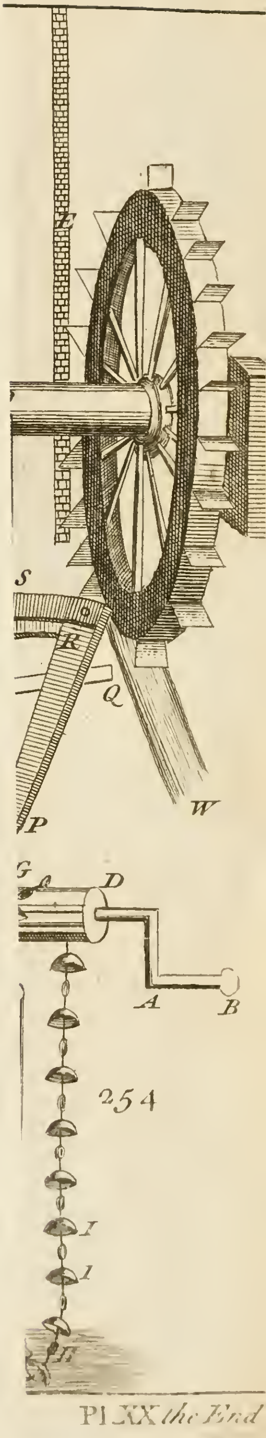

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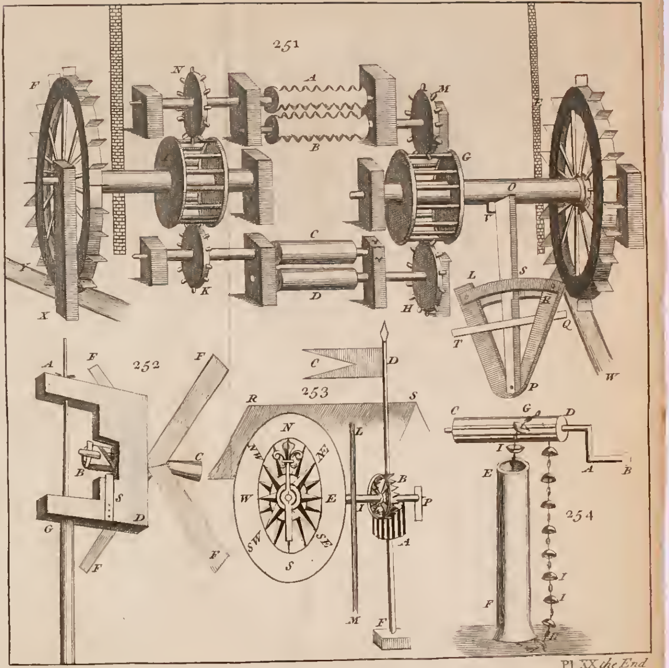
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Pl XX the End

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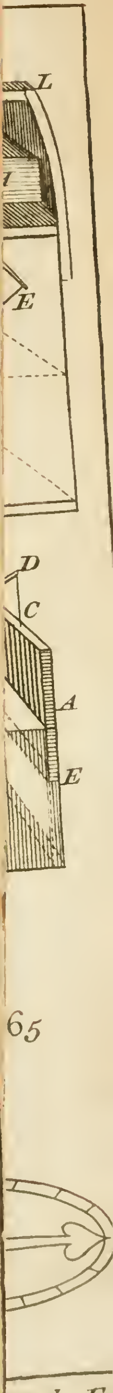

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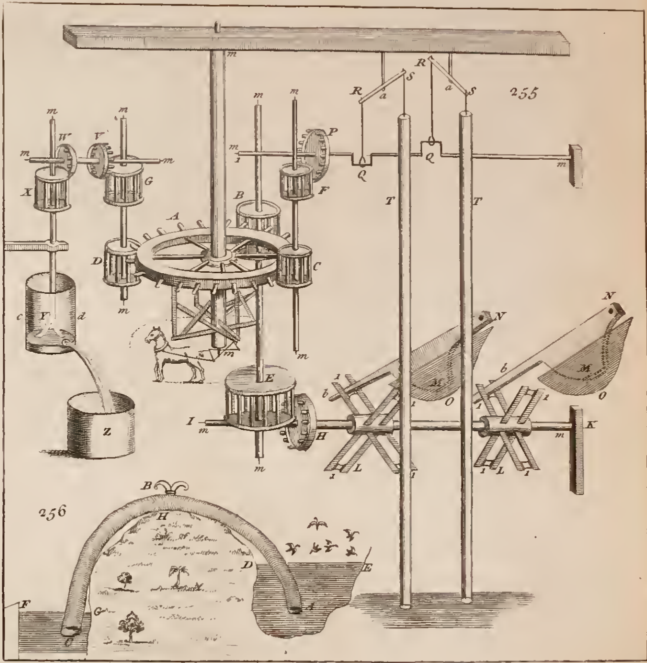

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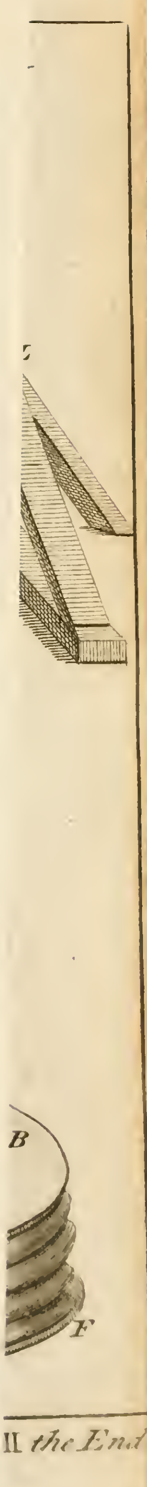

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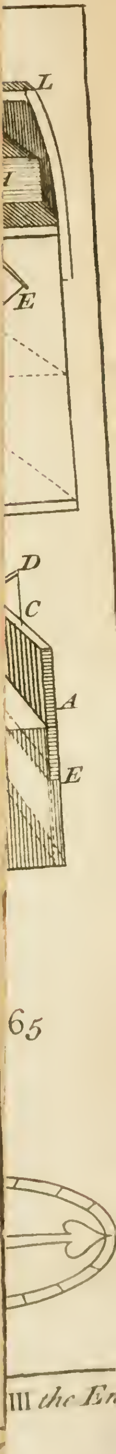

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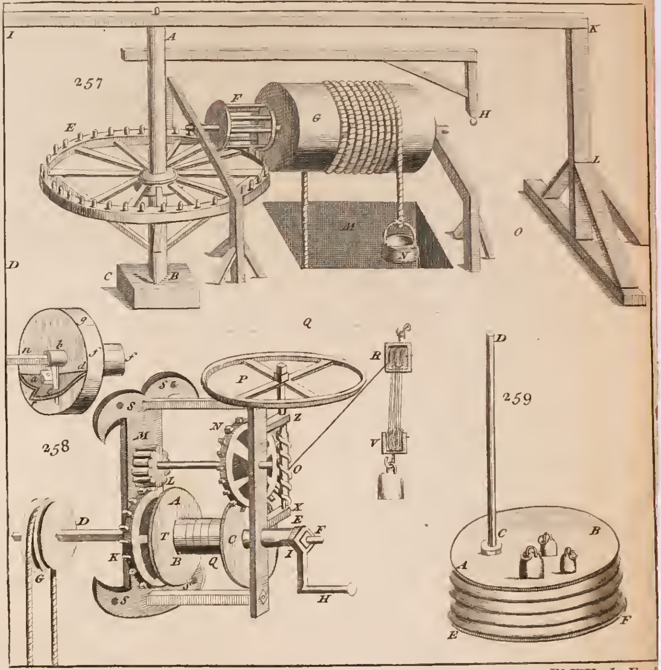

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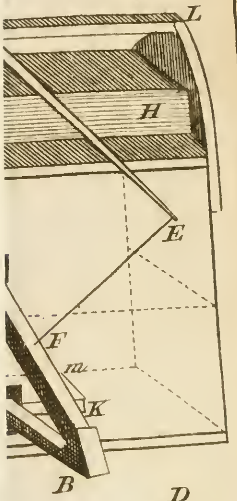
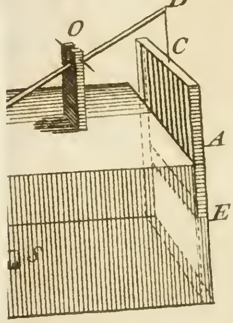
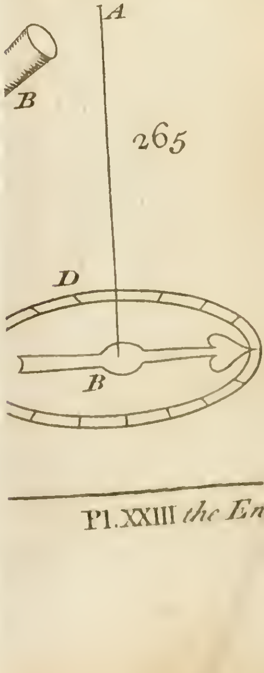


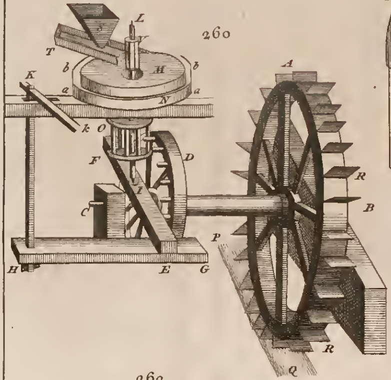




265

Pl.XXIII *the En*

260



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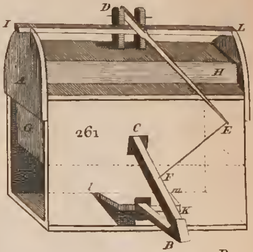

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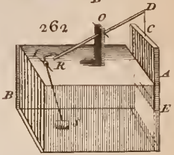

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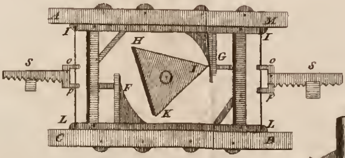

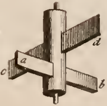

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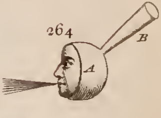

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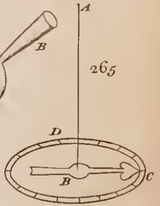

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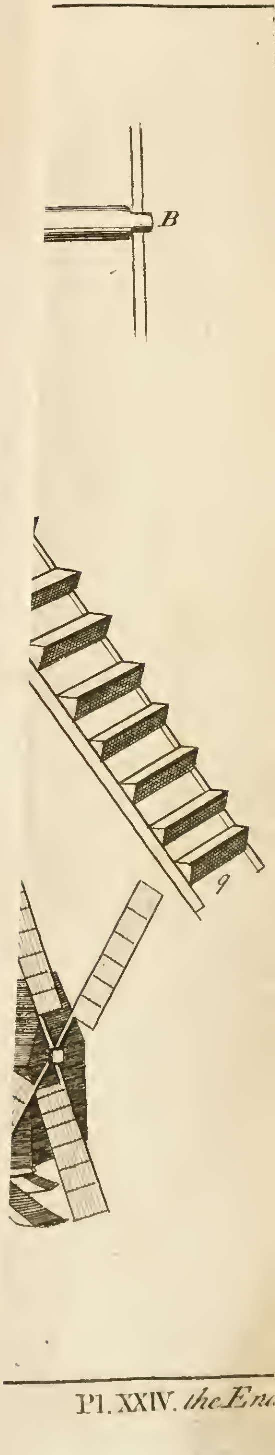

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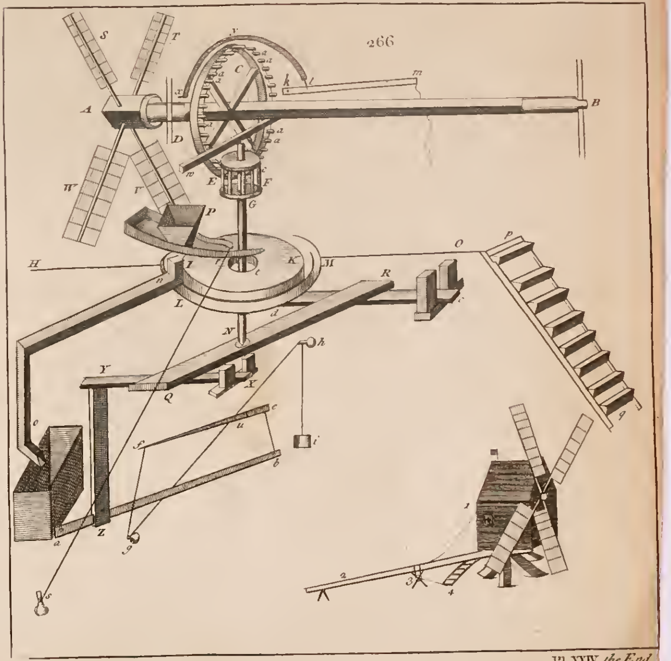

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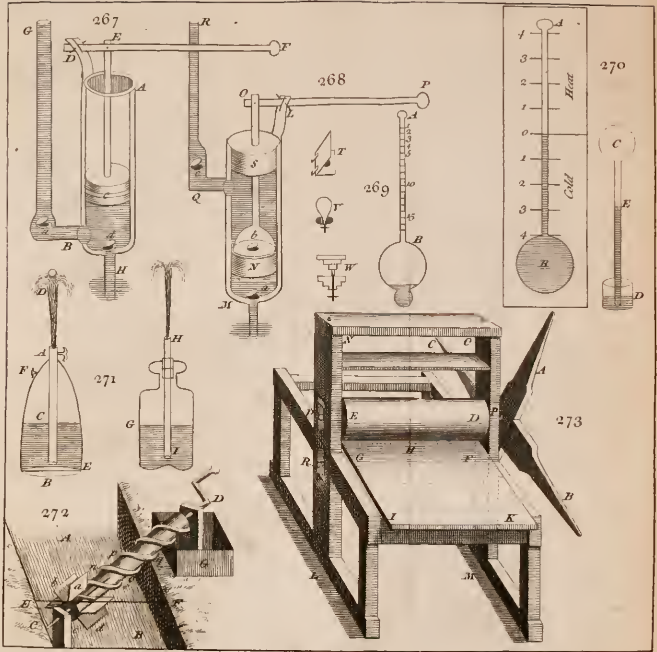

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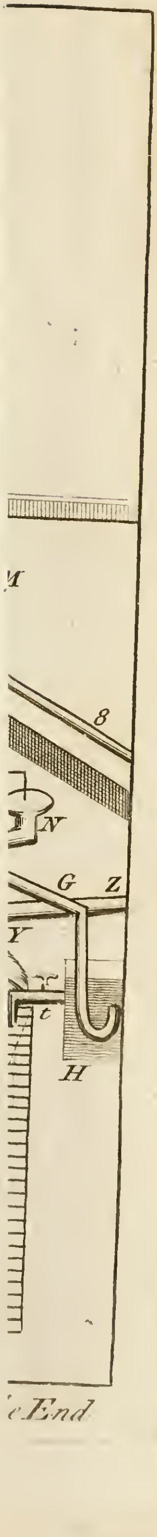

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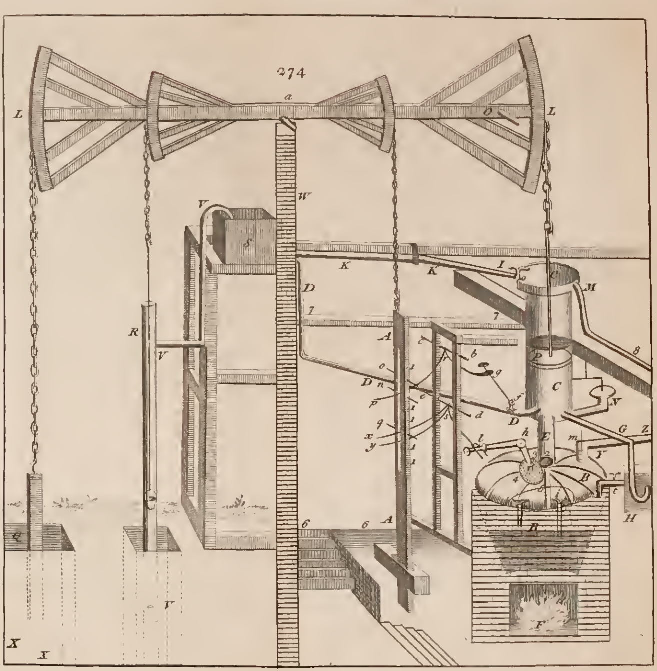

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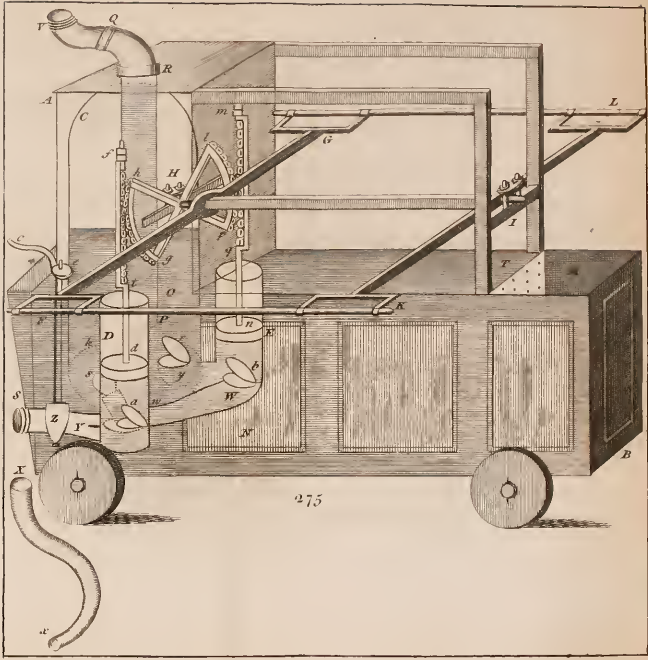

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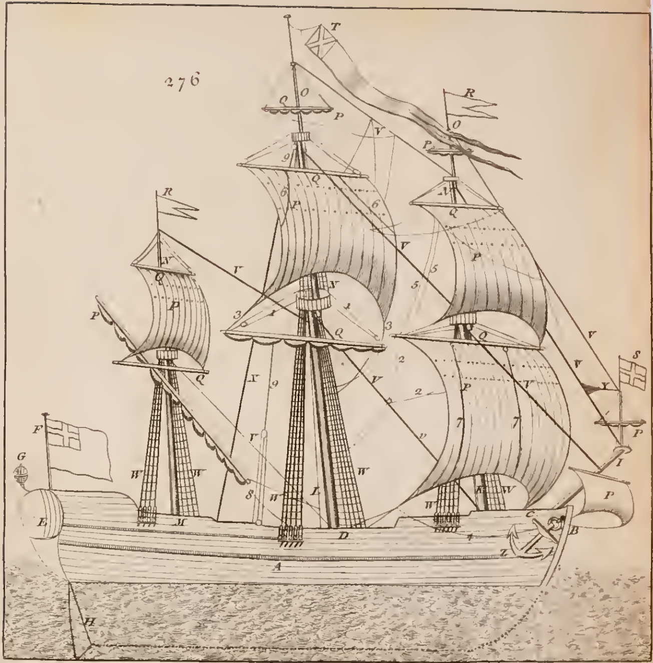

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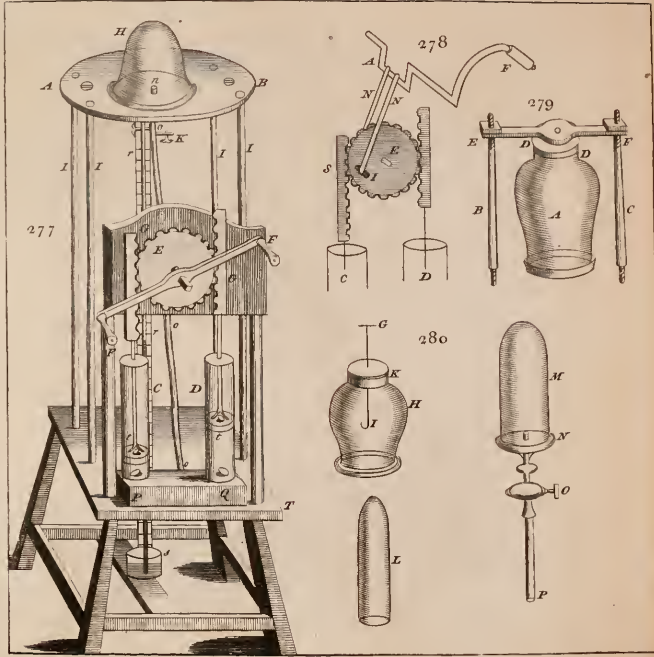

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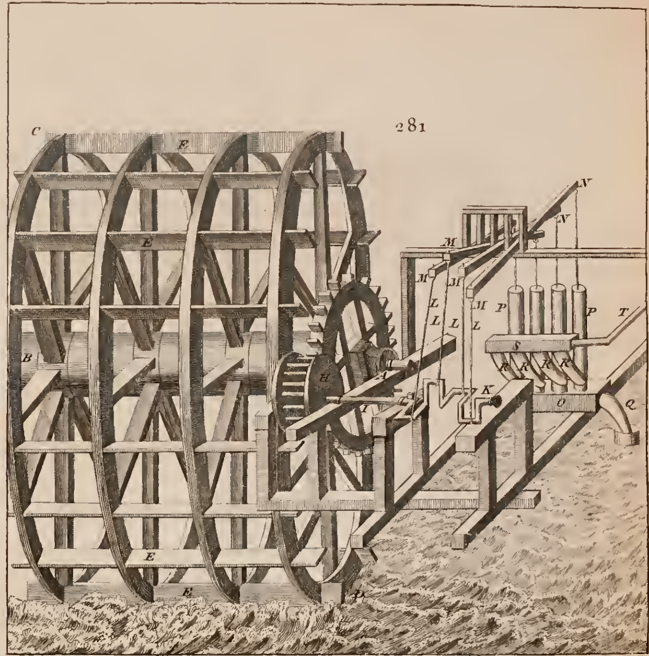

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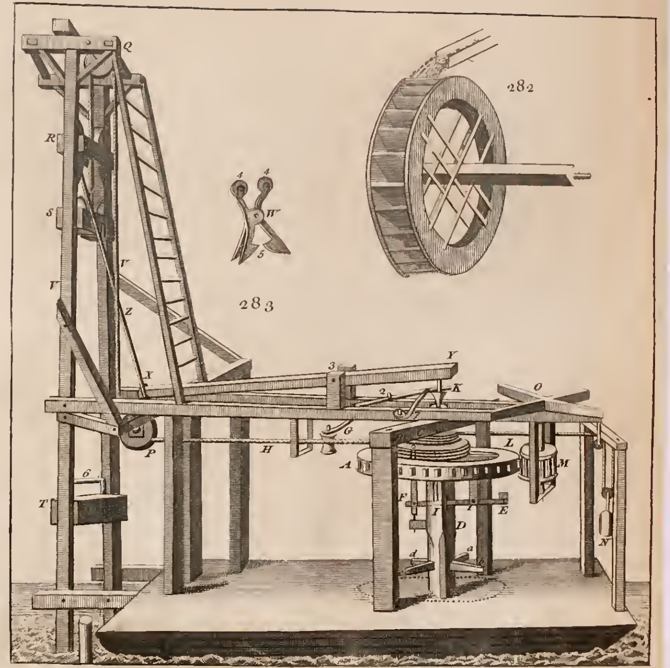
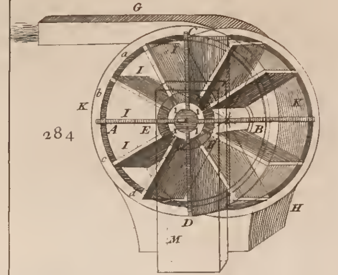


Pl. XXXI *the End*

284



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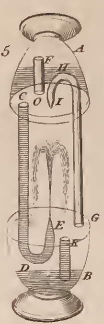

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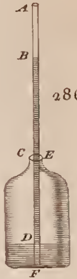

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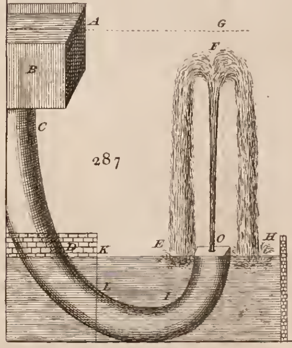

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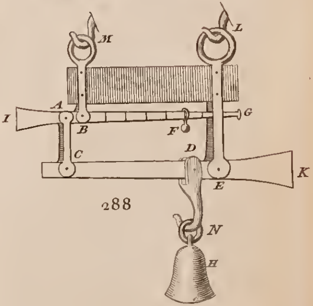

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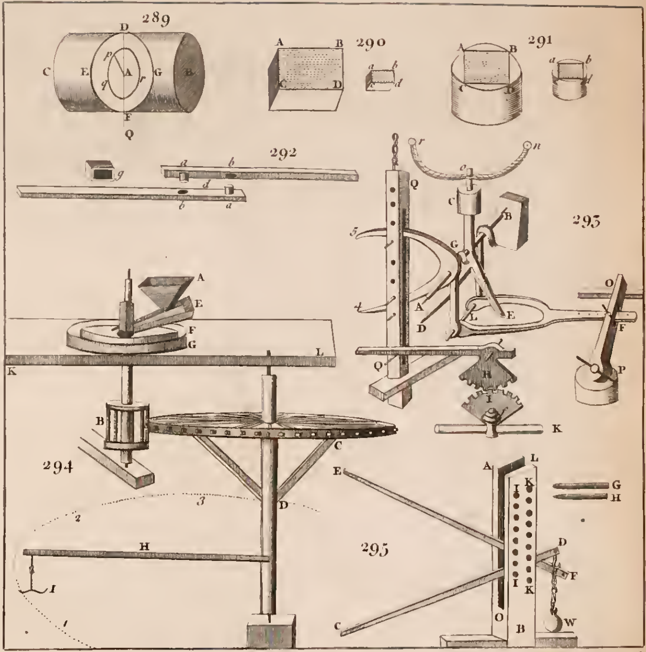

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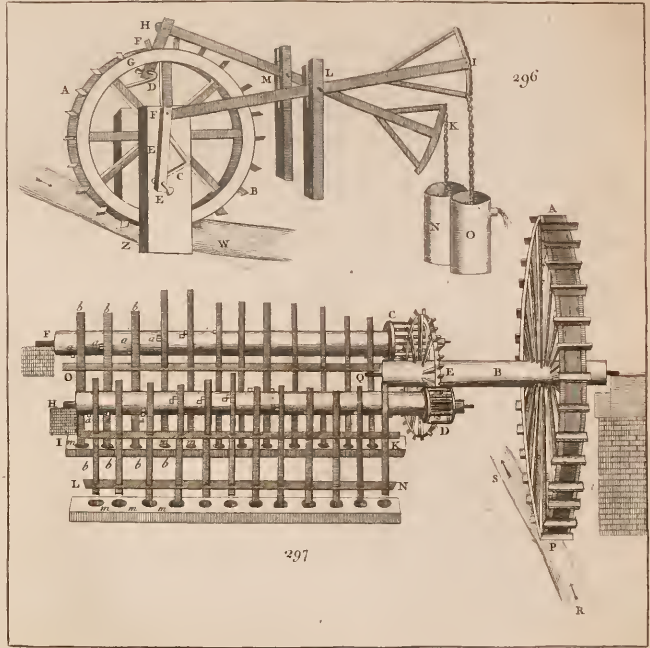

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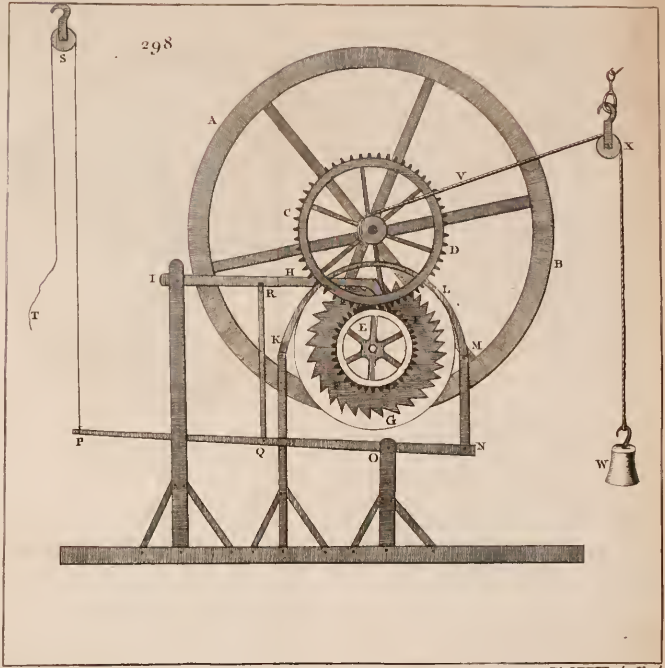

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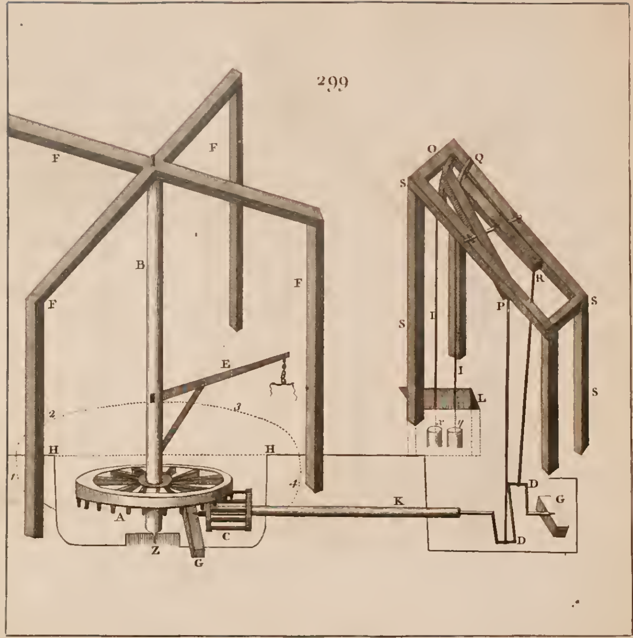

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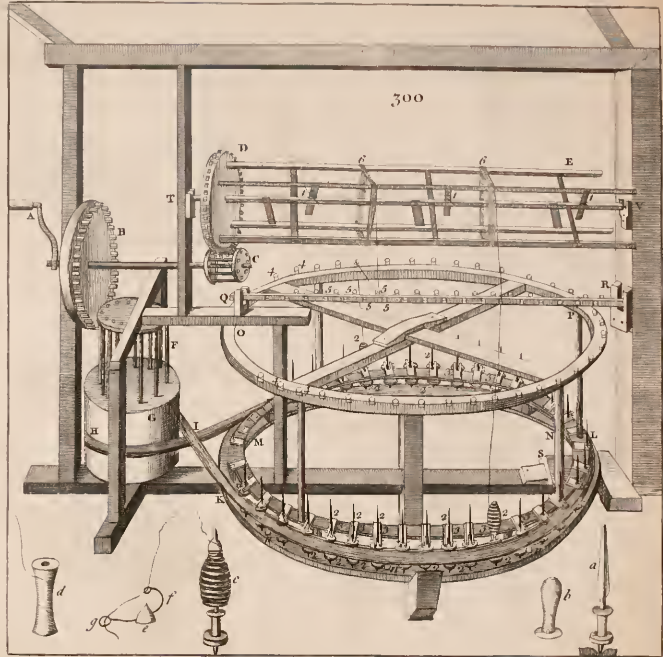

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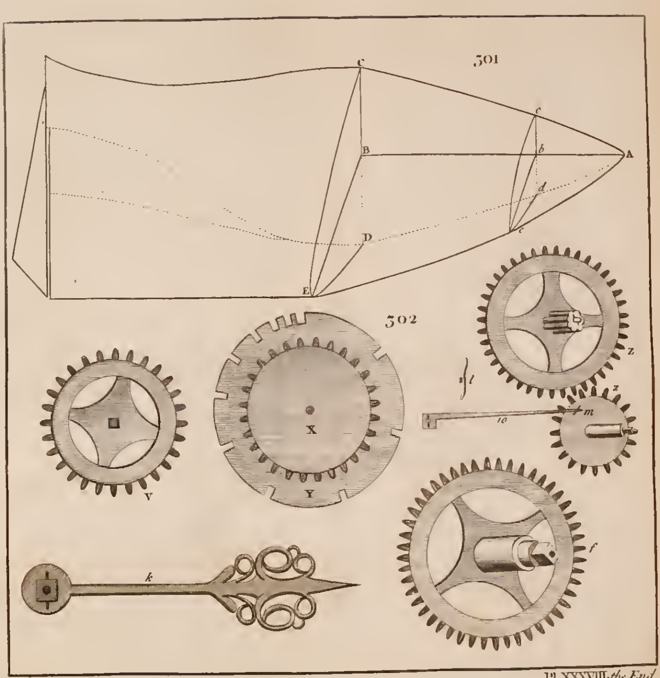

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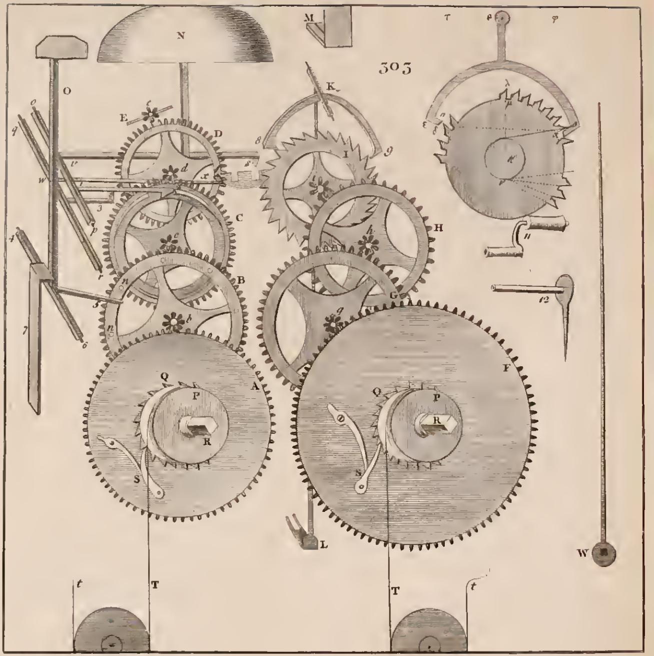

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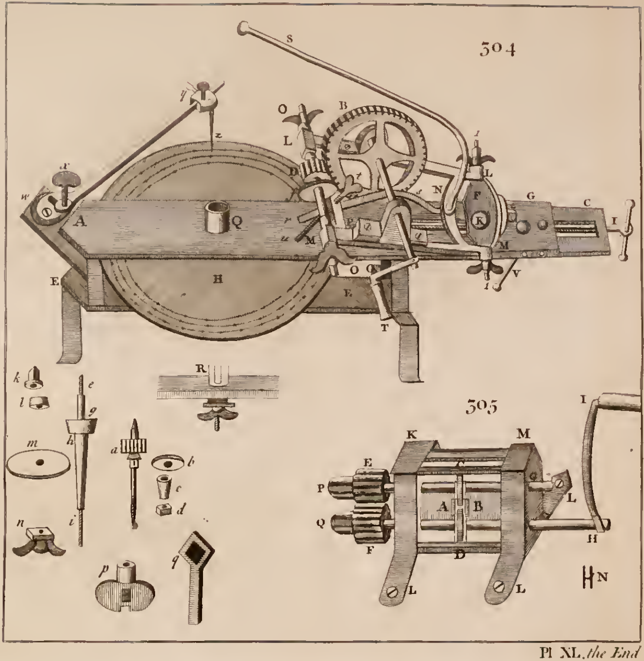
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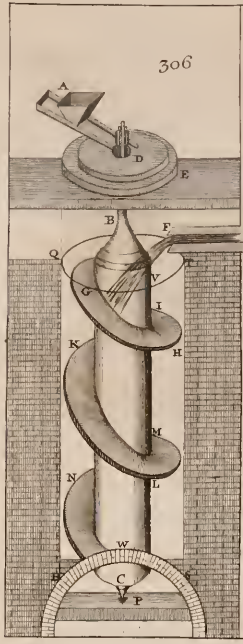
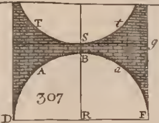
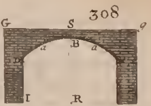
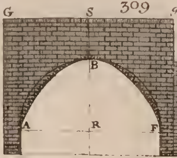
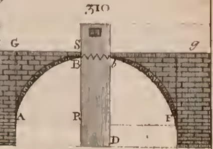
Pl XL. *the End*

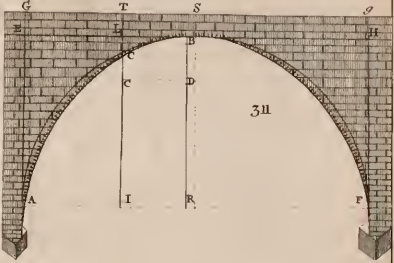




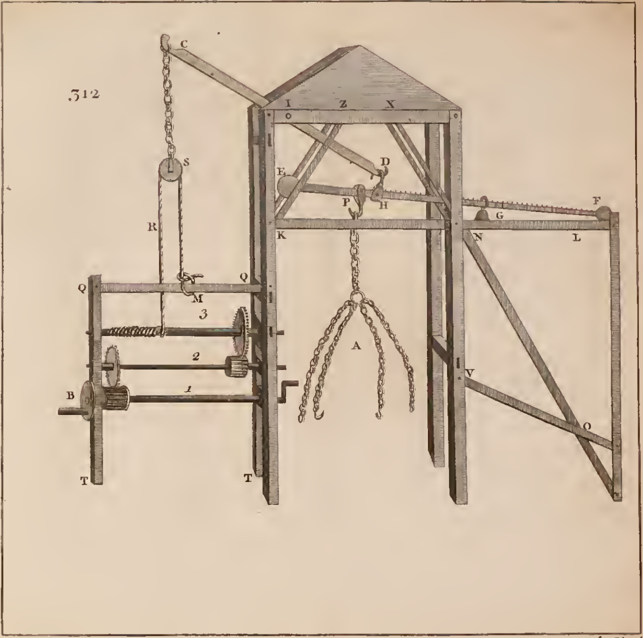








Pl XLI *the End*



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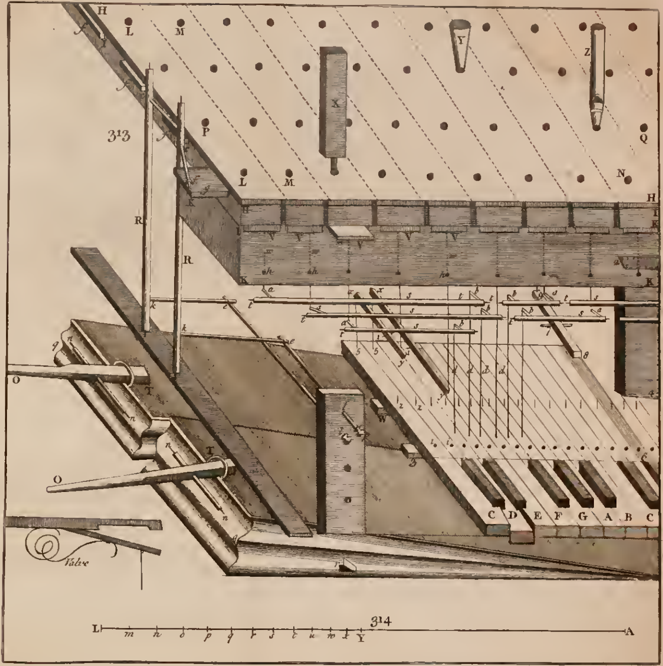

Pl. XLIII. *the End.*

ERRATA.

Page 2. line 3, *for* whence *read* where. Page 12. line 22. *dele* and in theſe directions. Page 23. line 14. *for* are the ſquares, *read* are as the ſquares. Page 87. line 2. *for* their weights, *read* any weights. Page 107, lines 19, 20. *dele* the power of all. Ibid, line 25. *for* $= \frac{1}{2}d$ *read* $= \frac{1}{2}r$.

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